

**An Integrated Water-Electricity Market Design for  
Multi Reservoir, Mixed Operation**

Indrajanaka Mahakalanda

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## **Abstract**

Water markets are often regarded as the most promising method of managing this increasingly important natural resource, but the literature on water market concepts is only emerging. Most of the focus is on physical trading arrangements, but financial property rights appear both conceptually and practically appealing, as a way to develop commercial and organizational arrangements to improve liquidity and ultimately increase efficient resource use. This thesis focuses on market arrangements to manage hydrology dependent surface water supplies, where consumptive and/or non-consumptive use occurs in a network with storage. Binding resource constraints create temporal and locational price differences. Moreover, the uncertainty about price differentials creates barriers to trade. Participant bids, reflecting their marginal use values, are assumed to be cleared by a benefit-maximising optimisation, such as Stochastic Linear Programming. This also creates price differences between locations, and time periods, and causes the market to accumulate a “settlement surplus” of rents associated with resource constraints. This thesis draws on the Financial Transmission Right (FTR) concepts developed for electricity markets to outline a general structure of financial hedging instruments that could be used to deploy this settlement surplus to hedge against price risks, across space and time. We also consider a swing option based approach, which bundles the above rights to create a virtual “slice of system” model that could be practically and conceptually appealing to both aggregated and disaggregated hydro reservoir systems. While only preliminary, our discussion of these options suggests that developments along these lines may be important in creating a water market environment that is acceptable to potential consumptive and non-consumptive participants.

The remainder of this thesis is about the problem of intra-period consumptive and non-consumptive water allocation in a mixed-use catchment. We develop a deterministic nodal Constructive Dual Dynamic Programming (CDDP) procedure which implicitly clears a market determining both consumptive and non-consumptive water allocations, across all nodes in a catchment with a single reservoir. Consumptive users extract water from the system, so each unit of water flow can only be used for a single consumptive use. A non-consumptive user transfers water from one node to another,

extracting some benefit, or incurring some cost. Arc flow bounds may limit the opportunities for using water at the nodes. Costs can be associated with arc flow bounds and distributary demands to represent in-stream and environmental reserve flows enforced using penalty costs. The algorithm constructs the intra-period demand curve for release by sequentially forming marginal water value curves at each node, passing these curves towards the reservoir.

This approach can generate net demand curves representing *all* possible market-clearing solutions at nodal and user levels. It can also be used to construct net demand curves for water release from the reservoir, in each period, which could then be used in a stochastic inter-temporal CDDP model to construct marginal water value curves stored in the reservoir over an appropriate time horizon. Several variants on this approach are explored.

We discuss extending the procedure to assess the marginal value of water stored in two inter-connected reservoirs in a mixed-use catchment. A “lower level” intra-period CDDP is applied to construct a two dimensional “demand surface” for transfer, representing the marginal benefit from net release into either end of the inter-reservoir chain between the two reservoirs. Then a higher level inter-period CDDP demand-curve-adding method could be deployed to strike the optimal trade-off between the current release demands for the inter-reservoir chain and other sub-trees leading from the two reservoirs and the future storage demands.

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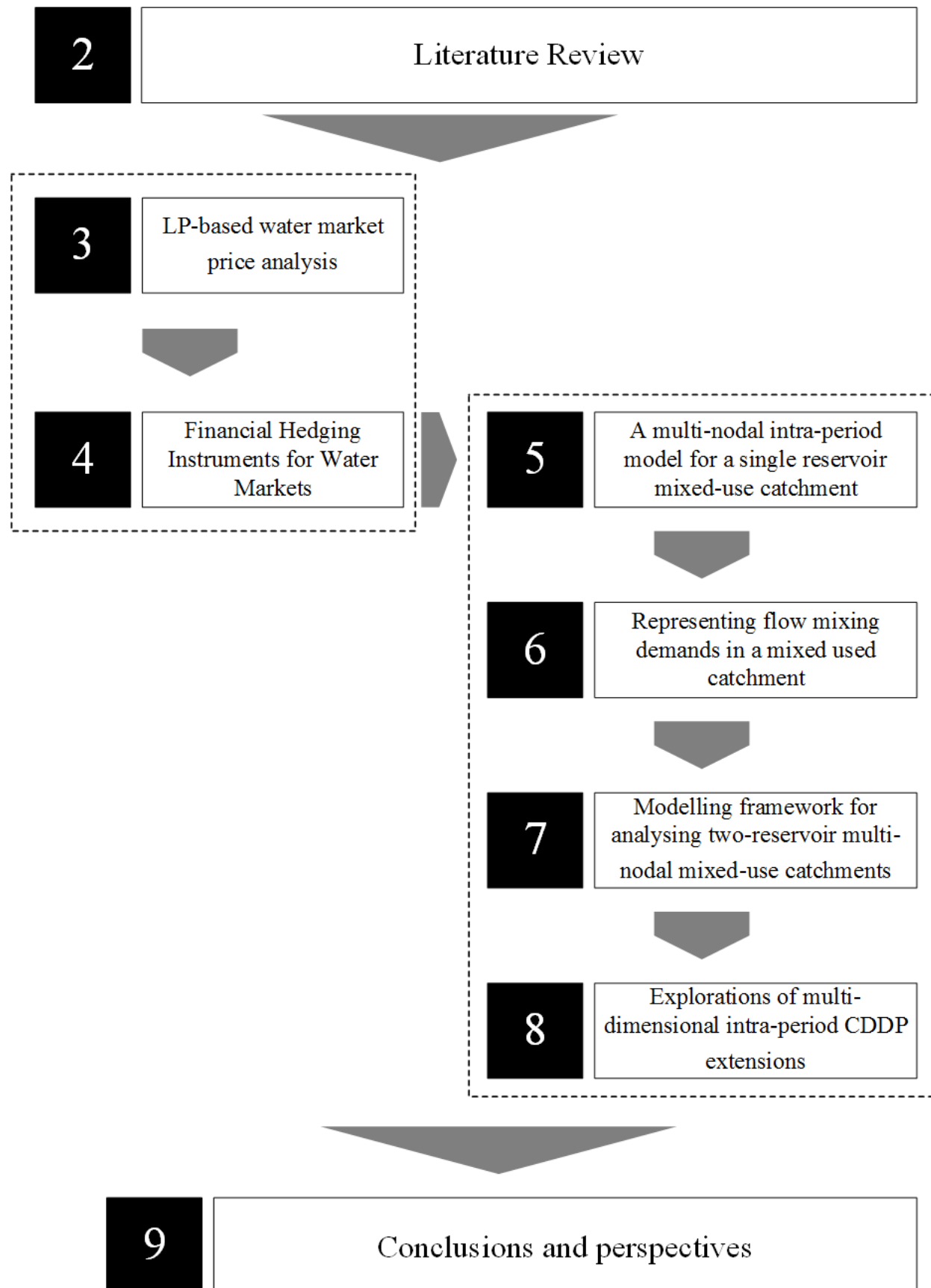
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## Research Road Map



## Notations, Acronyms and Abbreviations

<b>Symbol</b>	<b>Definition</b>
ACR	Arc Capacity Right
ACO	Arc Capacity Option
CDDP	Constructive dual dynamic programming
<i>cdr</i>	Conditional demand curve for release
<i>cdcs</i>	Conditional demand curve for storage
<i>dcr</i>	Demand curve for release
<i>dcs</i>	Demand curve for storage
<i>dcw</i>	Demand curve for water
<i>dct</i>	Demand curve for transfer
<i>dsr</i>	Demand surface for release
<i>dss</i>	Demand surface for storage
<i>dst</i>	Demand surface for transfer
DP	Dynamic programming
DDP	Dual dynamic programming
FTR	Financial transmission right
FDR	Financial Delivery Right
FGR	Flow gate right
FIR	Financial Inflow Right
FSR	Financial Storage Right
FSO	Financial Storage Option
LMP	Locational marginal price/pricing
LP	Linear programming
MC	Markov chain
NCB	Net nodal consumptive benefit function
NCF	Non-consumptive benefit function
NLP	Non-linear programming
NZ	New Zealand
RSR	Retrospective Storage Right
SDP	Stochastic dynamic programming

SDDP	Stochastic dual dynamic programming
SLP	Stochastic linear programming
SO/ISO	System operator/Independent System Operator
SP	Stochastic Programming



# 1.

## Introduction

### 1.1. Missing markets for power and water

The inspiration for this study pertains to the problem of efficiently allocating and valuing storable surface water among hydroelectric, environmental, urban and irrigation uses in time and space. Water supply and demand both vary based on location and time of year, and significant mismatches are common. As a result, reservoirs and water distribution systems are operated to distribute to different consumptive and/or non-consumptive uses in most parts of the world, often under some form of water sharing rules. Moreover, ideally, reservoirs are expected to fully meet inter-temporal consumptive and/or the non-consumptive demands. A consumptive user (beneficiary) “permanently” removes water from the supply system. This includes water for agricultural purposes, urban water treatment and supply plant, or re-hydrating wetlands.<sup>1</sup> A non-consumptive user (beneficiary) derives value from water flowing in the system without “consuming” it. Typically, this is water used for hydropower generation, but there are other (typically environmental or recreational) benefits to be derived from in-stream flow.

Water markets are often touted as the most promising way of managing this increasingly critical resource. However, the literature on water market concepts is only now emerging. Market initiatives have been reported in literature in various parts of the world (Bauer, 2004; Easter et al., 1998; Bjornlund & McKay, 2002; Bjornlund et al., 2007; Wheeler et al., 2010). Nevertheless, a standard

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<sup>1</sup> We will ignore the complication that some such users may return some proportion of the water “consumed”, perhaps indirectly and with some delay.

market design that involves participants bidding to buy and sell water stored in a common storage, now and in the future, with structured contractual mechanisms, has not yet fully emerged. Thus, the broad goal is to develop a set of market mechanisms, including spot and financial products, to support trading of storable surface water in such multi-use catchments, carefully re-interpreting traditional understandings, in this new situation.

This study takes, as its principal analogy, those wholesale electricity markets which are now actively traded based on constraint-based pricing theory. Generally, these electricity markets are cleared using deterministic optimisation models. Water storage may play a significant role in electricity systems with a significant hydro component. In most cases, though, the storage infrastructure is owned by a market participant, effectively outside of the market. Thus, it is common to find hydro-dominated electricity markets with separately managed hydro-reservoir systems under different ownership regimes. As a result, there is an offset between the profit-maximising and welfare-maximising solutions. However, well-structured contract mechanisms at the reservoir level to trade water could improve the operation of the electricity market to get closer to welfare-optimization outcomes.

In a water system though, water may move through both natural and artificial channels, and storage may be available in both natural/artificial reservoirs, and natural aquifers. Any or all of these may be “owned”, or at least managed by the agency responsible for determining water allocations. As a result, in a water market, the operation of water storage might, instead, be managed as a part of the market. Since the hydrology cycle of the catchment creates a stochastic environment in which the market designer faces the *intra-period* problem of how much water should be allocated within a period of time, and also the *inter-period* problem of how much should be stored from one period to the next. As a result, an inter-period water management model must account for storage and supply (inflow) uncertainty as it constantly evolves over time. While the physical nature of water (for example, fugitive and uneven flows and other externalities) may make it difficult to define tradable “commodities” participants are likely to demand at least indicative future water allocations to be produced by inter-temporal market clearing models. In that context, dealing with uncertainty becomes a major issue for market clearing. Moreover, there are delays within the surface water transportation network, over which

the market operates. This leads us to consider a number of pricing and hedging issues that may be disregarded in the case of electricity.

## **1.2. Research Objectives**

The primary goal of this thesis is to explore a situation in which market arrangements are used to manage hydrology-dependent surface water supplies, where multiple consumptive and/or non-consumptive uses occur in a tree network, with storage. This research is a theoretical study on conflicting consumptive and non-consumptive uses in a single catchment, where consumptive and non-consumptive participants are both assumed to express their willingness to buy water (or flow) through market bids. As such, it expands on the work of Starkey (2014), which represented the entire intra-period market using a simple constant elasticity demand curve. The present study largely ignores the inter-period optimisation issues addressed by Starkey and merely assumes one of the market designs he studied. The scope has also been limited in other ways. The issue of gaming incentives cannot ultimately be ignored because it significantly impacts on participant bidding strategies, and hence, real life outcome. But while it was initially intended to be a primary focus area, a detailed study has proved to be beyond the scope of this thesis. Thus, it is only briefly discussed in the literature survey of Chapter 2 and in relation to the hedging instruments of Chapter 4, which can be used to significantly ameliorate problems of that kind. Auction design, trading water under uncertainty, locational and temporal management of storages, infrastructure investment and asset ownership/management are also outside the scope of this study.

The current research contributes to the literature on this large and complex topic by studying some aspects of the market situations which have not yet been explored by previous authors. In particular, the present research focuses on intra-period market arrangements to manage surface water supplies, where consumptive and/or non-consumptive use occurs in a network, and also studies, aspects of risk management, by developing a general structure of financial instruments to hedge against price risks across space and time.

### 1.3. Manuscript Organisation and Thesis Contributions

This manuscript is organised as follows.

The first section of Chapter 2 discusses existing water issues such as scarcity, valuation, and pricing, describing water as a commoditised natural resource. The second section looks at existing markets for natural resource allocation, followed by a survey of electricity and natural gas markets. The third section describes the background of reservoir optimisation models. The chapter then concludes by highlighting gaps in the existing research.

Chapter 3 describes market arrangements to manage hydrology-dependent surface water supplies where consumptive and/or non-consumptive use occurs in a single catchment network, represented as a tree, with storage. Participant bids, reflecting their marginal use values, are assumed to be cleared by a benefit-maximising optimisation.

Section 3.3 formulates a detailed deterministic LP model for that market-clearing problem. This section adds to the literature (for example Starkey (2014)) in the following ways:

- A trading mechanism is developed for a mixed-use catchment, based on constraint based (primal/dual) pricing theory, using deterministic Linear Programming (LP) optimisation to clear markets, while ignoring (long-term) inter-temporal linkages. This model includes representation of physical limitations such as arc capacity limits (for example, minimum in-stream flow levels and spill bounds) in a networked water system, as well as demand-side (private) restrictions (such as maximum consumptive and non-consumptive bid levels).
- A Shapley value-based method is described to apportion infra-marginal benefits between multiple non-consumptive users on the same arc in the water market context, so as to minimise incentives for some users to take advantage of their physical positioning in the network relative to users who are prepared to pay more to increase flows.

Section 3.5 then goes on to briefly describe a stochastic market model in which bids for future water use determine the demand for stored water for the deterministic spot market. Section 3.5's contribution



is to set the stage for the following chapter, and to extend Starkey's formulations by including constraints that describe surface water transportation, storage, and operational limitations.

Chapter 4 examines the formulation and interpretation of financial rights and obligations in a market context. In particular, it describes and explains their possible role in establishing the framework of contracts required to manage inter-temporal storage marginal price differences and the interplay between consumptive and non-consumptive values. The benefit-maximising LP/SLP described in Chapter 3 creates price differences between locations and time periods, and causes the market to accumulate a "settlement surplus" of rents associated with resource constraints. Uncertainty in the rents associated with resource constraints (for example, channel capacities, storage bounds) may make some participants unwilling to engage in trading unless insurance is available. As with Financial Transmission Rights (FTRs) in electricity markets, financial "hedge" products, defined with respect to spot water market prices, would provide users with the required insurance, and could be traded in secondary markets, or in an integrated market.

Chapter 4's contributions are to develop and discuss the support and practical use of:

- Financial Water Rights (FWRs) as a generic form of market trading instrument, for a fixed volume of water.
- Financial Inflow Rights (FIRs) giving rights to a proportion of future inflows, but leaving the holder facing volume uncertainty.
- Financial Storage Options (FSOs), and obligation-inclusive rights (FSRs), to hedge against inter- temporal price risks associated with the storage system constraint rents.
- Arc Capacity Rights (ACRs) and node-to-node Arc Capacity Options (ACOs) to hedge against transportation congestion rent risks on arcs with, and without, non-consumptive costs/benefits, or leakage losses.
- Node-to-node path rights formed by combining obligation-inclusive ACRs.

Finally, this chapter extends the work of Barrosso et al., (2012) by describing an alternative "virtual reservoir" approach to creating a water market, in situations where common resources must be shared

between several parties who want to act independently of each other, rather than accept solutions produced by a central market-clearing optimisation model.

In Chapter 5 we shift our focus to algorithm development. The chapter commences with a description of the relationship between intra-period and inter-period reservoir optimisation problems highlighted in Chapter 3. Then it discusses how to optimise intra-period consumptive and non-consumptive water use in any river catchment that can be represented as a tree radiating from a single reservoir. This chapter then develops an intra-period pre-computation method to efficiently clear the intra-period water market and describes how to construct an intra-period “demand curve for release” that can be used to construct beginning-of-period marginal value curves for water stored in the reservoir over an appropriate time horizon, using Starkey’s (Starkey, 2014) inter-period stochastic Constructive Dual Dynamic Programming (CDDP) model. This chapter adds to the CDDP literature in the following ways:

- The intra-period CDDP procedure is developed to clear a networked water market under deterministic setting. Unlike the previous CDDP versions, this CDDP pre-computation procedure processes over space rather than time.
- A net demand curve for water release from the reservoir in each period is constructed in a recursive manner and the CDDP explicitly includes both consumptive and non-consumptive demands. The intra-period CDDP applies over a general tree network and not just an ordered sequence of stages.
- The intra-period CDDP implementation is described in terms of the general operations of vertical and horizontal curve addition allowing a range of demand curves structures.

Chapter 6 then shows how the multi-nodal CDDP market model of Chapter 5 can be extended to cover situations where flow must be split between parallel arcs, e.g., to meet cooling and in-stream temperature control mixing flow requirements. Chapter 6’s contributions are:

- To adapt the intra-period multi-node CDDP algorithm by enforcing node/arc level manipulations to address uses with water-mixing requirements.
- To develop an offer-based flow splitting method for environmental flows in a water market context.

Chapter 7 further expands the intra-period CDDP algorithm described in Chapter 5, developing a framework for creating the demand surface for release from two inter-connected reservoirs in a tree network. The intra-period problem is decomposed into a set of sub-problems, corresponding to market clearing on a spot basis.

The demand surfaces corresponding to the sub-problems for each reservoir are shown to be only one-dimensional, and easily produced by the one-dimensional CDDP algorithm of Chapter 5. But, the demand surface for “transfer” between the two reservoirs is two-dimensional, and could be constructed in several different ways. This chapter develops a general structure of a two-reservoir intra-period CDDP procedure and then shows how to implement the procedure.

Chapter 8 then explores some further intra-period CDDP model extensions. At the outset, the issues of modelling transportation and release delays in a water market setting using an LP formulation are explained. The chapter discusses a time-of-use CDDP procedure to model the release delays, and shows that this implies a requirement to construct multi-dimensional nodal marginal value surfaces describing the incremental sub-period flow arrivals under the intermediate storage capacity constraints. Chapter 8’s contributions are:

- To develop a time-of-use intra-period model that includes multiple cyclic (non-sequential) sub-periods.
- To explain how to include short-term storage, and its application in CDDP.

Chapter 9 then provides a summary of the present research, conclusions, areas for further research, and avenues for model extensions.

## 2.

### Literature Review

#### 2.1. Introduction

The previous chapter introduced the premise of this research and outlined the research goals. It is the purpose of this chapter to review some literature relevant to our research themes. This chapter is laid out as follows:

- Section 2.2 focuses on consumptive and non-consumptive use of water in a catchment.
- Section 2.3 provides a catalogue of research articles on the classical reservoir benefit maximization problem of allocating water among various uses in a river catchment.
- Section 2.4 discusses literature on hydro reservoir planning and optimization models. In particular, we focus on stochastic reservoir optimisation models used to solve the hydroelectric scheduling problem, with a focus on the kind of techniques used in this thesis.
- Section 2.5 reviews smart market models for electricity, and gas, and catalogues articles about contracting and hedging, and agent behaviour in electricity markets.
- Section 2.6 reviews the limited literature on model based natural gas markets.
- Section 2.7, reviews literature on some existing water market arrangements. Then we shift our focus to model-based water market proposals.
- Finally Section 2.8, discusses lessons learnt from electricity markets and water market proposals, and highlights open research issues and opportunities.

## **2.2. Consumptive and Non-consumptive Use of Water**

A consumptive or “off-stream” user (e.g., agriculturalist, urban water supply plant) “permanently” removes water from the natural supply system. A non-consumptive user extracts value from the water, but returns water<sup>2</sup> into the system without “consuming” it (Young & Loomis, 2014). These include hydropower generators, but also recreational and environmental (e.g., wildlife) minimum flow type water requirements.

In some cases, non-consumptive users are competing with consumptive users. In others, the two are complementary, as non-consumptive use extracts value from the water as it flows down the river chain to be consumed eventually by the consumptive users such as urban consumers and agriculturalists, for example producing “*the joint product of irrigation water and hydropower*” (Howitt et al., 1998).

This problem of allocating water among consumptive and non-consumptive users becomes even more complex in the presence of the hydrological uncertainty of inflows, and various socio-economic complexities (Berger et al., 2007).

## **2.3. Combined Benefit Maximisation Problem**

This section briefly discusses literature on integrated basin-wide modelling. Several authors have considered the problem of multi-objective optimization of non-consumptive use. For example, Shane & Gilbert (1982) outline a set of operating objectives such as flood control, transportation, urban/agricultural water supply, power generation and recreational use for the Tennessee Valley Basin. This literature is relevant to some later sections, which are chiefly concerned with allocating water for non-consumptive hydro-power production. But the bulk of this thesis is concerned with maximizing the combined social benefits of consumptive and non-consumptive use. So this section reviews several reservoir optimization models that seek to provide non-consumptive versus consumptive trade-offs (in particular energy versus irrigation trade-offs). Ideally, a convex social welfare maximization problem equalises the marginal benefit from the use of water resource for all uses. If the marginal benefits are

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<sup>2</sup> In general, this thesis assumes that a non-consumptive user returns all water ‘used’ to the water system and all water is returned to the same location. However, later in the thesis we relax the idea of water used by a consumptive user all being lost by investigating models that include partial or delayed return flows.

not equalized, the resource may be redirected to the highest value unmet use in order to improve benefit maximization solutions (Dinar et al., 1992). The issue of welfare gains from reallocation of water using various mechanisms has been extensively studied by a number of researches (e.g., see Rosegrant & Binswanger (1994), Becker (1995), Hearne & Easter (1995) Miller (1996) and, Garrido (1998).

In general, the optimization studies in this area mainly highlight potential welfare gains from diverting water from low valued use to high valued use during low inflow periods. For example, Oven-Thomson et al. (1982) studied agriculture versus hydropower trade-offs at the High Aswan Dam in Egypt. Allocation of water between hydropower generators and agriculturalists in the Snake Colombia River system has been studied by McCarl & Parandvash (1988) and Hamilton et al. (1989).

Howitt et al. (1998) model intra-year water release in a long-term planning model to determine a suitable trade-off between irrigation and hydropower production. They reported that provisioning water for agriculture in the spring can lower the hydropower benefits because of the loss of reservoir head for subsequent releasing. More recently, Tilmant et al. (2009) applied a stochastic programming (SP) method to assess agricultural-to-hydropower water transfers in a multipurpose system of reservoirs in the Euphrates river basin. Tilmant et al. propose dynamically adjusted water allocation across time and space so some catchment users could be temporarily re-allocated with downstream water. According to their findings, dynamic allocations can increase the economic efficiency if downstream uses have sufficiently high values. Goor et al. (2010) investigate provisioning of water from energy production to irrigation use under hydrological inflow uncertainty in a long-term reservoir planning problem. Their model seeks to maximize the net benefits from non-consumptive, consumptive use and penalties for not meeting a list of other constraints. In addition, river basin level hydrologic-economic models could be used to analyze issues such as infrastructure expansion, competitive water [re]allocation, and the design of institutional policies, and/or used as a basis for implementation (Harou et al., 2009; Goor et al., 2010).

#### **2.4. Hydro Reservoir Planning and Optimization Models**

The water resources literature to a great extent focuses on long term reservoir planning studies. Although much more could be added here, we have chosen to review the literature on optimising hydro

reservoir operations, because the long term reservoir planning problem (e.g., planning horizon of five years) is outside the scope of this study. The most studied reservoir optimisation problem, by far, has been the maximisation of benefits from hydroelectricity production over time, assuming known electricity demands for each period.

In this section, we trace the evolution of optimisation models, for hydro-reservoir scheduling. The majority of such models have been used to perform centralised optimisation of system. But, in an area so filled with marginally varied techniques, the main objective of this section is to briefly review the most relevant water resource economic and optimisation literature as a modelling tool to set up a water market. In particular, the focus is on stochastic dynamic programming and stochastic programming based techniques, with a focus on duality.

Hydroelectric optimization models have different optimisation objectives, long and short term planning horizons, different system sizes and configurations, and different treatments of uncertainty. Loucks et al. (1981) have reviewed general reservoir management models and techniques. Yeh (1985) and Wurbs (1993) review optimization and simulation modelling approaches. In Wurbs (1993), reservoir optimization models have been evaluated on the basis of their usefulness in various decision-making contexts, so as to help reservoir managers select the appropriate technique. More recently, Labadie (2004), presented a comprehensive survey of reservoir optimization approaches.

Generally, these optimization models determine optimal periodic releases to minimise expected operating costs. In simple terms, the reservoir operator has to decide the optimal trade-off between releasing now and saving for later release. These decisions are made in a stochastic environment with spatially and temporally correlated tributary flows and capacity-constrained reservoirs. A convex maximization problem equalizes the marginal benefit from all water uses. This means that the manager of a single hydro-reservoir should release water if, and only if, the marginal value of releasing it is at least equal to the expected marginal value of storing it. More generally, the marginal value of releasing should generally equal the expected difference between upstream and downstream marginal water values.

A network reservoir optimization model typically allocates water across the distribution network satisfying buyer-seller water delivery relationships. Again, Labadie (2004) provides, a useful and

succinct network representation, and emphasises the importance of explicit stochastic modelling of hydrology uncertainty for better estimation of the system performance. According to Cheng et al., (2009), these network structures represent distribution systems implicitly (Israel and Lund, 1999; Hollinshead & Lund, 2006; Lund et al., 2008) or explicitly (e.g., Labadie, 2004; Wang et al., 2007; Barros et al., 2008; Hsu et al., 2008). Seyam (2002) used a simulation-based nodal “value-flow concept” and this was used to drive marginal water values. In Harou et al., (2009), a node represents confluences, reservoirs, abstraction points etc. Arcs denote inflows to the system, canals, and river reaches.

#### **2.4.1. Stochastic Dynamic Programming**

##### **2.4.1.1. History and Focus**

Dynamic Programming (DP) has been widely applied to derive time-sequential optimal decisions. The DP, based on Bellman’s optimality equation (Bellman, 1957), breaks down a multi-period decision problem into a finite set of “stage” sub-problems, each having only a few decisions. A state variable fully describes the current state of the system at a stage without any knowledge of a system’s past behaviour. An optimal policy implies a set of future cost functions for a number of state variables at each stage or sub-problem. The system state can change as a result of each decision made. As described by Stedinger et al. (1984) and recently by Nandalal and Borárdi (2007), Massé & Boutteville (1946), Little (1955), and Hall & Buras (1961) applied DP in water resource systems studies in the early period. Harboe et al. (1970) derive an optimal single reservoir operating policy using deterministic DP to serve multiple purposes: power generation, urban water use, and flood control. Young (1967) generated synthetic inflows using Monte Carlo simulation and implicitly used a deterministic DP optimisation with regression methods to construct an optimal reservoir operation policy. Later, DP and DP-like methods were extensively used for water resource systems analysis because they could handle non-linear objective functions and constraints (see Reznicek & Cheng, 1991). Labadie’s (2004) overview and studies by Yakowitz (1992), and recently Nandalal and Bogárdi (2007), on SDP based hydro reservoir optimisation techniques are referred to in the following discussion. Unlike deterministic DP, the cost/benefit function in an SDP is uncertain. Yet, an SDP based release policy can be useful as a guide for decision making under uncertainty.



Note that this thesis is not about the theory of SDP, and nor does it develop or use classical SDP models<sup>3</sup>. So we are really only interested in three specific aspects of the large SDP literature, namely:

- Stochastic CDDP reservoir optimisation models of the type developed by Starkey et al. in as much as they form a context in which our own models may be applied,
- The CDDP concepts themselves, which are here re-interpreted and applied to form optimal deterministic intra-period release policies for a catchment with no-cyclic topology, and
- The insights provided by Stochastic CDDP about the relationships between current and expected future marginal water values, and prices.

#### 2.4.1.2. Classical SDP Formulation

This section formulates the classical SDP formulation. We will refer to this formulation in later chapters when we discuss the intra-period optimization problem. Typically, the reservoir optimisation formulations assume Markov process-dependent serial correlation of inflow probabilities (e.g., see (Loucks et al., 1981)). For example, a temporal link between the current period inflows and the previous period inflows can be formed using a lag-one Markov model.

Let  $f_{h,t}$  denote the stochastic inflow (a random variable at each stage) which is described using a stochastic process. Here,  $h$  and  $t$  represent hydrology state and time indices. Now the optimal release might depend on the Markov state as well.  $r_{t,h}$  is the release volume in period  $t$  corresponding to hydrology state  $h$ .

Generally, SDP reservoir optimisation models use a first order Markov chain:  $\text{Prob}(f_{h',t+1}|f_{h,t}, f_{h,t-1}, \dots, f_{h,0}) = \text{Prob}(f_{h',t+1}|f_{h,t})$ . This means that the inter-temporal information is carried forward to the next stage through an extra dimension. Inflow to the reservoir in any time period is related to the preceding inflow. The SDP breaks inflows and the storage into a number of discrete states. Let  $h$  and  $h'$  be the inflow states in periods  $t$  and  $t + 1$ . Given the inflow state  $h$  in period  $t$ , the probability of inflow state  $h'$  in the next period  $t + 1$  is written as:  $\rho_{hh'}$ . This is known as

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<sup>3</sup> Instead, the models used in this thesis are first formulated as LP models and then effectively apply CDDP to optimise the LP models.

the Markov state transition probability. Thus, 1-lag Markov chain based beginning-of-period SDP objective function  $V^{t,h'}$  for current inflow conditioned by the previous period inflow is:

$$\mathbb{E}_{f_{t+1}|f_t} \left[ \max_{r_{t,h}} \left\{ NB^{t,h} \left( s_t, r_{t,h}(f_{t,h}) \right) + V^{t+1,h} \left( W^t(s_t, r_{t,h}, f_{t,h}) \right) \right\} \right] \quad \forall t \quad [2-1]$$

Subject to:

Flow balance at the storage,

$$s_t = s_{t-1} + f_{t,h} - r_{t,h} \quad \forall t, h \quad [2-2]$$

Here, inflows in the current period could be assumed to be known at the beginning of the period. This could be either stored or released to meet demand within that period. This is known as the “observation-first” type of release policy or “informed” inflow allocating policy (Starkey, 2014). So the planner looks forward from the beginning of period storage level. In a stochastic setting, there will be multiple inflow realizations for a period. Then each of the above realizations leads to a specific release decision. Unlike the deterministic case, there will be different end-of-stage storage levels, one for each inflow realization. In Equation [2.1], the release decision is unique for each hydrology state. Alternatively, a single release decision  $r_t$  in the continuity equation can apply for all hydrology states as follows:

$$\max_{r_{t,h'}} \left( \mathbb{E}_{f_{t+1}|f_t} \left[ NB^{t,h} \left( s_t, r_{t,h'}(f_{t,h}) \right) + V^{t+1,h} \left( W^t(s_t, r_{t,h'}, f_{t,h}) \right) \right] \right) \quad \forall t \quad [2-3]$$

This can be thought of as a “look ahead” or decision-first type of release policy which is similar to the conservative release policy (e.g. Read, 1982; Archibald, 1995; Starkey et al., 2012).

An end-of-horizon Marginal Water Value (MWV) function is used in the above problem. By doing this, it is ensured that enough water is carried over into the subsequent ‘year’ as a value is assigned to water stored beyond the end-of-horizon (e.g., end of the year). In general the marginal water value reflects how much water to be released and/or stored in each period and the expected marginal water value depends on time, reservoir level, and inflow. This observation forms the basis for the CDDP technique originally developed in 1983 for the PRISM model described by Read (1989).

#### **2.4.2. Constructive Dual Dynamic Programming**

Read (1976) considered the first derivative of the future value function of the backward recursion SDP to construct the marginal water value function. Read (1979) discussed applying mixed LP/DP dual methods to produce marginal water value functions given that benefit and value functions are continuous and differentiable. Later, the works of Boshier et al., (1983) and Read & Boshier (1989) on a marginalistic iterative variant of SDP lead to the development of the constructive dual dynamic programming technique. Read & Hindsberger (2010) summarize the CDDP literature since then, which mainly focusses on the development of (expected) marginal water value functions under uncertainty. However, our focus in Chapter 5 is on the construction of an intra-period marginal benefit function of the CDDP. That is, determining the marginal value of release for a single period.

Constructive dual dynamic programming (CDDP) solves the dual of the standard DP reservoir optimisation problem. It implicitly optimises an operating policy similar to SDP for the planning horizon, but focuses on determining the primal storage state levels corresponding to a pre-defined grid of dual “state space” variables, typically marginal water value levels (and inflow states). With no bounds, the marginal value of water at the beginning of period is equal to the marginal value of releasing water during that period, which equals the marginal value of storing water at the end of period. Using this, a backward recursive CDDP procedure constructs the marginal value for water function for the current period using the end of period marginal water value curves. As a result, there is no need to search for optimal solutions to each sub-problem as in standard DP. The marginal water value for release and marginal water value for storage can be thought as the primal variables in the dual DP problem. Read (1989) describes an early implementation of dual DP in a two reservoir optimisation model (RESOP), embedded in the PRISM planning model of Read et al. (1987). Read shows that, provided the intra-period benefit function is convex and differentiable over a continuous state space, the SDP transition function can be re-cast as a dual problem to determine the beginning-of-the-period value function.

The original PRISM model relied heavily on running an intra-period model, independently of the DP optimisation, to pre-compute optimal releases for each critical combination of marginal water values.

Read & George (1990) show that the DP formulation could be “*implicitly defined as a whole set of LP problems starting from each possible initial storage level in each period*”. The beginning-of-period marginal water value curve is formed by augmenting the end-of-period marginal water value curve and marginal value curve for releasing water in the current period. Read et al. (1994) formed expected beginning-of-period marginal values for storage by considering the step-wise change in the end-of-period marginal value curve as a result of discrete inflow probabilities. Yang and Read (1999) further developed the above CDDP approach by accounting for inflow correlation. Casseboom & Read (1987) applied similar method to optimize the coal stockpiling problem.

According to Read & Hindsberger (2010), initial CDDP models constructed monotone piece-wise linear derivatives of intra-period and future value functions. These were denoted as the marginal value of releasing water, and the marginal value of storing water, as functions of release and storage respectively. This type of CDDP can be very efficient, provided the structure of the intra-period optimisation produces relatively wide “flats”. That is storage /release ranges over which the marginal value of releasing water is constant. This is typically true where water is being used to generate power in competition with a limited set of thermal stations.

This kind of CDDP produces an optimal release policy as a set of time dependent guidelines for maintaining storage levels. The marginal value of water in storage equals the marginal thermal fuel costs. Baninister & Kaye (1991) performed a very similar deterministic single reservoir optimisation using a similar technique. Travers & Kaye (1998) generalised the theory, so as to apply to more general multi-dimensional economic dispatch problems, and introduced the term “constructive” DP to distinguish this approach from the popular “Stochastic Dual DP” method of Pereira & Pinto (1991) described in the following section.

Later models typically dispensed with the piece-wise linear assumption, though, and used a process referred to as the “demand curve adding” approach in (Read & Hindsberger, 2010). Scott & Read (1996) introduced this approach in a context where marginal water value curves were no longer piece-wise linear, because participants were assumed to be “playing games” in an electricity market. So they employed a Cournot gaming model to pre-compute intra-period marginal benefit functions, and then

used the inverse function of the end-of-period marginal water value curve to incorporate the hydrological uncertainty to compute expected end-of-period marginal water value curves. In general, CDDP can employ any type of intra-period sub-model that produces a monotone marginal benefit function for the current period (Scott & Read, 1996).

Chapter 5 of this thesis contributes to the CDDP literature in two ways. First, it develops a methodology for the intra-period problem. This pre-computation procedure constructs an integrated “demand curve for release” for a set of consumptive and non-consumptive users interacting in a single catchment. Using this demand curve for release in Starkey’s inter-period stochastic CDDP formulation, the inter-period market clearing model can assess market benefits, and expected market prices. Second, it produces this demand curve for release by developing a deterministic nodal CDDP intra-period model, working towards the storage reservoir(s) along river chains, rather than backwards through time. Like all DP models, though, it does suffer from the “curse of dimensionality” if multiple reservoirs need to be considered.

### **2.4.3. Stochastic Linear Programming**

#### **2.4.3.1. History and Focus**

Stochastic Linear Programming (SLP) was originally developed by Dantzig (1955), and is discussed in detail by Kall & Wallace (1994). Jacobs et al. (1995) developed an SLP based power generation scheduling system for PG&E. Watkins et al. (2000) developed a multi-stage SLP reservoir optimization model for the Highland Lakes, Lower Colorado River Authority. Yeh (1985) and later Wallace & Fleten (2003) have reviewed several applications in the field of reservoir management. The water resource literature mainly refers to the SP models with recourse.

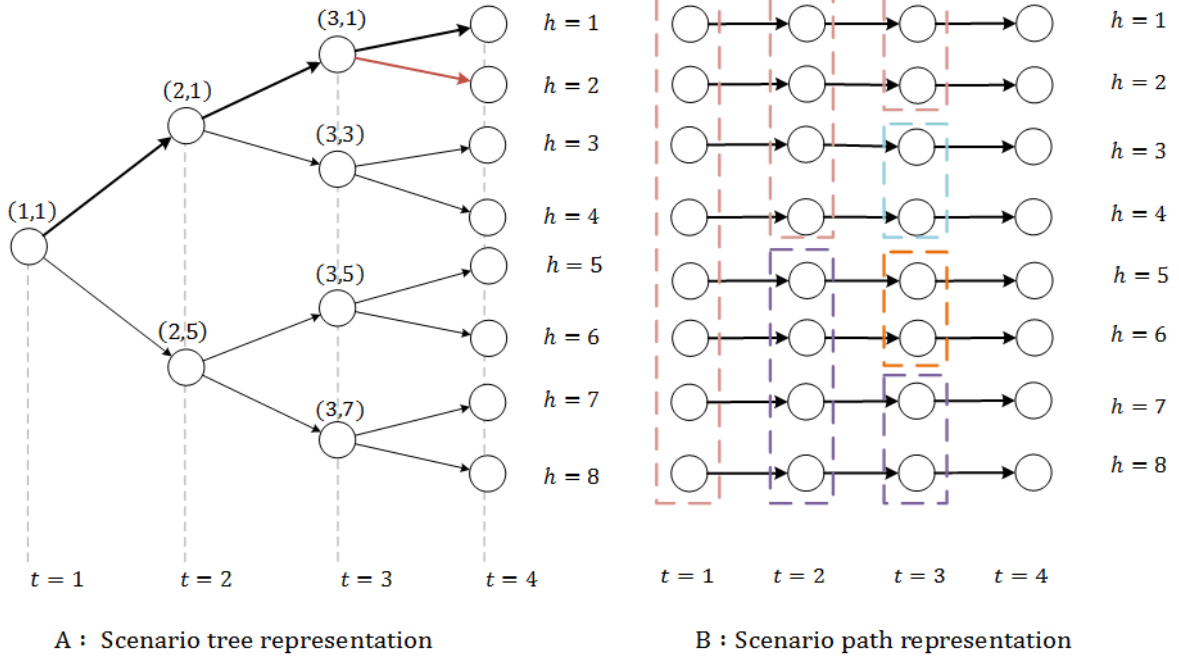
This thesis is not about the theory of SLP, and nor does it develop a computational SLP model. So we have no intention of discussing SLP computational routines. But SLP concepts are relevant inasmuch as Chapter 3 formulates a basic multi-nodal reservoir benefit maximization model as an SLP, in order to exploit its dual (pricing) structure in defining and analysing inter-nodal and inter-temporal hedging options for water markets.

#### **2.4.3.2. SLP Representations**

The central feature of an SLP model is a “scenario tree”, starting from a single point and showing how the uncertainty evolves over time. Unlike SDP, SLP does not suffer from the “curse of dimensionality” because it only constructs solutions for this scenario tree, which effectively represents a limited sample of possible future flow/storage/release states, rather than constructing solutions for every point in the multi-dimensional flow/storage/release state space potentially available. See Høyland & Wallace (2001) for a full discussion about constructing multi-stage scenario trees for SLP modelling.

For a reservoir optimization problem, starting storage and inflows in the first period are assumed to be known, thus decisions can be made with certainty. Future period decisions can be conditional on observed random inflows. In a two-stage decision problem, the first stage performs the action, the outcome of uncertainty is revealed then the second stage suggests the corrective action. A multi-stage problem alternates between performing actions that adjust for past revelations and having more uncertainty revealed. The SLP objective function maximizes the first stage net benefits, plus the total expected net benefits associated with future decisions subject to stochastic future inflows, as described by Kall & Wallace (1995).

Sen & Hingle (1999) provide a simple tutorial, showing how to formulate a problem using both “event tree” and “path based” representations. See also Labadie (2004). This discussion focuses first on the former representation as we follow the event tree formulation approach to formulate the stochastic water market-clearing model in the next chapter.



**Figure 2-1:** Scenario tree and scenario path representations

Following Gassmann & Ireland (1995), primal variables are indexed by event node,  $(t, h)$ , where each node in period  $t$  has a unique hydrology index,  $h^4$ . The set of event nodes immediately subsequent to event node  $(t, h)$  for  $t = 1, 2, \dots, T-1$  are the set of child nodes  $\mathfrak{c}(t, h)$  at level  $t+1$ . Each node  $(t, h)$  at level  $t = 2, \dots, T$  is linked to a single ancestor  $\mathfrak{a}(t, h)$  node at level  $t-1$  (see Diagram A in Figure 2-1)<sup>5</sup>. The scenario tree divides into branches corresponding to different realizations of the random inflow events. In this formulation the historical path from root to node  $(t, h)$  in stage  $t$  in the event tree can be denoted by  $(h_1, h_2, \dots, h_{t-1})$ , omitting the period indices.  $\mathbb{p}^{(t,h)}$  denotes the probability of reaching event node  $(t, h)$ . The nodal probability is the product of corresponding conditional probabilities along the historical path.  $\mathcal{H}_t$  denotes the set of all event nodes in time period  $t$ ,  $\forall t = 1, 2, 3, \dots, T$ . If we let  $NB^{t,h}(r)$  be the benefit derived from the release made at event node  $(t, h)$  and  $V^{T,h}$  end-of-horizon value function, then the stochastic reservoir optimization objective function can be written as:

<sup>4</sup> Note that the indices are not the same as in the SDP above. Later in the tree there could be different nodes with the same inflow (and so the same hydrology state) but a different  $h$ .

<sup>5</sup>  $\mathfrak{a}(t, h)$ ,  $\mathfrak{c}(t, h)$  notation are required as every combination of  $h$  and  $t$  do not appear in the event tree.  $\mathfrak{a}(3, 3) = (2, 1)$  and  $\mathfrak{c}(3, 3) = \{(4, 3), (4, 4)\}$ .

$$\max_{r,s} \sum_{(h,t) \in \mathcal{H}_t} \mathbb{P}^{(h,t)} \left( \sum_t NB^{t,h}(r_{t,h}) + V^{T,h}(s_{T,h}) \right) \quad \forall t \quad [2-4]$$

And, the storage balance constraint as:

$$r_{t,h} - s_0^{a(t,h)} + s_0^{h,t} = f_0^{t,h}, \forall (t,h) \quad [2-5]$$

Here the information process within the decisions making sequence is explicitly modelled by the scenario tree representation. Both the scenario tree information structure and the data difference contribute to the optimal solution.

The path-based formulation instead starts by defining “paths”. For example, in Figure 2-1, path  $\{(1,1), (2,1), (3,1), (4,1)\}$  and path  $\{(1,1), (2,1), (3,1), (4,2)\}$  represent scenario 1 and scenario 2 respectively. Note that these paths need to go to the last stage. The two scenarios share a common scenario path and also the same information until  $t = 3$  giving identical decisions for the two scenarios through to period  $t = 3$ . Many scenarios share a common sequence of outcomes and decisions for some periods, in this way. In order to ensure the decisions are the same for these shared outcomes, the formulation requires a set of the “non-anticipativity” (implementability) requirements. These can be stated in various forms, see, for example, Dempster (1988), Rockafellar & Wets (1991) and Mulvey & Ruszczyński (1995). This “*non-anticipativity constraint ensures that decisions honor the information structure of the problem*” (Higle, 2005). One form of these requirements is as follows:

$$s_{t,h} - s_{t,h'} = 0 \quad \forall h \in \mathcal{H}_{t,h'}, \quad t = 1, 2, \dots, T, \quad h = 1, 2, \dots, H \quad [2-6]$$

$\mathcal{H}_{t,h}$  denotes the set of scenarios that pass through node  $(t,h)$ . The former formulation is used in Section 4.3.1 to model the inter-temporal FSRs. Section 4.3.3 refers to the path based formulation for an alternative representation in its discussion of “retrospective” rights.

#### 2.4.3.3. Stochastic Dual Dynamic Programming

Irrespective of the representation, solution of SLP problems can be challenging, because the straightforward LP formulation of the multi-stage problem has a large state-space to represent all the scenarios. In reality, techniques such as Bender’s decomposition, Monte Carlo sampling, and nested decomposition are generally used to reduce the problem into a manageable set of sub-problems. For example, refer to Pereira & Pinto (1985), Jacobs et al. (1995) and Seifi & Hipel (2001), Nemirovski et al. (2009), and Shapiro (2011).



In particular, stochastic dual dynamic programming (SDDP) has been utilised for a range of problems such as reservoir optimization, hydro-thermal coordination and capacity planning. Broadly similar methods have been developed and applied by others, including De Matos et al. (2015).

Pereira & Pinto (1991) applied SDDP to a hydro scheduling problem with an objective of minimizing the hydro-thermal system operational cost (Pereira & Pinto, 1985, 1991; Gorenstin et al. 1991; Granville et al., 2003). The SDDP as described by (Pereira & Pinto, 1991) splits the optimization problem into a series of master problems and sub-problems. A sub-problem can have further sub-problems because of the hydrologic uncertainty. The model determines the expected value of the dual of a sub-problem and a corresponding Bender's cut. Pereira & Pinto approximate the expected value of the original sub-problem using the Bender's cuts<sup>6</sup> of sub-problems. The value function can be approximated using values of the sub-problems subject to bounds (Pereira & Pinto, 1991). In this way an approximate solution is determined for the future cost. SDDP then successively refines this approximation within the likely state-space.

Lee & Labadie (2007) considered SDDP as another variation of SLP with recourse. But, while SDDP is clearly not just a variation on classical SDP, it may be seen as similar in the sense that it deals with water value functions that reflect expectations with respect to future water values. In fact its piecewise linear future water value functions are conceptually equivalent to the (step-wise) "demand curves for storage" built up by some implementations of the CDDP technique discussed in Section 2.4.2<sup>7</sup>. But SDDP avoids some of the dimensionality issues associated with the classical DP, or CDDP, by determining marginal water value curves that are only accurate near the storage levels reached in some assumed scenario tree, and then iterating to improve those approximations of the future value functions, (Velasquez et al., 1999). SDDP reduces the issue of excessive scenario branching in classical SLP. Recently, Löhndorf et al. (2013) employed a modified SDDP method to solve a multi-reservoir optimization problem when hydropower generators participate in a wholesale electricity market.

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<sup>6</sup> Benders decomposition can be thought of an outer approximation method as it adds linear constraints to reduce the search region while safeguarding the original feasible region.

<sup>7</sup> Other CDDP variants build up piece-wise linear marginal water value curves, corresponding to derivatives of piece-wise quadratic water value curves.

SDDP has been used extensively to optimise and/or analyse hydro systems around the world. In particular it is used to clear “cost-based” markets operating in many Latin American countries with hydro dominated power systems, as discussed in Section 2.6.2.

In this thesis, we apply both SLP and SDP techniques. Chapter 3 discusses deterministic, and then stochastic, LP-based water market clearing formulations whose dual solutions can be used to explore water market pricing implications spatially and temporally. Chapter 4 then uses those formulations to develop water market hedging instruments.

The rest of the thesis deploys CDDP based methods to model the intra-period problem. A practical water market development would most likely employ an SLP formulation like this, because SLP can readily deal with a general catchment configuration involving multiple reservoirs. But each SLP solution only provides a decision policy starting from a single starting storage pattern. But the original motivation for this development was to provide a sub-model that could be used to perform pre-computations for the longer term stochastic CDDP developed by Starkey (2014). And that, in turn was to provide a modelling environment within which an agent-based simulation model could be developed.

## **2.5. Smart Markets for Electricity and Gas**

### **2.5.1. Background**

This thesis is about the development of model-based “smart” market approaches to improve coordination of some aspects of the water sector beyond what has been achieved, to date, by traditional optimization. So, having surveyed the achievements of traditional optimization in the sector, we now turn to consider what has been achieved by the model-based smart market approach to other sectors, particularly electricity and then, to a lesser extent, gas. The privatization/market reform process (see Gilbert & Kahn, 1996; Newbery, 2002; Jamasb & Pollitt, 2005, Sioshansi & Pfaffenberger, 2006, Joskow, 2008), institutional design (e.g., Armstrong et al., 1994), and market design issues (Bunn, 2000; Green, 1996; Day et al., 2002) lie outside of the scope of this thesis. Here we focus on the core model-based market clearing processes, in an attempt to take advantage of what can be learnt from market reforms in those sectors to develop relevant models for water markets.

Both sectors have developed competitive “optimisation” markets which are configured as auctions and cleared using a mathematical program (Bichler et. al., 2010). McCabe et al., (1989) describe these auction-based commodity/natural resource markets as “smart markets”. A smart market model reduces transaction costs by increasing the amount of information available to the bidders and omitting the need to search for partners for pair-wise trading. The purpose of a smart market is to allocate commodities to maximize some social objectives. A market is created with price offering agents and a market clearing agent maximizes the economic surplus at the offered prices.

This maximization problem is typically solved using mathematical optimisation techniques such as Linear Programming (LP), and improved computational facilities have enabled authorities to develop smart auction markets to efficiently allocate non-traditional commodities in a centralised pool. Generally, in the optimisation world, a non-linear optimisation problem with linear constraints may be approximated using a convex piecewise objective function, and solved in an LP fashion. Piecewise linearization has been commonly used to obtain feasible solutions in natural resource markets, too. For example, LP based commodity markets or electricity and gas determine dispatch schedules and locational marginal prices that maximize the benefits from trade while satisfying a set of physical network and economic constraints. These markets generally operate in networks, within which supply must equal demand at each node (Hobbs & Helman, 2003).

### **2.5.2. Model-based Electricity “Spot” Markets**

Electricity is a public commodity often subject to uncertainty in supply, and transportation. Today, electricity in many countries is traded in competitive markets as a result of the recent thrust towards industry liberalisation and market reforms. Modern electricity market prices are generally determined in real-time, and self-adjusted to balance supply and demand. So load patterns, supply characteristics, network constraints and market forces influence these electricity “spot” prices.<sup>8</sup> But electricity markets have distinct differences to other commodity markets. This is mainly due to non-storability of electricity and the way in which electricity flows strictly in accordance with physical laws, within

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<sup>8</sup> The current market price at which a commodity is bought or sold for immediate payment and delivery is known as the spot price.

transmission networks. In that context, markets can be cleared using deterministic optimization models, but they must correctly model the physical nature of the network, and flows within it.

These complexities have drawn the attention of many research communities to analyzing the issues related to electricity sector. Schweppe et al., (1988) incorporate physical laws into the transmission pricing to form an economic dispatch model that allocates energy, accounting for losses and network constraints. In electricity markets, laws of electricity movement and the market must closely interact as a result of non-storability, the need for instantaneous physical transportation from a source to one or several load points, and the need for near real-time coordination (Ventosa et al., 2005; Hogan & Toulouse, 2008). Thus, physical supply and demand must balance in real time for a reliable grid operation. However, there could be real-time and/or ahead of time trading of power in the market<sup>9</sup>. Read & Chattopandhayay (1999) describe the structure of LP based electricity market clearing models. Buyers and sellers at each node in the network express offers and bids. An LP based auction model clears current, or near future (e.g. day-ahead) bids and offers across the network. Typically these markets are cleared at fixed time intervals. The LP solution matches the offers and bids submitted for a particular market clearing period and this provides both the optimal dispatch schedule and the nodal prices. The locational prices indicate the system marginal generation costs, marginal losses, and congestion. Wu et al., (1996) and Hsu (1997) describe the principles of nodal pricing, while Stoft (2002) and Kirschen & Strbac (2004) discuss the economics of such markets. All current electricity markets use a deterministic market-clearing formulation. A typical market formulation can be written as follows:

$$\max_{q_d, q_g} (\sum_{d=1}^D P_d q_d - \sum_{g=1}^G P_g q_g) \quad [2-7]$$

Here  $b$  and  $\bar{b}$  denote demand index and supply index respectively.  $P$  and  $q$  denote offer/bid price and quantity expressed by participants. The above objective function seeks to maximize the net benefits from trading in a spot market and will be subject to both physical and economic constraints such as generator capacities, consumption, transmission line capacities, and system security and reliability

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<sup>9</sup> Here we describe a typical “gross pool”, “uniform price” market. Other variants exist, such as the “pay-as-bid” UK (net) balancing market (Dettmer, 2002), but these lie outside the scope of this thesis.

requirements (Hogan, et al., 1996; Alvey et al., 1998; Ma et al, 1999). For the interest of simplicity, it was chosen not to express the constraint equations. Market settlement usually takes longer time intervals (e.g., end of month account balancing).

Several recent studies discuss stochastic market clearing mechanisms. The need for an efficient stochastic market clearing mechanism has been discussed in the presence of a relatively large fraction of renewable capacity such as wind (Khazaei et al., 2013) and hydro. An SLP based market clearing model would consider the future clearing scenarios in the current decision. Such a market clearing model would depend on agents agreeing on probability distributions of future outcomes to yield the expected value of the solution (Wong & Fuller, 2007; Pritchard et al., 2010). The welfare maximization problem remains equivalent to a perfectly competitive market cleared using a multi-stage SLP when all participants are risk neutral and they have perfect access to information about the future hydrology conditions.

### **2.5.3. Contracts and Hedging Mechanisms in Electricity Markets**

#### **2.5.3.1. Trading Future Electricity**

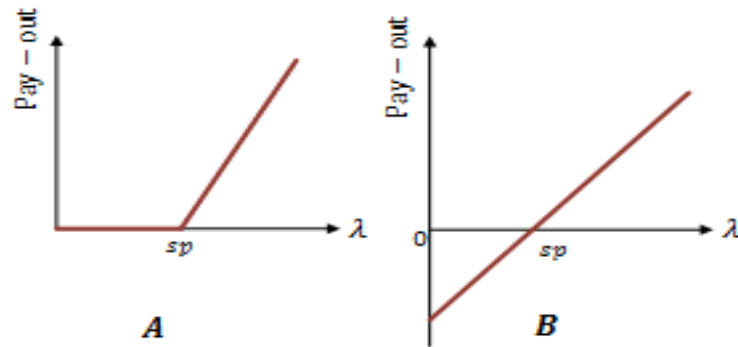
The electricity prices produced by spot markets of the type described above can be quite volatile. The desire to avoid being exposed to spot price variations has led to the development of forward markets and various contracting mechanisms. Hull (2006) and Wystup (2007) provide a detailed description of options and various other structured products/derivatives applicable for stock markets and commodity markets. Stoft et al., (1998), Deng & Oren (2006), provide details on electricity futures and derivatives. Perez-Arriaga (2010) explains various forms of financial contracts traded in electricity markets such as forward and futures, financial swaps, and contracts for difference (CfDs).

Here we focus on CfDs and option products as we will develop analogous instruments for water markets in Chapter 4.

A call option gives a participant the right, but not the obligation, to buy a pre-specified amount of power,  $Q_t$ , at an agreed “strike”, (or exercise) price,  $SP_t$ . If  $\lambda_t$  denotes the spot price, mathematically, the payout in time period  $t$  is

$$\max\{(\lambda_t - SP_t), 0\}Q_t \quad [2-8]$$

Diagrammatically this option may be represented as in Figure 2-2A. Here the holder is not exposed to the risk of market clearing price rising above the strike price.



**Figure 2-2.** Graphical illustrations of a call option pay-out and a CfD pay-out

Second, a “put” option gives the holder the right to sell a given amount of MW at the strike price. Mathematically, the payout is:

$$\max\{(SP_t - \lambda_t)Q_t, 0\} \quad [2-9]$$

Then, a contract for difference (CfDs) can be thought of as a combination of a call and put option. Participants negotiate a strike price and a quantity. The buyer pays the seller the difference between strike price ( $SP_t$ ) and the spot price, ( $\lambda_t$ ) times the quantity, ( $Q_t$ ), if the former is higher. Otherwise the seller pays the buyer the difference between the two prices times the quantity.

Mathematically, the payout is:

$$(\lambda_t - SP_t)Q_t \quad [2-10]$$

Diagrammatically, this option may be represented as in Figure 2-2B.

Note that there is no physical delivery of energy associated with any of these financial products. They operate in parallel with the spot market, which can be cleared without any knowledge of who has contracted with whom, for what quantity, or at what price. Valuation and settlement follow normal capital market processes, and this attracts more participants into the market which helps to increase the liquidity. But the commercial effect of a CfD is essentially the same as if the buyer actually took physical delivery of the agreed quantity, at the strike price, then traded the difference between that quantity and their actual consumption at the spot price. Mathematically we have:

$$(SP_t - \lambda_t)Q_t + \lambda_t q_t \quad [2-11]$$

By re-arranging the above we can see that the commercial effect for the seller is essentially the same as if they actually had to trade the difference between the agreed quantity and their actual output at the spot price, as on the RHS below:

$$(SP_t - \lambda_t)Q_t + \lambda_t q_t = SP_t Q_t + \lambda_t (q_t - Q_t) \quad [2-12]$$

The key contribution of CfD-type instruments is that they can be traded, valued and settled without requiring a precise alignment with real-time physical reality. Economic equivalence does not mean that financial products deliver the physical energy. Thus Starkey (2014) discusses a range of wholesale water market design options, in which participants could also trade financial products informed by a centralized market-clearing. In Chapter 4, we assume the existence of a CfD-type market for water. We investigate deploying CfD-type instruments to hedge against inter-locational and inter-temporal price risks. But those instruments are more akin to the Financial Transmission rights discussed in the next section.

### **2.5.3.2. Financial Transmission Rights**

Like any other tradable property right, a Financial Transmission Right (FTR) can be employed in variety of ways to raise market liquidity, provide security or hedge risk associated with the locational marginal prices (LMPs) (see Wu et al. (1996) and Hsu (1997)). When clearing the spot market, the ISO (Independent System Operator) typically receives a “settlement surplus” as there is a positive difference between consumer receipts and the generator payments. That surplus can be decomposed into loss rentals associated with convex rising marginal loss functions, and congestion rents generated by binding transmission capacity constraints are normally collected by the ISO during transmission congestion.

A common approach to manage these surpluses is by defining virtual property rights such as FTRs (Hogan, 1992), stakeholder “shareholdings” (Read & Sell, 1989) or virtual shares (slices) of the system capacity (Barroso et al., 2012). In all cases the surplus is redistributed in some way to property right holders.

In particular, Hogan (1992) introduced the concept of “Financial Transmission Rights” (FTRs) defined on a point-to-point basis to enable participants to hedge against inter-locational price risks in a constrained electricity spot market.

FTRs are path independent, but direction dependent. That is, an FTR defined from node  $i$  to node  $j$  is of equal value, but opposite sign, to an FTR defined from node  $j$  to node  $i$ . The FTR pay-off is for the entire contract quantity and thus independent of actual “usage”. They can be either obligation-inclusive (that is the FTR holder has the right to receive compensation but at the same time has an obligation to pay compensation when the locational marginal price difference is negative) or of (call) option type. That is<sup>10</sup>:

$$FTR_{ij} = (\lambda_j - \lambda_i)Q_{ij} \quad [2-13]$$

An FTR is a financial instrument that provides the holder no priority in physical transaction scheduling. Allocation of FTRs could be done in various ways (Lyons et al., 2000). Generally, FTRs are sold in a periodic auction (primary auction) and provide congestion insurance for a period sometime in the future. The need for revenue adequacy is a major consideration when allocating FTRs. That is, the revenues collected with locational marginal price differentials in the form of congestion payments in a particular period should be sufficient to support the payments to the FTR holder in that period. The simultaneous feasibility test<sup>11</sup> ensures revenue adequacy of the re-configured FTRs (Harvey et al., 1996). Hogan (1992) proves revenue adequacy for simultaneously feasible FTRs for lossless networks<sup>12</sup>. Bushnell & Stoft (1996) extend those results for quadratic losses, in which case the new feasible dispatch feasibility is determined using the appropriate “lossy flow equation” (Bushnell & Stoft, 1996). Later, Hogan (2000) includes non-linear constraints.

FTR prices are updated continually as trading occurs in secondary markets. But there can be a large number of potential point-to-point combinations in an electricity network, and it will often be

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<sup>10</sup> An FTR option is defined as:  $FTRO_{ij} = \max\{(\lambda_j - \lambda_i - SP_{ij})Q_{ij}, 0\}$

<sup>11</sup> This ensures that all FTR issuances never exceed the physical capabilities of the transmission system.

<sup>12</sup> A fundamental property ensuring revenue adequacy is convexity of the dispatch model (Philpott & Pritchard 2004).



impossible to match buyers and sellers for rights between exactly the same pairs of points. Fortunately any set of FTRs may be decomposed into, and replaced by, any other set of FTRs, as long as the aggregate MW power flows are matched. Revenue adequacy can be guaranteed provided a net balancing of flows across the network can be achieved. To do this, the SO solves an LP to determine successful offers and bids for FTRs ensuring that the transmission system can support all FTR issues under normal system operating conditions.

Chao & Peck (1996) later introduced the related concept of Flow Gate Rights (FGRs), which allow the holder to recover (some part of) the rents collected as a result of congestion for a specified line, or other “flow gate”<sup>13</sup> (see, Chao et al., (2000)). The value of the FGR is determined by the “shadow price”<sup>14</sup> associated with the maximum transmission capacity of the physical flow gate.

In chapter 4, we draw on these FTR and FGR concepts developed for electricity markets to describe and discuss similar water market hedging instruments.

#### **2.5.4. Agent Behaviour in Electricity Markets**

Although outside the scope of this study, issues such as the exercise of market power, market incompleteness, and risk aversion are somewhat related. So we briefly review some recent literature on these areas.

##### **2.5.4.1. Classical Market Power Modelling**

Researchers have used classical economic equilibrium models, and, more recently, agent based models to analyse market power issues, particularly the ability of a generating company to produce and dispatch supply offers above its marginal cost of generation. In this section, our focus is on economic equilibrium models.

Day et al., (2002) provide a nice discussion on the use of equilibrium models for power markets. These models attempt to define a market equilibrium considering supply/demand bids and quantities, and sometimes transmission network conditions, subject to the first order conditions of each participant’s net benefit maximization problem. Day et al use a numerical method to analyze market power issues in

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<sup>13</sup> They are, in fact, closely related to the line-by-line shareholdings proposed by Read and Sell (1989).

<sup>14</sup> This is the marginal benefit (cost) of relaxing (strengthening) the constraint.

large networks. Classical equilibrium models are based on different economic conjectures of competitive behaviour between a firm and its rivals. Widely used are pure competition (no market power), Bertrand (game in prices), Cournot (game in quantities), collusion, Stackelberg, and supply function equilibria (Tirole, 1998; Day et al., 2002; Li et al., 2004). Cournot models are commonly used because they are relatively tractable. Read & Chattopadhyay (1999) conclude that useful approximations to market behaviour can be obtained, even though the assumptions are clearly quite unrealistic.

Transmission constraints make Cournot modelling more difficult, but models have been developed to study the exercise of market power at particular locations in a constrained power network (Oren, 1997; Rudkevich, 1999; Hobbs, 2001; Momoh, 2008). In particular, Rudkevich (1999) and Hobbs (2001) discuss strategic interaction on networks as the market power issues that can be caused by transmission limits.

Hobbs & Helman (2003) provide a detailed tutorial on complementarity-based energy commodity market models and then compare alternative specifications of oligopolistic power markets subject to transmission constraints. These models represent the constrained optimization problems of electricity generators, consumers, and transmitters. The authors discuss the mixed complementarity problem (MCP) by defining the first-order optimality conditions for participants and market clearing conditions. Supply function equilibria provide a better conceptual model of market behaviour, but are much more difficult to solve. Still, Klemperer & Meyer, (1989) investigated this using continuously differentiable supply functions, in the presence of uncertain demand, while Green & Newbery (1992), Green (1996), and Anderson & Philpott (2002), studied the existence of supply function equilibria.

We see considerable scope for future studies to develop similar models to analyze water market behaviour. But we do not attempt any such development in this study, because it seemed premature in a situation where more basic issues such as the mathematics and mechanics of market clearing and hedging have not yet been established.

#### **2.5.4.2. Hydro Generator Behaviour**

Given that this thesis is about water markets, the treatment and behaviour of hydro generators in electricity markets seems particularly relevant. Section 2.7.3 below discusses centrally-optimized or

cost-based” market structures, in which all hydro storage is effectively managed by the market operator, much as in the water markets discussed by Starkey et al (2012). But, in many deregulated electricity markets, hydropower firms generally manage their own storages in electricity market that is water storage being outside the market. Such a firm’s decision to trade depends on hydrology conditions and spot market prices and, of course, water storage levels. While the modelling and analysis of agent behaviour lies outside the thesis scope, we briefly survey the literature on two topics which will eventually prove relevant, if water markets of the kind discussed in this thesis are to be implemented.

### **Market Power**

Scott & Read (1996) appears to be the pioneering article that incorporates the optimal reservoir scheduling problem into a market context, where hydro firms possess significant market power. In a competitive market, the marginal value of storage is determined through electricity spot market prices subject to long-term contract prices. Scott & Read model Cournot game equilibrium strategies for generating firms in the presence of uncertain inflows allowing water demands to be represented by monotone non-increasing “demand curves”. Bushnell (1998) shows that the hydro generator enjoys a competitive advantage as a result of its storage capacity. Later Crampes & Moreaux (1999) proposed a model of competition between a thermal power station and a hydro generator. More recently, Garcia et al., (2001) have focused on the issue of strategic competition in electricity markets. Lino et al., (2003) noted that different ownership/management of price taker hydro firms in the same river cascade can distort the net welfare.

### **Risk Aversion**

Steege et al (2014) provide a literature review on the bidding problem in hydro-dominated electricity markets. They observe a few studies on the problem for price-taker hydro producers, but risk dimensions, and price-maker hydro producers, have attracted little attention. Read & Kerr, (1996) and Kerr (2003) adapted the CDDP model to deal with risk aversion by adding a dimension for accumulated wealth. This development was then incorporated, along with the gaming model of Scott and Read, into the two reservoir RAGE optimization model developed for ECNZ (see Read & Hindsberger, 2010). Pritchard et al., (2005) apply a dynamic programming approach to model a price taking hydro firm in a

pool electricity market. They represent each stage with many trading intervals to manage the time-step gap between the market trading periods and the reservoir planning periods.

As noted above, there has been less attention paid in the literature to the issue of competitive equilibrium in hydro-dominated markets with risk-averse agents. But Philpott et al discuss the social welfare problem when the agents are risk averse with respect to hydrological conditions, concluding that hedging instruments could reduce welfare losses in a competitive equilibrium (Philpott et al., 2013). They discuss some of the consequences of risk aversion on market outcomes, using a simple two-stage competitive equilibrium model with three agents. An agent's attitude to risk is formed according to the theory of coherent risk measures. However, this approach may not fully represent situations such as high inflows favouring hydro-power agents and low inflows favouring thermal generators. As a result, agents must be willing to trade off risks, possibly through contracts, in order to bring the social planning problem closer to its optimality, Baker (2014). However, Philpott et al argues that the market for risk is incomplete in a similar way to the case identified by Lino et al., (2003), so actual market solutions will remain sub-optimal, from a societal perspective.

#### **2.5.4.3. Agent-based Methods**

During the last decade, several authors have attempted to model electricity markets using agent-based approaches. Our own interest in surveying this literature was motivated by the observation that, while agent behaviour would be an important issue in any future water market implementation, large scale game theoretic models were likely to prove intractable for systems involving complex interactions over both space and time. Thus it was originally intended to develop a simulation model along these lines, as part of this thesis. That has not proved possible, though, given the complexity of the issues uncovered in analysing the basic structure of market-clearing solutions, and potential hedging arrangements.

Agents are autonomous decision-making units, who are assumed to behave strategically within their rational boundaries. Agent based models reveal interactions between agents who are often assumed to know nothing about other agents and the state of the market at the start. They are assumed to learn and adapt to new environment settings created by other agents. Thus agents may themselves select bidding strategies, rather than rely on those provided by the researcher.

For example, Tully & Kaye (1996) developed an algorithm based on agent-based techniques to select reservoir management strategies for generating companies. Visudhipan & Illic (1999) demonstrated an agent-based simulation for a spot market with uniform-price market clearing. They used a price-elastic demand curve to model electricity demand. Bower & Bunn (2000) performed an agent based simulation of the England and Wales electricity market. They compared various market mechanisms such as pool versus bilateral trading, daily versus hourly bids, learning through repetition and availability of market information. Generating agents bid on a single electricity market. A price-inelastic aggregate demand curve was used to determine the electricity demand. Bunn & Oliveira, (2001) assumed that agents were capable of assessing their market share, contract cover, retail price, and forecasting errors. Bower & Bunn, (2000) and Bower et al., (2001) demonstrated the ability of generating agents to achieve maximum profits at a given plant utilization using a simple reinforcement learning algorithm. Micola, et al., (2008) used an agent based simulation approach to model natural gas and electricity markets. Weidlich & Veit (2008) reviewed different agent learning algorithms developed by Richter & Sheblé, (1998), Richter, et al., (1999), Sun & Tesfatsion, (2007), and Tellidou & Bakirtzis, (2007) used in the electricity market literature to analyze agent behaviour. However, they highlighted that no clear justification was given for using certain algorithms, while most agent-based studies employed simplified system models, such as neglecting transmission grid constraints, and treating demand as fixed.

Why have researchers searched for alternatives to the classical modeling approach? One criticism of existing models was not being able to capture system dynamics. Static models are ill-suited to model intelligent behavior of participants (agents) in the market. Furthermore, traditional economic models *“tend to look at an industry in aggregate, from the ‘top down’”* (Bower, et al., 2001). On the other hand, Weidlich & Veit (2008) emphasized the need of more rich and flexible models to capture network complexities and market dynamics. Saguean et al (2001) compared game theoretic and agent based economics approaches for assessing market power in electricity markets, concluding that the agent based approach is a “useful tool to evaluate and compare results with other approaches”.

## **2.6. Model-based Markets with Storage and Delays**

Our discussion of model-based electricity markets in Section 2.5 focussed solely on “spot markets”, in which electricity, as the product being traded, is transported instantly, and cannot be stored. In that context, a deterministic market clearing model is clearly adequate. While long term trading is important, it can be accomplished by trading financial instruments, independently of the spot market. The situation is clearly different for water, though, because it is not transported instantly, and can be stored. Before we consider water markets, though, there are already two types of model-based market in which inter-temporal optimisation is performed: the Victorian gas markets, and several Latin American hydro dominated electricity markets.

### **2.6.1. Model-based Gas Markets**

There has been much less development, or analysis, of model-based markets for gas than there has for electricity. But we briefly survey the limited literature because, like water but unlike electricity, gas flows through both space and time. Like electricity, compressed gas flows through a network according to a set of physical laws, in this case from higher to lower pressure. And the flow may even split, at uncontrolled junctions, in a manner similar to electricity. But, unlike electricity, the transportation system itself can become a storage medium for gas (Bushnell, 1998), and this “line-pack” is used extensively to control the way in which gas is delivered when and where required, just as in water storage systems. The equations also imply that there can be uncontrollable time delays for gas flows between nodes, just as on channels between nodes in a water transport system. For both reasons, we can and must relax the real time coordination requirement usual in electricity systems.

According to Zheng et al. (2010), previous research studies on natural gas market models have focused on issues such as regulation (O’Neill et al. 1979), social welfare maximization in natural gas equilibrium models (Gabriel et al. 2003), gas system analysis models (Brooks, 2003), gas pricing (Tzoannos, 1977; Pepper & Lo, 1999), gas production optimization, gas transportation network development and operations. Several articles study problems such as gas compressor operation optimization and physical gas flow transportation optimization. For example, De Wolf & Smeers (2000) and Pepper et al. (2012) discuss a range of linearization approaches to be applied to physical gas flow dispatch problem.

The Australian state of Victoria seems to have been the only jurisdiction to introduce a model-based gas market, similar to those described above for electricity, and proposed here for water. Read et al. (2012) describe primal and dual LP inter-locational and inter-temporal market clearing models analogous to LMP models used in electricity sector. That model could actually be adapted to provide a short term market-clearing model for water flowing in pressurised pipes. Fortunately, this thesis relates to the dynamics of water flow over longer periods, and/or in open channels, which is much simpler. Thus, while the basic concept of the market-clearing formulation developed in Chapter 3 is derived from Read et al, it is much simpler.

Read et al also propose several types of financial products to hedge against locational and temporal price differences. In particular, they highlight the need to find simultaneously feasible combinations of inter-locational/temporal equivalents to the FTRs employed in electricity markets in an integrated market, instead of clearing separate markets in inter-locational and inter-temporal instruments. In Chapter 4 we discuss the creation of water market hedging instruments to hedge against price differentials across both space and time differentials. In our case, though, those instruments relate to separate system components (storage reservoirs and transportation channels), and can be traded separately.<sup>15</sup>

## **2.6.2. Treatment of Hydro in Model-based Electricity Markets**

Hydroelectric production accounts for the largest share of power production in Norway (99%), Brazil (90%), New Zealand (65%), Canada (62%), and many other countries. The presence of hydroelectric generators means that inter-temporal optimisation must be handled somehow in the electricity market context, as reservoirs can store water for later production of energy. When electricity markets are cleared using deterministic real-time models they must ignore this aspect of the situation and assume that any internal constraints of generators have already been accounted for within bid offers.<sup>16</sup> For

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<sup>15</sup> But we do propose a similar approach when a single arc can transport water from one node and time to another node and time (see Section 3.4.4).

<sup>16</sup> “Spot markets” that employ inter-temporal optimization over, say, the day ahead represents an intermediate case. Such markets routinely perform inter-temporal optimization of thermal unit commitment etc, and could, in principle, do the same for short term management of water in river chains. Participants would offer a river chain model, plus end-of-day marginal water value curves, with storage targets for small head ponds as a special case.

example in New Zealand, both hydro and thermal participants submit their offers irrespective of the fuel type (Alvey et al., 1998). Supply offers and demand bids are cleared a day ahead producing pre-dispatch schedules and price forecasts. This implies that the hydro generator could be facing a restriction on the amount of stored water available for use over the horizon.

But here we discuss the treatment of hydro power generators under two alternative approaches. The “centrally optimised” (cost-based) approach has been extensively employed in Latin America. And various forms of the “virtual reservoir” approach, have been implemented, or proposed, in some places.

#### **2.6.2.1. The Centrally Optimised Approach in Latin America**

In contrast to bid based markets, most Latin American markets are cleared using a centralised inter-temporal optimisation, which Mastropietro et al (2014) and Mastropietro et al (2015) describe as “cost-based”. This description is partly justified by the fact that many of these markets do actually regulate the costs that participants are allowed to offer. But actually the same centralised optimisation approach could be taken with participants freely offering thermal cost curves, end-of-horizon marginal water values, reservoir capacity, and river chain models, as was originally proposed for the day-ahead New Zealand market.<sup>17</sup> The central issue driving the perceived need for inter-temporal optimisation is the difficulty in managing inter-connected large hydro systems. That difficulty is compounded by the fact that, in many of those countries, cascaded and multi-party owned hydro reservoirs are spread over several river systems, and are able to provide storage capacity for power production for several years. The SO seeks to optimize long term system operations considering the availability of hydro and thermal plants, reservoir levels, and forecasted future inflows. The value of water use in all reservoirs over time has to be accounted for when deciding the marginal value (non-bid based) for water in any one reservoir. The SDDP model is popularly applied to construct a large number of hypothetical future marginal water value curves for all reservoirs, and determines hypothetical future releases for a large

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The market-clearing LP would then endogenously determine marginal water values for each storage in the chain, in each market dispatch interval, and hence implicitly offer curves for each station and period. The effect would be somewhat similar to the Latin American market regime described below, only over a much shorter time scale within which deterministic optimization may be acceptable. This approach does not seem to have been adopted anywhere, but the concept was extensively tested by Drayton-Bright (1997), prior to development of the New Zealand electricity market described here.

<sup>17</sup> See footnote above.



number of inflow scenarios. And those releases are driven by upstream/downstream differences in those values, and thus often relate to reservoirs outside the owner's control. In reality, those hypothetical future marginal water value curves and releases play no direct role in the market.

Detailed study of Latin American Electricity markets are beyond the scope of this thesis, but interested readers are directed to the following literature: long-term markets of reliability products (Moreno et al. 2009 & 2010), capacity expansion (Porrua et al, 2007; Batlle et al., 2010; Rudnick et al, 2012), regulation (Mastropietro et al, 2015). But the key point is that this centralized and cost-based optimization model can be thought of as a “water market” clearing model, except that here water only has one assumed use, and may be represented in equivalent energy terms. It is also similar to the water markets described below, in both situations the dispatch instructions are centrally made with the objective of maximizing the net benefit. But the focus of this thesis is mainly allocating storable surface water for both consumptive and non-consumptive uses in a catchment via a market based approach.

#### **2.6.2.2. The Virtual Models for Reservoir Management in Competitive Markets**

Barroso et al., (2012) describe general contracting mechanisms that help to develop “virtual reservoirs” to be managed by competing traders. Then authors analyze some existing regimes such as the Brazilian Energy Reallocation Mechanism, a virtual power plant agreement between Argentina and Uruguay (cited in Barroso et al (2012)), and the Churchill River catchment in Canada (Churchill River Water Management Agreement, 2009) and proposed regimes such as upstream/downstream coordination in New Zealand, competing traders in Tasmania, and in Brazil.<sup>18</sup> Perhaps the clearest example of such a regime working in practice is the Columbia River catchment agreement described by (BPA, 1999).

Such schemes could operate as an alternative system-wide market concept to any of the above, but they have mostly been designed to allow competitive coordination of particular river basins, in the context of a broader electricity market. An example is the Tasmanian proposal of Hunt et al., (2012).

Chapter 3 extends the work of Barroso et al, (2012) by describing an alternative “virtual reservoir” approach to creating a water market, in situations where common resources must be shared between

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<sup>18</sup> Hughes et al. (2013) discuss similar regimes already in operation in the Murray-Darling Basin in Australia, but those are not in an electricity market context.

several parties who want to act independently of each other, rather than accept solutions produced by a central market-clearing optimisation model.

## **2.7. Markets for Water**

### **2.7.1. Pre-market Water Exchange Mechanisms**

Many water professionals have utilized an integrated water resource management approach, which depends on the four principles expounded at the Dublin conference on Water and Environment in 1992, for managing the water resource (Tilmant et. al., 2008). The idea of commoditizing water resources is supported by the fourth principle: “Water has an economic value in all its competing uses and should be recognized as an economic good...” (United Nations, 1992) and this debate has been featured frequently in the water resources literature for the last several years. Economists have repeatedly pointed out the need for transferable private property rights for environmental resources such as water which is usually treated as common pool, publicly owned, resources (Coase, 1960; Tietenberg, 2003)<sup>19</sup>.

Hence, the task of assessing water value has become an important aspect in almost every activity of water use. Economic valuation of a finite, but renewable resource (e.g. water) may not be that simple or straightforward. The marginal water value represents the utilization of the next unit of water from any type of use (such as consumptive or non-consumptive, public or private). Interested readers are referred to the works of Gibbons, (1986), Loomis, (1987), and Colby, (1989), who have studied the issue of water values in alternative use

### **2.7.2. Implemented Water Market Developments**

According to Saliba & Bush (1987) a market for water is defined as the “*interactions of actual or potential buyers and sellers over one or more interrelated water commodities.*” There are formal and informal water trading arrangements already in operation in various parts of the world (Easter et al., 1998; Calatrava & Garrido, 2006).

These include informal water markets in South Asia (Thobani, 1997), a market for ground water abstraction in Pakistan (Jacoby et al., 2001), an indirect water market in France (Dinar & Subramanian,

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<sup>19</sup> Coase (1960) pioneered the theory of tradable permits. Coase argued that the tradable property rights approach better handles negative externalities over the Pigouvian solution method (Chao & Peck, 1996).

1997), a market for irrigation supplies in Israel (Tsur, 2009), markets for water in Chile (Hadjigeorgalis & Lillywhite, 2004; Bauer, 2015; Hearne & Donoso, 2014), and water markets in the U.S. (Michelsen et al., 2000). A formal market normally trades long-term water entitlements (Bjornlund, 2004) and they mainly focus on allocating water for agricultural use. Easter et al. (1998) provide evidence for both spot and option trading in the rural United States, while Hanak (2002) and Griffin & Characklis (2002) further describe water market arrangements in California and Texas. Bjornlund (2004) describes water market developments for agricultural use in Australia, while Anderson & Snyder (1997), Bjornlund & McKay (1998), Young & McColl, (2007) and Zaman et al. (2009) all report on the growth in water markets in Australia. For example, Waterfind, established in 2003, facilitates trading permanent and temporary water allocations across major irrigations regions in Australia via an online trading portal, Waterfind (2013). In Chile, the water sector reform process has been reportedly a significant success. It gave an opportunity for agriculturalists to acquire water for higher value agricultural products (Schleyer, 1992; Hearne & Easter, 1995). Regional water markets reported significant gains from trading water/water rights (Hearne & Easter, 1995).

### **2.7.3. Water Markets Research**

The previous section describes some real experiences with water markets around the world. However, much of the literature is still focused on defending water market proposals over existing water allocation regimes as the preferred method from the point of view of economic efficiency. Some papers evaluate economic effects of non-market re-allocations and resource transfer from one user to another. This is generally done through centrally planned social welfare LP/DP optimization models (see Vaux & Howitt (1984), Rosegrant & Binswanger (1994); and Calatrava & Garrido (2006)).

The works of Easter et al., (1998), Lino et al., (2003) and Ambec & Doucet (2003), attempt to draw lessons for water market developments from the electricity market literature. Second, according to Ambec & Doucet (2003), the absence of markets for water could sometimes lead to sub-optimal management of water resources and exercise of market power. Similarly, Lino et al., (2003) studied the issue of market incompleteness and noted the importance of remunerating reservoirs for their economic contribution to the hydro system. On the other hand, there is a strong possibility of developing a

spectrum of product trading opportunities such as spot, forwards, futures, and options, as in electricity markets, to trade storable surface water due to a number of analogies between the two resources (Easter et al., 1998).

Some articles propose various institutional arrangements to set up hypothetical water markets that trade either spot water or rights (Vaux & Howitt, 1984, Garrido, 2007). Rosegrant et al. (2000) report intra-seasonal gains from trade Maipo basin in Chile. Recently, Qureshi et al. (2009) provide evidence of economic gains from water trading in Australia. Garrick et al (2009) discuss the enabling conditions for water market policy reform and implementation. Other authors including Brennan (2008), and Hughes & Goesch (2009), discuss the reform process of water storage rights in Australia.

Smart market models for water seems to have pioneered by Dinar et al (1998) and Murphy et al (2000). Murphy et al. (2009) represented in-stream flow via an environmental agent who submits demand bids in the water market. Several detailed smart market design options for water, and associated “commodities” have been proposed by the Water Management Research Group (WMRG) at the University of Canterbury, in New Zealand. These include market-clearing models for nitrates transported via subterranean water (Prabodanie & Raffensperger, 2007), water pollution (Prabodanie et al., 2010) and sediment discharge (Pinto et al., 2012). These models all seek to allocate the natural resources via respective transportation networks, with trading at prices determined by shadow prices on the critical resource constraints. Most of them employ deterministic optimization models, but the last uses SLP as it considers uncertainty of rainfall events.

With respect to water markets, as such, Raffensperger et al (2009) illustrate a low cost market based approach to allocate ground water rights among participants who draw irrigation water from a joint aquifer. That model employs deterministic optimisation. Starkey et al., (2011) and Starkey (2014) outline several types of simple to sophisticated water market design options that might employ stochastic optimisation to clear a market for surface water in a reservoir-based system with variable hydrology driven by variable weather changes, in surface water. In all cases dual prices on water availability constraints are used as market clearing prices. The authors discuss how each market interface and/or clearing option could handle uncertain future inflows and demands. Participant’s bids

reflect their future demand for water, and could be binding or merely indicative. They could be for fixed quantities, or depend on a hydrological index. They could trade physical contracts or financial instruments. And quantities could be determined directly by the optimisation, or the market manager could employ statistical methods to determine the final allocation. Or the whole market could be merely indicative, merely informing participants who would arrange their own trades, perhaps directly, or through an independently managed futures market.

In a sense, the proposal that “financial instruments” be traded, rather than “future physical water” builds on some earlier work proposals in the literature. Hamilton et al. (1989) discuss long-term option contracts formed between agriculturalists and hydropower users to allocate water from agriculture to power generation during low flow periods. Michelsen & Young (1993) propose option contracts as a “less expensive institutional arrangement” to temporarily allocate irrigation water rights to non-irrigation uses during drought periods. Early studies such as Miller (1996) and Howitt et al (1998) discuss how different spot and rights markets could help to stabilize water availability. Brown & Carriquiry, (2007) highlight potential welfare gains from diverting water from low valued use to high valued use during low inflow periods using an option-contract arrangement.

For our purposes, Chapter 3 and Chapter 4 will assume that a water market exists and is cleared by solving a stochastic formulation such as that described in Starkey (2014). Starkey actually favours a centrally cleared market trading a mix of fixed and proportional “futures” products. But we will assume a simpler arrangement, under which the market trades spot water and financial contracts for fixed volume long term water and index based inflows as further discussed in Section 4.2.1. The focus of those chapters is then on developing financial instruments to complement such markets, by hedging price differences over space and time, just as FTRs complement markets in financial contracts for future electricity.

## **2.8. Chapter Conclusions**

This Chapter has noted the importance of water allocation issues, and discussed traditional approaches to optimization of water resources allocation. But it has also focused on the way in which similar commodities such as electricity and natural gas are traded through “smart markets”, which are, typically

cleared using an LP based optimization model, subject to constraints representing a transportation network. Thus, we surveyed the literature on model-based electricity and natural gas market models, along with the emerging literature on applying similar approaches to trading storable surface water under uncertainty.

From that survey, it would appear that no consideration has yet been given to developing, model-based markets for surface water in mixed-used catchments involving consumptive, non-consumptive, in-stream, and environmental users. Nor has there been any detailed analysis of the structure of solutions that would emerge for such mixed participants, or the locational/temporal structure of the prices in such a system. So the major part of this study (Chapter 5 to Chapter 8) develops intra-period CDDP pre-computation models that clear mixed-use water markets in catchments with one or two reservoirs. This work builds on several CDDP developments previously implemented in New Zealand. It produces marginal water value functions that can be interpreted as demand curves for water released from the long term reservoir storage, and these could be employed within a long term stochastic market-clearing model such as that of Starkey (2015). The way in which consumptive and non-consumptive use value functions combine in forming these curves also provides insights into how economic incentives operate in a multi-user nodal river catchment.

One other obvious gap in the literature is that no hedging instruments have been proposed to deal with the inter-temporal and inter-locational risks that would arise when trading in such a catchment. But we expect that such instruments will be required to support efficient water trading between locations, and between periods, just as FTRs are required to support efficient trading between locations, in electricity markets. So the earlier part of this thesis (Chapters 3 and 4) draw on electricity market analogies, and particularly the FTR literature, to develop an SLP formulation for the water market clearing problem, broadly following Starkey (2015), and a set of financial instruments to hedge against the inter-locational and inter-temporal price differences in a mixed-use nodal water market.

# 3.

## Primal and dual water market optimization models

### 3.1 Introduction

The objective of this chapter is to illustrate how to clear a market for hydrology dependent surface water supplies to optimise both consumptive and non-consumptive water use across space and time. It does so by developing a general LP formulation that could be used to clear markets in which water is traded between water users of various types, both consumptive users and non-consumptive users, or by a catchment manager to optimise integrated operations. The benefits these users receive can be modelled directly, or as market bids at points within a river catchment, which forms part of the market clearing model. A market bid reflects users' willingness to pay as a measure of her benefit from water consumption, or flow. Water can be traded in the present, on the "spot market", but that leads us to ask how much water to save in storage for future release? The market framework discussed here assumes that both release and storage decisions are taken by the market, which is periodically re-cleared to account for changes in participant bid/offers, and in hydrological conditions.

This chapter is organised in the following way. Section 3.2 highlights relevant literature on LP based commodity market models, while Section 3.3 presents a deterministic multi-period nodal LP-based water market model. Section 3.4 describes how to allocate costs associated with arc flows yielding joint benefits. Section 3.5 presents a general form of stochastic multi-nodal multi-stage benefit maximising model, following the event tree formulation approach. Finally, Section 3.6 presents Chapter Summary.

### 3.2 LP-based Market Models

The literature on LP based *surface* water network market modelling starts with MaCabe et al. (1991) and Dinar et al (1998). Murphy et al (2000) illustrated how to achieve an efficient water allocation using auction based mechanisms. Recently, the same authors have studied some important issues such as environmental use and in-stream flow values in water markets (Murphy et al., 2009; Murphy & Stranlund, 2003).

On the other hand, Labadie (2004) surveys several non-market network reservoir optimization models (see for example, Bertsekas & Tseng (1994), Hsu & Cheng (2002), Labadie & Baldo (2001)). These models use nodes to represent storages or non-storage points (e.g., confluence, flow off-take) and arcs to represent reservoir releases, conveyer flows, and storage transfers over time. In the majority of water cases, piecewise linearization of the non-linear cost (benefit) functions is believed to give a good enough approximation to model the problem with an LP/SLP (Dupacová et al., 1991). More recent literature discusses a number of water market issues such as locational marginal prices, revenue adequacy (Raffensperger, 2009) and in-stream flow values. However, except for Starkey et al. (2011), these papers rarely focus on hedging instruments such as arc flow capacity rights and contracts.

Unlike the water sector, the electricity market literature refers to many LP based auction models (e.g., Hogan, 1992; Alvey et al, 1996; Read et al. 1998). Schweppe et al. (1988) first modelled nodal energy prices for power electricity networks. The economic dispatch model allocates electricity generation optimally, accounting for the costs of losses, and network constraints. Hogan (1992, 1993) studied the full complexity of the AC power system, based on the AC optimal power flow formulation. His *ex post* pricing approach determines the locational marginal prices which can be used in a single period settlement model for wholesale electricity market transactions. Later, Hogan et al. (1996) presented a generalized dispatch based pricing framework which could explain active and reactive nodal prices basing upon Hogan (1992, 1993). The locational electricity prices reflect how the physics of current flow in a network, and the economics of trading a commodity in a network, influence the marginal cost of supplying energy. Even though the physical attributes of electricity and water are quite different, we consider an analogy between water and electricity markets, and also gas markets, in terms of financial trading.



Section 3.3 presents a *basic* LP model (multi-period, deterministic) to demonstrate some important price relationships in the market. This helps us to understand wider network flow issues later.

### **3.3 Multi-period Deterministic Nodal LP for a Water Market in a Tree Structured**

#### **Catchment**

Storable surface water supplies are uncertain and differentiated spatially and temporally. Typically, water is stored in reservoirs and within the transfer network. This stored water would have opportunity values for consumptive use and sometimes for non-consumptive use. For example, many power systems rely, in part, on hydro-electric power generation, which often occurs in catchments where other parties also value water for competing or complementary uses. Typically a hydro reservoir system includes water stored for later use in interconnected storages and a complex network of water ways with different users distributed geographically (e.g. Waitaki river catchment in New Zealand, Mahaweli Scheme in Sri Lanka).

In this chapter, though, we formulate a multi period mathematical programming model for a simplified network model based on the following assumptions. It clears a nodal water market in a catchment with a single long term storage reservoir, over a planning horizon, assuming known bids and inflows. Our market clearing model described here relates to the operation of the network in the short to medium term. Typically, an annual time horizon is assumed for the following developments<sup>20</sup>. A typical market optimization model seeks to maximise the producer (seller) and consumer (buyer) surpluses to maximise the net welfare to the society. The purpose of our mathematical programming based market is to allocate water resource to maximise the net welfare<sup>21</sup>.

We assume the catchment network has a single long-term storage reservoir (at the root) and a tree structure, which might extend downstream and/or upstream from the reservoir. Due to the tree-structure, each non-root node has a parent node, being the neighbouring node on the unique path towards the reservoir. Even though this chapter assumes a set of interconnected nodes radiating in a tree

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<sup>20</sup> System performance in the long run equilibrium is outside the thesis scope.

<sup>21</sup> A market is formulated assuming price taking agents and a market clearing agent maximises surplus at the marginal prices. Market water flow is determined by the joint impact of bids and gravity. However, the impact of the latter is not accounted for in the model.

structure from the reservoir, we illustrate some limited applications of this procedure for non-tree structures within a nodal network in Chapter 5. A node's parent may be either upstream or downstream, depending on how it is situated with respect to the reservoir. Moreover, assume there is no loss of water between periods (e.g., evaporation, leakages).

In a perfectly competitive market, consumers and producers would express bids reflecting their, marginal benefit of consumption, and offers reflecting their marginal cost of supply. For this thesis we assume that all bids and offers do this. The costs and benefits of consumptive and non-consumptive use at different locations are assumed to be independent of one another. That is, water use in one location does not affect costs and benefits in other locations. This single reservoir mixed-use multi-nodal problem can be formulated as an LP model<sup>22</sup>, given the user bids and offers form monotone decreasing piecewise constant marginal benefit functions as in electricity market clearing models. Following Starkey (2014), we form a set of net demand bids by concatenating the nodal demand bid set  $nb_D$  and the nodal supply bid set  $nb_S$  such that  $nb = nb_D \cup nb_S$ <sup>23</sup>. Then the final bid set,  $nb$ , is stacked in monotone, non-increasing price order to construct the net nodal demand curve. In Chapter 5, we will show how to construct a nodal net demand curve when several types of consumptive users (e.g., agricultural, distributary) exist at a node.

Note that the following formulation does not consider “consumptive” supply, although that extension is simple. Non-consumptive pumping bids can be either represented by assigning appropriate signs or declaring new variables for the marginal water value (to reflect the cost) and the quantity (to reflect flow direction). We follow the former approach when we discuss risk management aspects of pumping applications in the next chapter.

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<sup>22</sup> The LP is formulated as a “gross pool”, “uniform price” market which is independent from the initial position of property rights. That is the market requires the participants to bid for quantities starting from zero, ignoring any initial holdings. There will be equivalent bid and offers created for each participant based on her contract position including in the case of a participant with existing rights choose not to trade them in a market clearing round. The issue of settlement surplus discussed in Chapter 4.2.3 arises as a result of these net differences in trading.

<sup>23</sup> Net inflows to the reservoir and tributary flows to the downstream nodes are treated as zero cost supply offers. We defer discussing the issue of ownership of inflows and other resource constants appearing in the RHS of the constraints in the below formulation to Chapter 4.

Linear constraints represent reservoir storage balance, and arc flow and storage capacity bounds. Future inflows are exogenous parameters which are really stochastic and unknown, but are assumed to be known in the following deterministic formulation. We assume a non-pressurised water transportation network (e.g., channel/canal system), in which it is assumed to take no time to send water from an upstream node to a downstream one. Chapter 8 discusses transportation delay issues in a networked water system, but this chapter does not model transfer delays. It is assumed that water is traded over a long enough period of time that intra-period transport can be treated as more-or-less instantaneous. Further, this thesis will not consider water quality issues, other than indirectly through constraints on water flow or storage levels.

The first part of Section 3.3.1 declares the indices, parameters and decision variables used in the LP formulation. The dual variable(s) associated with each constraint are listed to the right of the (primal) constraints. Section 3.3.2 explains both primal and dual formulations, and how they determine prices in the water market.

### 3.3.1 Primal Formulation and Primal/Dual Variables

#### Nomenclature

##### *Indices*

$b = 1, 2, \dots, B$  : Usage (consumptive or non-consumptive) bid tranche (or step).

$i, j = 0, 1, 2, \dots, n$  : Nodes, where  $(i, j)$  denotes an arc belonging to the set  $A$ , of arcs in the network. Node 0 is the only long term storage reservoir in the network.

$k = 0, 1, 2, \dots$  : Non-consumptive users.  $\Omega$  denotes the non-consumptive user set for the network, where  $k \in \Omega(i, j) \subset \Omega$  indexes those non-consumptive users on arc  $(i, j)$ . The empty set  $\Omega(i, j) = \phi$  denotes no non-consumptive users on arc  $(i, j)$ . The notation  $(i(k), j(k))$  is used to indicate the arc for non-consumptive user  $k$ , so that  $k \in \Omega(i(k), j(k))$ <sup>24</sup>.

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<sup>24</sup> Two distinct  $k$  indices on the same arc represent two non-consumptive uses in series.

$m = 1, 2, \dots, M$ : End of horizon storage “bid” tranche. Here each participant could submit a set of bid tranches.<sup>25</sup> Otherwise we assume that the market manager uses the previous price information to estimate the marginal water value of storing water beyond the current planning horizon.

$t = 1, 2, \dots, T$ : Time period. Period  $t = 1$  is the current period (e.g., week).  $T$  denotes the last period of the planning horizon.

### *Parameters*

$\underline{X}_{ij}$ : Lower arc flow bound of arc  $(i, j)$ .

$\overline{X}_{ij}$ : Upper arc flow bound of arc  $(i, j)$ .

$P_{i,b}^t$ : Consumptive bid price for water demand quantity  $q_{i,b,t}$  at node  $i$ , tranche  $b$ , period  $t$ .

$P_{k,b}^t$ : Non-consumptive bid price for water flow demand quantity  $x_{k,b}^t$  on some arc, tranche  $b$ , period  $t$ .

$V_m^T$ : End-of-period bid price for storage demand quantity  $s_m^T$ , tranche  $m$ , period  $t$ .

$f_i^t$ : Uncontrollable tributary flows coming into node  $i$ , time period  $t$ .

$\bar{S}$ : Upper bounds on  $s_0^t$ , assumed constant for  $t = 1, 2, 3, \dots$ . Let the lower bounds on  $s_0^t$  is  $\underline{S} = 0$ .

$\bar{Y}_{k,b}^t$ : Upper bounds for non-consumptive bid tranches.

$\bar{Q}_{i,b}^t$ : Upper bounds for consumptive bid tranches.

$\bar{S}_{0m}^T$ : Upper bounds for the end-of-horizon bid tranches.

### *Decision Variables*

$q_{i,b}^t$ : Flow off-takes for consumptive/distributary use valued at bid price  $P_{i,b}^t$ , tranche  $b$ , in time period  $t$ .

$q_i^t$ : Total nodal flow off-takes in time period  $t$ .

$y_{k,b}^t$ : Non-consumptive arc flow use valued at bid price  $P_{k,b}^t$ .

$x_{i,j}^t$ : Flow through arc  $i \rightarrow j$  in time period  $t$ .

$s_0^t$ : End-of-period storage volume at the reservoir node 0 for time period  $t$ .

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<sup>25</sup> Starkey discusses the possibility of indicative bids but, for simplicity we will assume all bids to be binding.

$s_{0m}^T$ : End-of-horizon reservoir storage volume valued at  $V_m^T$ :

$z_i^t$ : Spill at node  $i$  in time period  $t$ .

### Model

*Deterministic offer-based nodal water market model (primal)*

$$\max_{x,q,s} \left( \sum_t \sum_i \sum_b P_{i,b}^t q_{i,b}^t + \sum_t \sum_{(i,j) \in A} \sum_{k \in \Omega(i,j)} \sum_b P_{k,b}^t y_{k,b}^t \right) + \sum_m V_m^T s_m^T \quad [3.1]$$

Subject to:

Flow balance at nodes (net flow injection):

$$q_i^t + \sum_{(i,j) \in A} x_{i,j}^t - \sum_{(j,i) \in A} x_{j,i}^t + z_i^t = f_i^t, \forall i \neq 0, t \quad : \lambda_{i,t} \quad [3.2a]$$

Storage flow balance:

$$\sum_{(0,j) \in A} x_{0,j}^t - \sum_{(j,0) \in A} x_{j,0}^t - s_0^{t-1} + s_0^t + z_0^t = f_0^t, \forall t \quad : \lambda_{0,t} \quad [3.2b]$$

Storage bounds:

$$0 \leq s_0^t \leq \bar{S}, \forall t \quad : \gamma_t^-, \gamma_t^+ \quad [3.3a]$$

Initial storage conditions:

$$s_0^0 = S_0^0 \quad : \gamma_0 \quad [3.3b]$$

Nodal spill bounds:

$$0 \leq z_i^t \quad \forall i, t \quad : \xi_{i,t}^- \quad [3.3c]$$

End-of-horizon storage conditions:

$$\sum_m s_{0m}^T = s_0^T \quad : \varsigma_{T+1} \quad [3.3d]$$

Bounds on end-of-horizon storage bid tranches:

$$0 \leq s_{0m}^T \leq S_{0m}^T \quad : \vartheta_{m,T}^-, \vartheta_{m,T}^+ \quad [3.3e]$$

Upper and lower arc flow bounds:

$$\underline{X}_{i,j} \leq x_{i,j}^t \leq \bar{X}_{i,j} \quad \forall t, (i,j) \quad : \mu_{(i,j),t}^-, \mu_{(i,j),t}^+ \quad [3.4a]$$

Arc flow non-consumptive use limit:

$$\sum_b y_{k,b}^t = x_{i(k),j(k)}^t \quad \forall t, k \in \Omega \quad : \omega_{k,t} \quad [3.5a]$$

Bounds on non-consumptive use bid tranches:

$$0 \leq y_{k,b}^t \leq \bar{Y}_{k,b}^t \quad \forall b, t, k \in \Omega \quad : \rho_{b,k,t}^-, \rho_{b,k,t}^+ \quad [3.5b]^{26}$$

Consumptive and distributary use:

$$\sum_b q_{i,b}^t = q_i^t \quad \forall i, t \quad : v_{i,t} \quad [3.6a]$$

Bounds on consumptive use bid tranches:

$$0 \leq q_{i,b}^t \leq \bar{Q}_{i,b}^t \quad \forall i, b, t \quad : \sigma_{b,i,t}^-, \sigma_{b,i,t}^+ \quad [3.6b]^{27}$$

### Explanation

[3.1] The objective function seeks to clear the water market by maximising the release benefits, plus future storage benefits, assuming that the bids represent the true economic contribution of the water used by activities across the network. The objective function coefficients  $P_{i,b}^t$  and  $P_{k,b}^t$  (participant bids) denote how much each incremental unit of water is worth for the consumptive and the non-consumptive users<sup>28</sup>.

Discounting could be used to represent the present value of future water allocation. Hotelling's rule, which is frequently used in the environmental economics literature, would then imply prices for water rising over time, at the assumed discount rate.<sup>29</sup> However, in this formulation, this aspect is assumed to be an issue for the bidders. That is, it is assumed that participants would discount their own bids.  $\sum_m V_m^T s_m^T$  is also assumed to be an appropriately discounted piecewise linear, increasing, convex, end of horizon benefit storage function. It represents the

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<sup>26</sup> For simplicity, the non-consumptive tranche bounds in the nodal CDDP discussed in Chapter 5 are:

$$0 \leq y_{k,b}^t \leq 1$$

<sup>27</sup> Similarly, the consumptive tranche bounds in the nodal CDDP are:  $0 \leq q_{i,b}^t \leq 1$

<sup>28</sup> Note that later chapters are motivated by the solution of the intra-period problem of the standard dynamic programming problem, which amounts to the first term of the convex programming problem above, for some particular period,  $t$ . This represents both consumptive (node) uses and non-consumptive (arc) uses, across the network.

<sup>29</sup> Manning & Gallagher (1982) discussed the importance of time preferences in reservoir operating policies. They argued that the marginal value of stored water should be rising at a rate of prevailing interest rate, based on the rule introduced by Hotelling (1931) and later applied for exhaustible resources by Solow (1974). This rule is normally applicable over multiple years but, strictly speaking it should really apply whenever storage is not at its bounds. Water values will rise faster than the interest rate when the upper storage bounds are binding, and fall when the lower storage bounds are binding.

value that would be achieved by the market if water is carried forward, beyond the current planning horizon.

[3.2] This constraint describes the flow balance at any node.  $s_0^t$  in constraint [3.2b] represents the storage volume in the reservoir at the end of time period  $t$ , and features only in the reservoir nodal flow balance constraint equation for node 0.  $\lambda_{i,t}$  and  $\lambda_{0,t}$  indicate the shadow prices of the two constraints, that is the marginal water values at the node.

[3.3] Constraint [3.3a] sets the storage bounds, which have dual variables  $\gamma_t^-$  and  $\gamma_t^+$ . The initial storage level is set by constraint [3.3b], which may be seen as a special case of [3.3a] in which both  $\bar{S}$  and the minimum bound are set to equal  $S_0^0$ .

In reality, “spill” often flows down the same channels as “release”,<sup>30</sup> and the model could handle this type of spill by relaxing the upper arc flow limits, while perhaps setting negative environmental values. Alternatively, the SO, or an environmental agency, could be assumed to impose a penalty scheme on flows outside the desired bounds. But [3.3c] also allows for an unconstrained spill which is treated by 3.2a&b as exiting (or not entering) the system, and this avoids the possibility of negative water values if a flood occurs.

A final storage target could be represented similarly to the initial storage level but, once we consider inflow uncertainty, it will not generally be feasible, or desirable, to establish a fixed end-of-horizon storage target. So constraint [3.3d] instead assumes a set of (possibly notional) “bids” for the end-of-horizon storage. Hence,  $\varsigma_T$  is the dual variable of [3.3d] and it represents the marginal value of carrying water forward. Constraint [3.3e] represents the bounds on end-of-horizon storage bid tranches (e.g.,  $\vartheta_{m,T}^-$ ,  $\vartheta_{m,T}^+$ ).

[3.4] The total flow through any arc  $(i,j)$  in the network in time  $t$  must be less than the physical capacity of that arc.  $\mu_{(i,j),t}^-$  and  $\mu_{(i,j),t}^+$  represent the dual variables on the constraint [3.4]. This is the marginal gain to the market from an extra unit of capacity on arc  $(i,j)$  in time period  $t$ .

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<sup>30</sup> We will ignore the possibility that spill may flow down alternative channels to re-enter the system downstream. Spills are assumed to be non-utilisable flows and they have a zero marginal value.

- [3.5] Constraints [3.5a] and [3.5b] relate to non-consumptive use<sup>31</sup>. Constraint [3.5a] aggregates accepted bid tranches to compute the quantities of flow allocated to each non-consumptive use.  $\omega_{k,t}$  indicates the shadow price on [3.5a]. This implies that a bidder must express her bid curve for the total arc capacity range sometimes with zero bids. This constraint allows the model to accept both positive and negative bids. Negative bids (if they exist) can offset positive bids. Constraint [3.5b] represents the bounds on non-consumptive bid tranches, and has two dual variables,  $\rho_{b,k,t}^-$  and  $\rho_{b,k,t}^+$ . Different non-consumptive users on the same arc are allowed and assumed to each gain value from the same flow of water.
- [3.6] This denotes the set of constraints for consumptive and distributary demands. [3.6a] aggregates the total consumptive/distributary demands at any node  $i$ , and the associated dual variable is denoted by  $v_{i,t}$ . [3.6.b] sets the upper and lower bounds of the consumptive/ distributary bid tranches. The quantity accepted in each bid tranche cannot be negative.  $\sigma_{b,i,t}^-$  and  $\sigma_{b,i,t}^+$  indicate the respective dual variables on these bounds.

Note that, as stated, the lower bounds in equations [3.3c], [3.4], [3.5b], and [3.6b] would have negative dual variables. But we will implicitly assume that these lower bounds are re-stated in a canonical way, so as to produce non-negative shadow prices<sup>32</sup>. So we have:  $-z_0^t \leq 0 : (-\xi_t^- \geq 0)$ ,  $-x_{i,j}^t \leq L_{i,j} : (-\mu_{(i,j),t}^- \geq 0)$ ,  $-y_{k,b}^t \leq 0 : (-\rho_{b,k,t}^- \geq 0)$ ,  $-q_{i,b}^t \leq 0 : (-\sigma_{b,i,t}^- \geq 0)$ ,  $-s_0^t \leq 0 : (-\gamma_t^- \geq 0)$  and  $-s_m^T \leq 0 : (-\vartheta_{m,T}^- \geq 0)$ . Therefore, non-negative values for  $\xi_{i,t}^-$ ,  $\mu_{(i,j),t}^-$ ,  $\rho_{b,k,t}^-$ ,  $\sigma_{b,i,t}^-$ ,  $\gamma_t^-$  and  $\vartheta_{m,T}^-$  will be assumed in the following section.

Section 3.3.2 presents the full dual formulation of the above problem, and explains each dual variable and constraint, to give insight into the resource prices that match the optimal market clearing solution.<sup>33</sup>

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<sup>31</sup> This formulation accounts for arc non-consumptive use in a catchment network. But, we do not cover any non-consumptive use of reservoirs or at other nodes.

<sup>32</sup> The shadow prices multiplied by the respective quantities provide a basis for resource valuation which will be discussed in Chapter 4.

<sup>33</sup> We can form an LP dual here because we dealt with non-linearity by assuming piece-wise linear convex benefit functions and linear constraints. However, a Wolfe dual (Lasdon, 1970) could be used, had we persisted with the non-linear  $V^T(s^T)$ .



### 3.3.2 Dual Formulation and Shadow Price Analysis

The dual formulation of the above primal problem can be thought of as the commodity valuation problem. The primal variables associated with the dual constraints are listed on the right hand side.

$$\begin{aligned} \min_{\mu, \lambda, \sigma, \rho, \gamma, \xi, \nu, \omega, \varsigma} & \left( s_0^0 \gamma_0 + s_0^T \varsigma_{T+1} + \vartheta_T^+ \sum_m s_{0m}^T + \sum_t \sum_{(i,j) \in A} (\bar{X}_{(i,j)} \mu_{(i,j),t}^+ - \underline{X}_{ij} \mu_{ij,t}^-) + \sum_t \sum_i f_i^t \lambda_{i,t} \right. \\ & \left. + \sum_t \sum_{k \in \Omega} \sum_b \bar{Y}_{k,b}^t \rho_{b,k,t}^+ + \sum_t \sum_i \sum_b \bar{Q}_{i,b}^t \sigma_{b,i,t}^+ + \sum_t (\bar{S} \gamma_t^+) \right) \end{aligned} \quad [3.7]$$

Subject to:

$$-v_{i,t} + \lambda_{it} = 0, \forall i, t \quad : q_i^t \quad [3.8]$$

$$-P_{i,b}^t + \sigma_{b,i,t}^+ - \sigma_{b,i,t}^- + v_{i,t} = 0 \quad : q_{i,b}^t \quad [3.9]$$

$$-\sum_{k \in \Omega(i,j)} \omega_{k,t} + (\mu_{(i,j),t}^+ - \mu_{(i,j),t}^-) + \lambda_{i,t} - \lambda_{j,t} = 0, \forall i, j, t \quad : x_{(i,j)}^t \quad [3.10]$$

$$-P_{k,b}^t + \rho_{b,k,t}^+ - \rho_{b,k,t}^- + \omega_{k,t} = 0 \quad : y_{k,b}^t \quad [3.11]$$

$$\lambda_{0,t} - \lambda_{0,t+1} + \gamma_t^+ - \gamma_t^- = 0 \quad : s^t \quad [3.12a]$$

$$\gamma_T^+ - \gamma_T^- + \lambda_{0,T} - \varsigma_T = 0 \quad : s^T \quad [3.12b]$$

$$-V_m^T + \vartheta_{m,T}^+ - \vartheta_{m,T}^- + \varsigma_T = 0 \quad : s_m^T \quad [3.12c]$$

$$\lambda_{i,t} - \xi_{i,t}^- = 0 \quad : z_i^t \quad [3.13]$$

The following section analyses the above dual constraints and price relationships. Later, Chapter 4 uses these dual variable relationships to examine some market issues such as network revenue, settlement surpluses, and hedging.

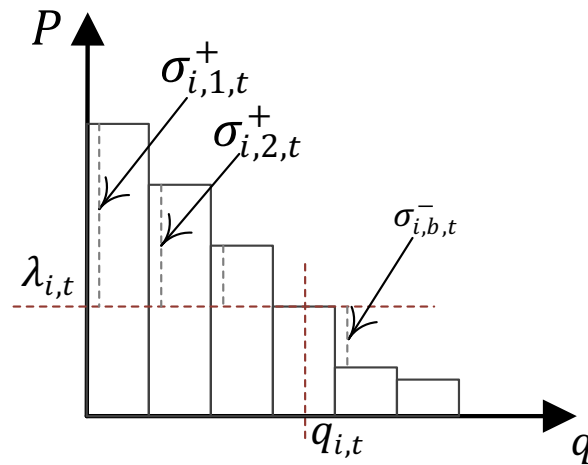
### 3.3.3 Deterministic Price Analysis

$\sigma_{b,i,t}^-$  and  $\sigma_{b,i,t}^+$  denote the shadow prices on the consumptive/distributary bid constraint [3.6b]. The bid price  $P_{i,b}^t$  ( $b^{\text{th}}$  bid of the consumptive use at node  $i$ ) will set the price for that period and location if, and only if, that bid tranche is marginal i.e. the optimal allocation lies between its upper and lower bound (ignoring degeneracy). In that case, the shadow prices on both bounds are zero. Otherwise, the dual variable  $\sigma_{b,i,t}^+$  is the amount of extra social benefit gained if a consumptive or distributary user at node  $i$  had use for an additional unit of water at a marginal value equal to the specified bid price  $P_{i,b}^t$ . In other

words, the dual variable  $\sigma_{b,i,t}^+$  is the social benefit loss if the consumptive use valued at  $P_{i,b}^t$  was restricted by one unit. Similarly, the shadow price  $\sigma_{b,i,t}^-$  of the consumptive bid lower bound constraint indicates how much society would lose if that bid was accepted. The dual constraint associated with the primal variable  $q_{i,b}^t$  describes the relationship between the consumptive bids and the nodal market prices:  $-P_{i,b}^t + \sigma_{b,i,t}^+ - \sigma_{b,i,t}^- + v_{i,t} = 0$ . Hence, the following price relationships can be derived from consideration of dual constraint [3.9]. For example, combining dual constraints [3.8] and [3.9], given complementary slackness, we can write the following relationship if the consumptive bid is fully accepted (i.e.,  $q_{b,i}^t = Q$ ) then  $\sigma_{b,i,t}^- = 0$ , so  $-P_{i,b}^t + \sigma_{b,i,t}^+ + \lambda_{it} = 0$ . If a bid is not accepted at all (i.e.  $q_{b,i}^t = 0$ ) then  $\sigma_{b,i,t}^+ = 0$ , so  $-P_{i,b}^t - \sigma_{b,i,t}^- + \lambda_{it} = 0$ . Finally, if the bid is partially accepted (i.e.,  $0 < q_{b,i}^t < Q$ ) then  $\sigma_{b,i,t}^+, \sigma_{b,i,t}^- = 0$ , and  $\lambda_{it} = P_{i,b}^t$ .

In summary,  $\sigma_{b,i,t}^+ > 0$  (only) for a fully accepted consumptive/distributary user bid, and  $\sigma_{b,i,t}^- > 0$  (only) for a fully rejected consumptive/distributary user bid. Here, the nodal price  $\lambda_{it}$  indicates the market price determined by the net nodal demands, and is thus applicable to the consumptive use at a particular node. Figure 3-1 illustrates the merit order allocation of the consumptive marginal water values ( $P_{i,b}^t$  bids) and the accepted/non-accepted bids. The market (nodal) price corresponding to  $q_i^t$  is shown by the checked horizontal line. According to Figure 3-1 consumer (buyer) surplus is:

$$\sum_i \sum_b \sigma_{b,i,t}^+ q_{i,b}^t.$$



**Figure 3-1.** Consumptive marginal water values (bid stack) at node  $i$ .

The flow off-take demands differ depending on the location in the network and the type (e.g., consumptive versus distributary) of the use. The dual variable  $\lambda_{it}$  for the constraint [3.2] represents the marginal value to the market (or system) from having an extra water unit available at node  $i$ , in period  $t$ , given the optimal water allocation. This is similar to locational marginal prices in an electricity or gas market. The locational marginal price (LMP) in an electricity market refers to the lowest price at which an incremental unit of power (energy) can be optimally dispatched to a node, without violating the binding network transmission constraints. In this context, the nodal (spot) price reflects how much the objective function would increase if an extra unit of (free) tributary flow was available at that node, in that particular time period. That value could be set by consumptive/ distributary use, there and then. But it could equally well reflect the value added if the water flowed on to be allocated to a downstream user, or was used to replace an extra unit consumed by an upstream user, in that time period. Or, it could reflect the marginal value of consumption in a later period, even at nodes where storage is not possible, because the extra unit could flow on down to the reservoir, or replace a unit that would otherwise have been released from the reservoir.

For non-consumptive uses,  $\rho_{b,k,t}^+$  and  $\rho_{b,k,t}^-$  denote the dual variables for the non-consumptive bid upper/lower bound constraints [3.5b]. The shadow price  $\rho_{b,k,t}^+$  of the non-consumptive bid upper bound constraint [3.5b] indicates how much the “social benefit”, as measured by the objective function, would increase if non-consumptive user for  $k \in \Omega$  could utilise an additional unit of water flow at a marginal value of  $P_{k,b}^t$ .<sup>34</sup> As above, these dual prices can be explained with reference to the full dual formulation.

The dual constraint [3.11] associated with the primal variable  $y_{k,b}^t$  explains the relationship between the prices of non-consumptive use and bid steps in the non-consumptive arc flow demand function:  $-P_{k,b}^t + \rho_{b,k,t}^+ - \rho_{b,k,t}^- + \omega_{k,t} = 0$ . Using duality and complementary slackness conditions we can write the following.

If the non-consumptive bid is fully accepted, i.e., if  $y_{k,b}^t = Y_{k,b}^t$  then  $\rho_{b,k,t}^+ > 0$  and  $\rho_{b,k,t}^- = 0$ . Hence,  $-P_{k,b}^t + \rho_{b,k,t}^+ + \omega_{k,t} = 0$ . Hence  $\rho_{b,k,t}^+$  is the price difference between the bid price  $P_{k,b}^t$  and the

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<sup>34</sup> Here, and elsewhere, the gain in “social benefit” from relaxing a bid tranche constraint would actually accrue to the individual party making the bid.

‘water flow’ price for  $k$ ,  $\omega_{k,t} \cdot \rho_{b,k,t}^- > 0$  implies the lower bounds on non-consumptive use bid tranche is binding. This indicates how much the society would lose if one unit were accepted from that bid. Therefore, if a bid is not accepted, i.e., if  $y_{k,b}^t = 0$ , then  $\rho_{b,k,t}^+ = 0$ ,  $\rho_{b,k,t}^- > 0$  and  $-P_{k,b}^t - \rho_{b,k,t}^- + \omega_{k,t} = 0$ . Finally, the price relationship  $\omega_{k,t} = P_{k,b}^t$  corresponds to a partially accepted bid, where  $0 < y_{k,b}^t < \bar{Y}_{k,b}$  and  $\rho_{b,k,t}^+, \rho_{b,k,t}^- = 0$ .

The dual constraints [3.12a] and [3.12b] correspond to the storage variables.  $\gamma_t^+ > 0$  corresponds to a binding upper storage limit (where  $\gamma_t^- = 0$ ). So  $\gamma_t^+$  positively contributes to the next period’s marginal water values, indicating a rise in the marginal water value as we pass a period in which we expect the upper storage limit to constrain the solution.  $\gamma_t^- > 0$  represents a binding lower storage limit ( $s_0^t \leq 0$ ) which then lowers the marginal water values in the next time period, as we pass a period in which we expect the lower storage limit to constrain the solution. Most of the time, we will have non-binding storage constraints, so  $\gamma_t^+, \gamma_t^- = 0$ , and the marginal water value does not change.

In dual constraints [3.12 b & c], unless storage limits bind we have  $\lambda_{0,T} - \varsigma_{0,T} = 0$  and  $-V_m^T + \vartheta_{m,T}^+ - \vartheta_{m,T}^- + \varsigma_T = 0$ . If an end-of-horizon storage target had been set, equation [3.12b] would merely set the marginal cost, in the current year, of meeting that target. However, because this study values end-of-horizon storage and storage volume that will be carried beyond the current planning horizon, that constraint says that the marginal water value in this year equals the marginal benefits of carrying water over beyond the current planning horizon, to the next year.

Equation [3.13] relates to the cost of spill to the market.  $\xi_t^-$  measures how much value is added to the system by relaxing (decreasing) the spill bounds by an extra unit (e.g., -1)<sup>35</sup>.  $\xi_t^- > 0$  implies the lower spill bound is binding. Thus  $\lambda_{0,t}$  is positive, unless water is being deliberately spilled to waste. When spill occurs we will have  $\xi_t^- = 0$ , and  $\lambda_{0,t} = \xi_t^-$  implies zero marginal water value.

The dual variable  $\mu_{(i,j),t}^+$  of the arc flow capacity constraint [3.4] for  $x_{(i,j)}^t$  measures how much the benefit maximising objective function would increase if the capacity of arc  $(i,j)$  could be increased by a unit, in period  $t$ . The dual variable  $\mu_{(i,j),t}^+$  is analogous to a “line shadow price” in an electricity

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<sup>35</sup> Note that  $\lambda_{0,t}$  could become negative if an upper bound on spill was to bind. But there is no such bound, and hence no possibility of a positive  $\xi_{i,t}^+$  shadow price being added on the LHS of 3.13 so as to force  $\lambda_{0,t}$  below zero.

market, which denotes the dispatch cost saving by increasing the transmission line flow capacity incrementally by a unit. The shadow price  $\mu_{(i,j),t}^-$  of the constraint [3.4] indicates how much the objective function would increase if the lower bound could be relaxed (e.g., by not meeting minimum in-stream flows) by a unit, in period  $t$ . Thus  $\mu_{(i,j),t}$  explains the difference between water values at connected nodes, driven by the fact that users on, or downstream of, the arc would make an incremental profit of  $\mu_{(i,j),t}^+$  if they were allowed to increase the arc flow capacity by a single unit of flow, without violating other binding constraints. Conversely, a penalty of  $\mu_{(i,j),t}^+$  per unit increase in flow would be just enough to discourage those users violating the upper arc bounds. Hence,  $\mu_{(i,j),t}$  is a market price which gives an efficient allocation of arc capacity, maximising the trading benefits.

$\sum_{k \in \Omega(i,j)} \omega_{k,t} = \lambda_{i,t} - \lambda_{j,t}$  implies that if there is more than one non-consumptive user on the same arc, there can be multiple values of  $\rho_{b,k,t}^+$  and  $\omega_{k,t}$  that satisfy this and the relationships here when the bids are fully accepted. This issue is addressed in the following.

The dual constraint  $-\sum_{k \in \Omega(i,j)} \omega_{k,t} + (\mu_{(i,j),t}^+ - \mu_{(i,j),t}^-) + \lambda_{i,t} - \lambda_{j,t} = 0$ , [3.10], indicates the relationship between nodal prices, arc flow non-consumptive use prices, and arc flow capacity constraint price. Let us ignore the non-consumptive rental component in the above equation for the moment. Then, it will provide us a simple relationship between the nodal prices and the arc flow constraints for an arc  $(\mu_{(i,j),t}^+ - \mu_{(i,j),t}^-) + \lambda_{i,t} - \lambda_{j,t} = 0$ . In this case, the shadow price on the arc flow limit is just the nodal price difference  $(\lambda_{j,t} - \lambda_{i,t})$ . Alternatively, in an unconstrained network, the constraint becomes:  $-\sum_{k \in \Omega(i,j)} \omega_{k,t} + \lambda_{i,t} - \lambda_{j,t} = 0$ . This implies that all non-consumptive users on the arc  $(i,j)$ , collectively, pay the price difference between the two nodes for the flow. The shadow price sum  $\sum_{k \in \Omega(i,j)} \omega_{k,t}$  represents the combined marginal benefit for all non-consumptive participants. This reflects the fact that their use of the arc flow is complementary, with all bidders gaining from the same flow unit (recall the non-consumptive users are assumed to be in series on the arc). As above, the model is forced to accept negative bids with the equality constraint. This can occur if negative bids are sufficiently offset by positive bids to create an aggregate gain, after accounting for the upstream/downstream water value differential. This implies that  $\omega$  has no sign restriction. As a result,  $\omega$

may become negative if the water value differential is positive. That is, the market could accept some cost of water transport, in order to move water to a node where it has a higher value.

While this shadow price sum needs to match the nodal price difference, when there are multiple non-consumptive users on the same arc there can be multiple optimal dual solutions each corresponding to a different allocation of the nodal price difference between different non-consumptive users.

The following section considers allocating costs in this situation, where arc flows yield joint benefits, creating a “free-riding problem” similar to one occurring in electricity markets.

### 3.4 Allocating Costs Associated with Arc Flows Yielding Joint Benefits

Non-consumptive users in series on an arc may gain more total benefit for water than reflected in the combined price they pay. This leads to multiple feasible allocations of the cost amongst those users.

Under the convexity assumption, equation [3.10] implies that each non-consumptive user should be paying in accordance with the marginal benefit the optimization allocates, irrespective of how much benefit they might have derived from infra-marginal release units. Thus in cases where  $\sum_{k \in \Omega(i,j)} \omega_{k,t} < \sum_{k \in \Omega(i,j)} P_{k,b}^t$  some non-consumptive bidders may pay nothing, because they become infra-marginal, in the sense that other users want, and are prepared to pay for, higher flow levels than such a user can economically utilise, or is prepared to bid for<sup>36</sup>. This is particularly likely if there are minimum stream flow releases for which the costs are implicitly incurred by, or on behalf of, the environmental users. The market implicitly charges the price only to the marginal user(s)<sup>37</sup>. This could provide gaming incentives for some parties to strategically evade their contribution to the dual prices associated with the

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<sup>36</sup> As a result of multiple dual solutions, there could be multiple ways of allocating the water values amongst non-consumptive users in series on the same arc. Preferably, the market is required to find a fair way to allocate water values consistently. A non-consumptive user in series with others on an arc can become marginal depending on the optimal dual solution chosen.

<sup>37</sup> The marginal user sets the price. Then all consumptive users at the same node pay the same price. However, with interior solutions, there will be no marginal users but each user pays non-negative prices less than their bid price.

capacity constraints, thus effectively taking advantage of their physical positioning in the network, relative to users who are prepared to pay more to increase flows.<sup>38</sup>

A market with a single non-consumptive user and a (large) set of consumptive users at each node could be expected to behave as a perfectly competitive market. Their bids are expected to reflect the marginal benefit of consumption to each participant. This creates a collection of prices at the nodes and arcs of a simple network that permit trades to take place. However, the correspondence of a perfectly competitive market with multiple non-consumptive users per arc cannot be guaranteed. This situation is somewhat analogous to the situation with respect to ancillary services (e.g., contingency reserve to cover breakdowns and frequency keeping) in electricity markets, in that common costs need to be allocated among the users. Many markets recognise energy and reserve as non-independent commodities (Read, 2010). For example, the New Zealand market accepts joint supply offers for energy and ancillary services (e.g., reserve) from the same capacity. Then an energy and ancillary service co-optimization model seeks to minimise total costs. In a co-optimization model, all ancillary service suppliers are recognised, and all receive the marginal price, but the optimisation only recognises one, or a few, largest contingencies as marginal ancillary service “consumers”.

Taken at face value the economic dispatch models alone would assign all ancillary service purchase costs to these few marginal contingencies, and would not provide any degree of cost sharing among the consumers in proportion to their benefits. However, in reality, participants (e.g., generators) are, in some sense, “consumers” of ancillary services, since all are liable to break down at some point. In reality, there is a common benefit situation, for which some kind of group-based cost sharing rule seems reasonable and the electricity market literature refers to several approaches to avoid the issue of “free riding”. Not surprisingly, these show affinities with group-based “incentive compatible” regimes discussed in the broader literature<sup>39</sup>.

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<sup>38</sup> Although note that this is already true, without a market, in that non-consumptive users have traditionally enjoyed free use of ‘environmental’ flows.

<sup>39</sup> Sometimes mutual cost sharing agreements may satisfy the requirement of being “fair and equitable”. For example, Read (1997) recommends freely negotiated settlements when establishing a transmission pricing regime with less regulatory intervention.

For example, in many electricity markets (e.g., Australia, New Zealand or Singapore), the reserve operating costs are allocated among the generators by a “runway” cost allocation approach, similar to the way in which runway construction costs are allocated among users according to a “club principle” (Littlechild & Owen, 1973; Littlechild & Thompson, 1977) depending on the air strip length use by different size carriers during their landing and taking-off. This is similar to a game theoretic approach that apportions the total cost/benefits of the shared resource among participants who cooperate on shared resources. For example, the Shapley value method is one popular method referred to in that literature (see Junqueira et al, 2007). It implicitly solves the cost-cause identifying problem (Tsukamoto & Iyoda, 1996), and is intended to give a “fair chance” of each participant facing both adverse and favourable situations<sup>40</sup>. That is, the right incentives will be offered to a group of participants, but not to an individual. In the runway charging formula, all plane types share the construction cost of the runway length required to handle the plane type with the shortest take-off/landing requirement. Then all but that type will face the incremental cost of construction for the incremental length required to handle the plane type with the next shortest take-off/landing requirement, and so on, until the plane type requiring the longest runway faces the incremental cost of runway construction, beyond the length required by any other plane type. Then, a fair and efficient landing fee schedule can be worked out based on the number of landings of each type.

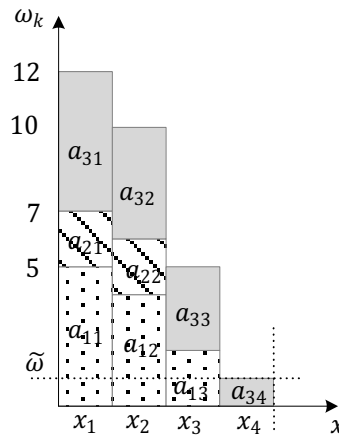
Similarly, in the electricity sector, the largest unit operating will face the incremental costs of contingency response cover, not required by any other unit, but will bear the common costs of covering lesser contingencies with smaller units. Thus, a fair and efficient fee schedule can be worked out for plant generating in a particular time period, based on the size and expected number of contingencies. Here we suggest a similar approach to apportion the non-consumptive infra-marginal benefits in the

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<sup>40</sup> For example, Faria *et al.*, (2009) studied a firm energy right allocation problem among hydro-power plants using the Shapley value method. Murphy & Rosenthal (2006) have cited several other applications of the Shapley value to apportion resources in telecommunication networks (Anandalingam & Nam, 1997; Archer et al, 2004), in shareholder voting games (Zingales, 1995), in pollution reduction and emission control (Carbone & Sweigart, 1976; Chattopadhyay, 1995) and in water resource allocations (Dinar et al, 1992; Kucukmehmetoglu & Guldmann, 2004; Becker & Easter, 2010).



water market context, as described in the following example. For illustrative purposes, let us assume a set of non-consumptive users  $k$  on the arc  $(i, j)$ .



**Figure 3-2.** Example of a non-consumptive arc flow bid stack belong to a set of arc flow users on the arc  $(i, j)$ .

Figure 3-2 shows the bids expressed by the non-consumptive users 1, 2, and 3 for the arc flow quantities  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$ . For example, the non-consumptive users 1, 2 and 3 value the first  $x_1$  units at  $a_{11} = 5$ ,  $a_{21} = 2$ , and  $a_{31} = 5$  respectively. According to this diagram, the non-consumptive user 3 becomes the only marginal user at the arc flow clearing price  $\tilde{\omega}$ , (That is with an arc flow clearing price of  $a_{34} = 1$ ). But all users enjoy consumer benefits on the first increment, in proportions,  $a_{11} : a_{21} : a_{31}$ . Therefore the purchase cost allocation implied by the dual of the optimisation problem could easily be considered “unfair”, and likely to invite gaming. That is, users will restrict their bids so as not to be the user demanding the ‘last flow’<sup>41</sup>, and hence deemed responsible for all costs. According to the above example, user 3 is the marginal user, and hence responsible for relevant costs, as she has exclusively bid for the  $x_4$  tranche. And that will be true if the market clears anywhere in this tranche. Alternatively, user 1 and user 3 would have shared the costs had user 1 reduced her bid so that the market cleared, say, just below the top of the  $x_3$  bid tranche. And cost burden faced by both these participants will reduce by another there will be another increment, if they can get the market to clear just below the top of the  $x_2$  tranche, and so on. Thus, users will set their demand curves so as to back away from being the marginal bidder. There is ambiguity as to how to apportion the clearing price (say “1”) among the three users. For example, if there was only one bidder for  $x_1$  bid tranche, she would

<sup>41</sup> Here the last flow (marginal flow) refers to the position of the user bids in the final bid stack.

simply pay clearing price “1”. However, in the above example, there are 3 bidders for  $x_1$  bid tranche and hence the payment of “1” can be made in one of the following ways. User 3 who is marginal at  $x_4$  will be required to accept the full amount if there are no other non-consumptive users in parallel. The market clearing price is payable by the highest bidder at  $x_1$  if there are no multiple highest bids. To avoid this effect, though, the flow purchase cost must be apportioned among the participants for the deterministic case in proportion to the consumer benefits for each flow increment, as shown below:

$$\left[ \left( \frac{5}{12}x_1 + \frac{4}{10}x_2 + \frac{2}{5}x_3 + 0x_4 \right), \left( \frac{2}{12}x_1 + \frac{2}{10}x_2 + 0x_3 + 0x_4 \right), \left( \frac{5}{12}x_1 + \frac{4}{10}x_2 + \frac{3}{5}x_3 + x_4 \right) \right]$$

For example,  $\left( \frac{5}{12}x_1 + \frac{4}{10}x_2 + \frac{2}{5}x_3 \right)$  denotes the flow purchase cost proportions for the first participant.

These bid apportionments dis-incentivise the participants to play a game for the last unit because the costs/benefits are now redistributed among each participant based on an accepted rule<sup>42</sup>. Thus the cost allocation adjusts continuously, and there is no distinct advantage for any bidder to restrict her bids so as not to become marginal. The above Shapley value based method can be used to determine a “fair cost” of each participant’s contribution in joint work. However, the participants may choose not to cooperate if the market outcomes do not meet their final expectations.

It may be argued that this weighted average consumer surplus approach is inconsistent with the uniform price market clearing assumption and that, in allocating costs based on infra-marginal rents, it possesses some features of the pay-as-bid auction method<sup>43</sup>. The choice between pay-as-bid and uniform price water auctions leads to theoretical and/or empirical questions that need deeper analysis, beyond the scope of this thesis<sup>44</sup>. And it will not explore the theoretical properties of the weighted average

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<sup>42</sup> Suppose User 1 reduces her bid for unit  $x_1$  to 4 from 5. Now her portion is 4/11. Still the circumstances could make it less likely for unit  $x_1$  to be marginal. She makes this gain at a lower risk. Similar game could be played by the other users to increase their gain but the level of risk of respective unit become marginal could be different.

<sup>43</sup> The financial contracts developed in the next chapter assuming uniform-price market clearing do not seem appropriate if multiple parties gain non-consumptive benefits from the same flow on the same arc as they bid to cover part of the cost.

<sup>44</sup> The literature refers to several studies about the choice between, and applicability and consequences of uniform price and pay-as-bid electricity market auctions. See, for example, Wolfram, 1999; Kahn et al, 2001; Klemperer, 2001; Rassenti et al., 2002; Fabra et al, 2002.

consumer surplus approach any further, either<sup>45</sup>. However, note that this problem, effectively, exists if there are multiple serial non-consumptive users on a single arc. In order to eliminate this problem, this arc can be split into multiple arcs by adding dummy nodes between the non-consumptive users on the original arc to accommodate a single non-consumptive user per new arc<sup>46</sup>.

The next section recasts the objective in Equation (3.1) to maximise net welfare temporally, spatially and under uncertainty. It considers net demand bids and it allows for state-dependent bidding, correlated to the system hydrology.

### 3.5 Impact of Uncertainty

The previous section formulated a single reservoir, mixed-use deterministic LP optimization model and provided some explanation of the shadow prices. A stochastic market formulation is more complicated, and we have to consider expected benefits from release and period-to-period storage.

We use an event tree formulation approach to set up the multi-stage stochastic market optimization problem. All possible realizations of inflows form a scenario tree which has nodes organized in levels that correspond to stages (time periods)  $1, 2, \dots, T$ . We assume a general scenario tree and thus its structure can be labelled using parent-child relationships. For each event node there corresponds a particular hydrology state. For example, the root node is denoted by  $h = 1$  at level  $t = 1$ . Thus, every event node in the scenario tree will have a unique label  $(h, t)$ , and all primal variables in the above deterministic formulation are indexed by event node,  $(h, t)$ . We refer to the set of event nodes immediately subsequent to event node  $(h, t)$  for  $t = 1, 2, \dots, T - 1$  as the set of child nodes  $\mathfrak{c}(h, t)$  at level  $t + 1$ . Each node  $(h, t)$  at level  $t = 2, \dots, T$  is linked to a single ancestor  $\mathfrak{a}(h, t)$  node at level  $t - 1$ .

The scenario tree divides into branches corresponding to different realizations of the random inflow events. It thus defines the historical path from root to node  $h$  in stage  $t$  in the event tree by  $(h_1, h_2, \dots, h_{t-1})$ , omitting the period indices. Let  $\mathbb{p}^{(h,t)}$  denote the probability of reaching event node

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<sup>45</sup> But we note that the problem at hand is not really an “auction”, in which each participant buys a distinct part of the total on offer at a price, uniform or otherwise. It is more of a joint-product situation, and such situations always require some method to allocate purchase/production costs among beneficiaries.

<sup>46</sup> There could be overlapping basic/dual solutions leading to degeneracy issues.

$(h, t)$ . The nodal probability is the product of corresponding conditional probabilities<sup>47</sup> along the historical path.

In reality, participants would not be willing to or be able to express contingent bids for every node in the scenario tree. We could think of notional bids estimated by the market manager, in situations where participants do not provide full information for all scenarios in all periods. These bids are hydrology dependent and conditioned on the previous events, as described by the scenario tree<sup>48</sup>. The details on developing these bid estimates are set aside as the scope of this chapter is to present a market clearing formulation suitable to discuss the issues such as risk management rather than to study various market design options and mechanisms for running a water market. The stochastic objective function in the primal model is a probability weighted sum of the terms, which now becomes:

$$\begin{aligned} \max_{x, q, s} \sum_{(h, t) \in \mathcal{H}_t} \mathbb{P}^{(h, t)} \sum_t \left( \sum_i \sum_b P_{i, b}^{h, t} q_{i, b}^{h, t} + \sum_{(i, j) \in A} \sum_{k \in \Omega(i, j)} \sum_b P_{k, b}^{h, t} y_{k, b}^{h, t} \right) \\ + \sum_{(h, T) \in \mathcal{H}_T} \mathbb{P}^{(h, T)} \sum_m V_m^{h, T} s_m^{h, T} \end{aligned}$$

$\mathcal{H}_t$  denotes the set of all event nodes in time period  $t$ ,  $\forall t = 1, 2, 3, \dots, T$ . We also need to re-state the primal constraints using the corresponding  $(h, t)$  index. In addition, we scale the dual variables associated with the primal constraints (e.g., from [3.2] to [3.6]) by multiplying each by the associated nodal probability. For example, the storage balance constraint can be re-written as follows:

$$\sum_{(0, j) \in A} x_{0, j}^{h, t} - \sum_{(j, 0) \in A} x_{j, 0}^{h, t} - s_0^{h, t} + s_0^{h, t} + z_0^{h, t} = f_0^{h, t}, \forall (h, t) : \mathbb{P}^{(h, t)} \lambda_{h, t}$$

At stage  $t$  the storage decision  $s_0^{h, t}$  can be thought of as a function of the history of storage/release decisions available at time  $t$ , and does not depend on future observations of the child nodes. As formulated, information regarding the intra-period inflow is assumed to be available for both future storage and present arc flow release decisions made in the current period. This corresponds to an informed inflow allocation policy (Starkey, 2014) and this implies that the release could be adjusted

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<sup>47</sup>  $\mathbb{P}^{(h, t)} = \mathbb{P}(h_1) \cdot \mathbb{P}(h_2|h_1) \cdot \mathbb{P}(h_3|h_2) \dots \dots \mathbb{P}(h_t|h_{t-1})$  describes the probability of a scenario  $h$  at level  $t$ .

<sup>48</sup> Demand for water increases as conditions get drier, and vice versa. As a result, high valued bids are likely to be allocated more water in a competitive constrained system.

accordingly as the inflow for the coming period is known. The only change to the price relationships comes in the inter-period connections (specifically, dual constraints [3.12a & b]) which, when generalised, link marginal water values for this period with probability weighted conditional average marginal water values for the next period.

Under informed policy, the stochastic version of the dual constraint [3.12a] corresponding to storage constraint  $s^t$  is:

$$\mathbb{P}^{(h,t)}(\lambda_{h,t} + \gamma_{h,t}^+ - \gamma_{h,t}^-) - \sum_{(h',t+1) \in \mathbb{C}(h,t)} \mathbb{P}^{(h',t+1)} \lambda_{h',t+1} = 0, \forall (h,t) \quad :s_0^{h,t} \quad [3.14a]$$

The probability ratio  $\frac{\sum_{h \in \mathcal{H}_{t+1}} \mathbb{P}^{(h,t+1)}}{\mathbb{P}^{(h,t)}}$  indicates the conditional probability of event  $h'$  occurring in time period  $t + 1$  given that event node  $(h, t)$  eventuated. Thus, the marginal water value of storage under scenario  $h$  in period  $t$  (i.e., event node  $(h, t)$ ) is equal to the probability weighted sum of all marginal water values of the event nodes in level  $t + 1$  that are linked to  $(h, t)$ . In general, under a stochastic setting,  $\lambda_{h,t}$  is the marginal water value in scenario  $(h, t)$  as a result of hydrology dependent inflows. This implies that the accounting occurs at the end of the period (that is at period,  $t$ ) as there is no prior knowledge during period  $t - 1$  about the scenario that will be occurring in period  $t$ .

Note that it is not necessary to have binding storage constraints to induce price differentials in a stochastic setting. The storage constraints still influence the (conditional) marginal water values for release irrespective of the type of formulation (stochastic versus deterministic), and the roles of the multipliers  $\gamma_t^+$  and  $\gamma_t^-$  remain essentially unchanged in a stochastic formulation<sup>49</sup>. In particular, the marginal water value can only change when the storage trajectory for that path reaches a storage bound. Apart from this, though, most of the price relationships can be derived from the deterministic LP model. For example, we could think of the scenario tree as populated with identical information for all forward

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<sup>49</sup> This may be seen more clearly if we apply the scenario decomposition to the SLP, in which copies for all variables and constraints are made for each scenario. We then need to declare the non-anticipativity constraints for the storage variable. If a pair of scenarios  $u$  and  $v$  in the tree, are identical up to time  $t$ , the non-anticipativity storage constraint is:  $s_u^t = s_v^t$ . We re-cast the associated dual variable as  $\lambda_u^t$ , and these dual variables link with those before/after in the scenario, via the storage bound multipliers, exactly in accordance with the deterministic dual equation, [3.12a] above. The path based modelling based revenue adequacy for financial storage rights is discussed in Read (2016).

paths nodes assuming  $\sum_m V_m^T s_m^T$  perfectly models the future value function. Alternate paths radiating from a node can be collapsed to a single path. Thus, temporal  $\lambda_0$  variables can be assumed to be correct with respect to the original model. We also limit ourselves to deterministic LP models to explain the network rents as it does not explicitly involve temporal variations of  $\lambda_0$ , and discuss some commercial and organizational arrangements which could be used to deploy “settlement surpluses” to hedge against trading risks, improve liquidity and ultimately increase efficiency of short-run use as discussed in the next chapter.

### **3.6 Chapter Conclusions**

In this chapter, we focused on presenting a market-clearing formulation to manage hydrology dependent surface water supplies, where consumptive and/or non-consumptive use occurs in a network, with storage. Participant bids, presumably reflecting their marginal use values, are assumed to be cleared by a benefit-maximizing optimization, such as Linear Programming. The aggregated net demand curve reflects the maximum quantity that each participant is willing to supply or consume, at a particular marginal cost/benefit. Section 3.3 lays out a deterministic multi-period LP primal/dual centrally coordinated water market clearing model. This helps us to present a price analysis and discuss issues such as allocating costs associated with shared resources and impacts of uncertainty. In Section 3.5, a centrally coordinated market clears water under uncertainty, based on marginal water values expressed by participants.

The above creates both locational and temporal price differences, and causes the market to accumulate a “settlement surplus” of rents associated with resource constraints such as storage bounds, arc flow bounds and inflow as further described in the next chapter.

# 4.

## Financial Hedging Instruments for Water Markets

### 4.1 Introduction

The previous chapter sets the background for the developments in this chapter by presenting a multi-period deterministic/stochastic water market model of the operation of a network in a series of relatively short intervals (e.g., weekly) within an annual time horizon. In the primal and dual LP formulations of that chapter, the resource (i.e., storage volume and inflows) was assumed to be traded in both current and future periods. Here we will simply assume that a market exists in financial water rights, at each location, and is cleared by solving a deterministic/stochastic formulation such as that described in Chapter 3. In reality, many decisions with respect to releasing and storing water are made across time and space, under uncertainty. As a result, markets make decisions regarding some resource components in the current period (e.g., flow delivery), knowing that subsequent choices regarding other resource components (e.g., storage) will be required in future, when the situation becomes clearer.

As noted in the previous chapter, a market for water can create a complicated situation given there is gaming and risk faced by participants due to the tight interaction between the upstream and the downstream parties. It is unlikely that participants accept the idea of trading water in a spot market unless there are contracts in place to deal with issues such as gaming, loss their current rights and risk

exposure. So this chapter explores some financial contracting ideas drawn from electricity market literature<sup>50</sup>.

Starkey (2015) ignores the locational detail introduced here, and hence any consideration of non-consumptive uses, but discusses many possible market options for trading some form of future water right. Ideally, contingent bids/offers for water seem conceptually appealing, and Starkey discusses affine bids comprised of fixed and proportional components. The proportional component (volume) can be scaled linearly with one or several indices depending on the inflows between the current and the future periods. Starkey also discusses the possibility of trading affine FWRs too. But, to avoid modelling difficulties, here we assume that, irrespective of the bid form, a market exists in “firm” (i.e. fixed volume) financial water rights, at multiple nodes, and is cleared by solving a deterministic multi-node formulation such as that described in Chapter 3, or the stochastic variant described in Section 3.5. Critically, it is assumed that water flow is strictly determined by the market clearing solutions. Thus water flows will only occur if downstream consumptive users express positive bids for water consumption, and/or non-consumptive (e.g. hydropower users express positive bids for water flow).

As discussed in Section 4.2.3, clearing such a market will generate a revenue surplus, implicitly capturing the rents associated with different types of constraints. For example, some limitations of allocation of water will occur as a result of transportation capacity constraints within a period, and other limitations will occur as a result of storage capacity constraints between periods. According to the market formulations in the previous chapter, such limitations will cause marginal price differences over the arcs across the network, and across time. And, as we have seen in Section 3.3.2, price differentials exist even over unconstrained arcs, because of non-consumptive arc flow use.

Thus the market accumulates rents associated with the resource constraints.<sup>51</sup> As in in other resource allocating network markets, the SO may collect the rents generated by some particular types of binding

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<sup>50</sup> Contracts can secure the existing position of participants transitioning into a market environment, and underpin market operations. This is similar to allocating vesting contracts to participants in electricity markets at the commencement.



constraints, thus creating a “settlement surplus”. This would generally expose both the SO and participants to inter-temporal risks depending on the amount of water carried forward.

Ideally participants prefer certainty about their future allocations and costs. But, in reality, with a constrained uncertain resource, these price differences will not be certain, as they depend on the hydrological conditions. Hence participants might wish to hedge against spatial and temporal price variations in the spot market by entering into some form of contract. Otherwise, uncertainty in the rents associated with network constraints may make some participants unwilling to engage in trading unless insurance is available.

To the extent that participants own their own physical systems they would manage those risks by taking physical actions, such as storing water, or moving it. To the extent that they rely upon a common system, though, they need to be provided with other “instruments” to manage those risks. This leads us to ask how to manage “settlement surpluses” and then to explore the possibility of introducing some commercial arrangement that could be supported by the settlement surplus, in order to hedge against trading risk<sup>52</sup>. The market is faced with the following issues regarding these rents and surpluses:

- Who to allocate the rents to?
- How to interpret different types of rents associated with market/system constraints? and
- How to utilise settlement surpluses to fully/partially insulate themselves from trading risks by “hedging”?

Briefly, the issue of who actually “collects” the rents largely depends on the market design, but the ultimate recipients should be the owners of system components. The interpretation of rents is determined by the physical and economic roles played by the corresponding system components. And, consequently, settlement surpluses can be decomposed into rental components that provide opportunities for the participants to hedge their trading risks in the market.

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<sup>51</sup> Technically, the RHS of each constraint represents the amount of some limited “resource”, such as channel or reservoir capacity, available to the system, and the rents represent the value of that resource, as determined by the dual solution discussed above.

<sup>52</sup> In the case of bilateral financial/physical contracts, the contract holders can swap their positions and effectively cancel each other out. The surplus is important in unbalanced situations where physical resources must be able to physically support the financial contracts.

The main objective of this chapter, then, is to establish a general structure of financial hedging instruments that could be used to deploy the settlement surplus arising as a result of water market clearing, to hedge against inter-temporal and inter-locational price risks in such a market. In particular, our scope is limited to exploring rent-based risk management concepts for water markets, in the form of (potentially tradable) financial property rights corresponding to physical system components. This chapter draws on the Financial Transmission Right (FTR) concepts developed for electricity markets, and discussed in Section 2.5.3, to describe and discuss similar water market hedging instruments. It is worth noting that the scope of this thesis does not include issues relating to the allocation, valuation, or trading of such instruments, or their impact on “gaming behaviour”.

This chapter is organised in the following way:

Section 4.2 sets the stage. It describes the water market environment, how financial water rights operate, and the market settlement process. We explain how marginal price differences across time and space causes the market to accumulate a “settlement surplus<sup>53</sup>” of rents associated with resource constraints (see Section 4.2.3). Then, Section 4.2.4 presents a range of financial property rights applicable to different components of the reservoir system that could be used to re-distribute rents accumulated in the settlement surplus among relevant parties.

Section 4.3 develops Obligation-inclusive Financial Storage Rights and Financial Storage Options for the storage system. Then we investigate the issue of inter-temporal revenue adequacy of these storage related hedging instruments.

Then, Section 4.4 describes transportation network rents (referred to as network rents). It describes both Options and Obligation-inclusive Rights for Arc Capacity Flow that would enable the right holder to be compensated for the flow transportation rents collected by the market. Four different scenarios involving consumptive benefits, consumptive benefits and non-consumptive costs, consumptive benefits and non-consumptive benefits and flow losses and delays, and investigates the issue of revenue adequacy for each scenario.

Section 4.5, considers an alternative “swing option” based virtual reservoir management approach in which a water swing option should cover initial storage, inflows, storage capacity and release.

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<sup>53</sup> A settlement surplus is created when there is a positive difference between receipts and payments.

Finally, Section 4.6 presents a chapter summary.

## **4.2 The Water Market Environment and Financial Water Rights**

The previous chapter describes a formulation for the multi-period deterministic storable water market clearing problem. Starkey (2014) extends that kind of formulation to cover general multi-period stochastic optimization problem and then explains a spectrum of modelling options. In all cases the market manager has to run the model to clear the water market in each period. The simplest market would just use a deterministic optimization model to allocate water between participants in the current period, once the market manager had determined an allocation to be reserved for future periods. In other cases water trading would also be conducted, or simulated, in advance. A deterministic multi-period optimization model could support a market trading both present and future water, or it could just estimate the future stored water value. A multi-period deterministic optimization with forward simulation could provide future price/quantity estimates for a range of scenarios, and Chapter 4 of Starkey (2014) explains how those estimates could be used as a basis for trading future water, or just to inform trading in futures markets. The same is true if the market manager runs a forward stochastic optimization given the hydrology information, to form a general scenario tree. Starkey discusses various ways in which such a regime might be made workable, perhaps using bids and/or contracts that depend on hydrology indices.

If a stochastic market clearing provides at least indicative price-quantity information for the possible future inflow realizations modelled in the scenario tree, then participants can estimate the price volatilities they expect to face. And they could make out-of-market arrangements (e.g., financial contracts) to hedge their risks using the market information, without the market manager's intervention or knowledge.

But Starkey also discusses using the stochastic optimisation more directly to clear a market in future water contracts. Key issues include the nature and form of the contracts that might be traded, and the ways in which the quantities to be traded could be determined from the stochastic "market-clearing" optimisation, if they are.

If not otherwise constrained, a stochastic optimisation of the form discussed in Section 3.5 will actually determine a different market-clearing quantity to be traded for every event node. But, while that kind

of “conditional” solution might indicate how the market could clear under various future scenarios, it would not really be implementable, in the form of a contract to be entered into now. While we could define the quantity to depend on the scenario, the reality is that none of those exact scenarios will actually occur. Starkey suggests that the best approximation would be a contract in which the quantities are defined in proportion to some hydrology index. He also discusses ways in which the stochastic optimisation could be constrained to produce market-clearing trades in such contracts, in a formulation utilising the hydrology indices implied by the scenarios.

A question remains, though, as to which hydrology indices might be used. Should it be a short term index representing current inflows, or a longer term exponentially weighted index representing cumulative inflows? Or might both be used in the same market? For simplicity we discuss only two variants here: Fixed volume Financial Water Rights (FWRs) for long term water trading; and Financial Inflow Rights (FIRs) defined in proportion to a short term inflow index.

#### 4.2.1 Financial Water Rights

While Starkey (2014) provides a more detailed discussion of options for trading water, as above, most of our discussion in this chapter will simply assume the existence of a market in “fixed volume” Financial Water Rights (FWRs). If the market-clearing price for water in storage 0 at the end of period  $t$  is  $\lambda_{0,t}$  then an obligation-inclusive FWR defined for a quantity of water  $Q_t$  in that storage, at the end of that period, at an agreed price  $SP_t$  pays out:

$$FWR(Q_t, SP_t) = (\lambda_{0,t} - SP_t)Q_t.$$

Thus this is a generic kind of (financial) “fixed” volume market trading instrument that can be used to protect against exposure to fluctuations in the price of water stored in the reservoir (as opposed to water released to the market) in any future time period  $t$ .<sup>54</sup>

Under the informed policy assumption, the exact amount of FWRs in the end-of-period and spot market clearing volumes will be known by the SO. Final settlement for a participant in a “single node” market holding an  $FWR_t(Q_t, SP_t)$  can be written as follows:

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<sup>54</sup> See Section 4.4.3 below for discussion of how that price might differ from that of water delivered to a node, and how that difference in value is covered by a FDR.

$$q_t \lambda_t - Q_t(\lambda_t - SP_t) = Q_t \cdot SP_t + (q_t - Q_t)\lambda_t$$

Here  $\lambda$  and  $q$  denote market clearing price and cleared quantity.

#### 4.2.2 Financial Inflow Rights

Inflow uncertainty makes future storage demands stochastic, but that is true of electricity flows, too. An inflow at the reservoir in time period  $t$  is similar to a nodal power injection in the electricity economic dispatch model. Hence these inflow risks will be hedgeable using an instrument based on inflow rents.

The term  $f^t \lambda_{0,t}$ <sup>55</sup> of the dual objective function in Section 3.3.2 refers to the surpluses that will accrue on inflow supplies at a node in time  $t$ . We will define property rights for the inflow related settlement surpluses, which we refer to as Financial Inflow Rights, FIRs.

Unlike other “capacity” volumes discussed in later sections, the inflow volume available is uncertain. In principle, fixed volume FIRs could be defined as analogues to CfDs in electricity markets, and used to hedge against spot price uncertainty at a particular time and place. Thus, a participant could have a financial contract to buy a pre-arranged quantity of inflow, at a pre-arranged strike price, and then buy matching FSR storage capacity rights (see below) to perfectly hedge a known withdrawal requirement in some future period. The pay-out from  $FIR_{0,t}(q_{0,t}, SP_{0,t})$ , giving the holder the right (and obligation) to “receive” quantity  $q_{0,t}$  of the flows arriving at reservoir node in period  $t$  will be defined as:

$$FIR_{0,t}(q_{0,t}, SP_{0,t}) = (\lambda_{0,t} - SP_{0,t})q_{0,t}$$

But, the problem with fixed volume FIRs is that the SO will be exposed if the “expected” inflows do not occur. While these hedge against spot *price* uncertainty at a particular time and place, they leave the holder facing *volume* uncertainty. In that sense they are like the line “shareholdings” proposed by Read and Sell (1989). Instead, this section defines index based FIRs of the type discussed by Starkey (2014). Those shareholdings were like FGRs discussed in Section 2.5.3, in the sense that they were defined in terms of the Lagrangian multiplier associated with particular flow constraints, but they were defined as proportions of available capacity, rather than as a fixed MW capacity. But FIRs are much simpler, and more intuitive. In general, they could be similar to weather derivatives, and be based on a variety of

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<sup>55</sup> But, to avoid modelling difficulties, we will only consider the inflows at reservoir node.

indices. But here we will assume that the “hydrological index” is just the reservoir inflow volume, so they simply apportion that volume amongst the FIR holders.

Note that these FIRs are being priced at the same value as FWRs, per unit of volume. They really only differ in that the volume is proportional to the inflow index,  $H^{h,t}$ . In other words the FIR is just a right to receive a volume of FWRs, with that volume being proportional to the inflow received. This is appropriate if all inflows can be totally directed to the storage without any non-consumptive benefit, and without turning any away as spill. Inflows arriving at another node could be priced differently, with FDRs being used to hedge the locational marginal price difference, as discussed in Section 4.4.3

These index-based FIRs ensure the inflows, and inflow risk, belong to the participants. Hence, the SO would be insulated from the risks of inflow fluctuations. The participants, on the other hand, would be accepting all the risks themselves, and would have to manage their physical consumption strategy, and/or exercise of the system capacity rights discussed below to manage those risks. And this creates a situation very much like that created by the “virtual models” approach discussed by Barroso et al (2012), and in Section 4.5.

#### **4.2.3 Settlement Surplus**

Before we proceed to in-depth investigations on inter-temporal marginal price and inter-locational marginal price hedging instruments, it is important to see how the settlement surpluses are created over the spatially and temporally distributed arcs and nodes. As discussed in Section 2.5, the balance of funds remaining in the electricity market pool account as a result of net receipts across the network is known as the settlement surplus or congestion rent. It accrues when the network is constrained or congested and the participants need to pay the marginal cost of congestion. Typically, an FTR pays part of the congestion rents receivable by the ISO back to the holder. As a result, a surplus of funds is created for the ISO if there is an excess of congestion rent over payments to FTR holders, and this may be re-distributed to market participants. In the case of a networked water market, the only “settlement surplus” arising from the purely intra-period market clearing would relate to the arrival of water into the system, and the movement of water within the system, in that period. But there would also implicitly be a profit, or potentially a loss, generated as the water carried forward was re-valued between periods.

That profit would not actually be collected by the market manager, though, until the water carried forward was actually bought by market participants, at those (hopefully) higher prices. So the market manager would face significant uncertainty with respect to the size of any profit, and possibly be in a position to lower that risk by issuing hedging instruments.

If future water was bought and sold in an integrated inter-temporal market, and the reservoir operated in accordance with market outcomes, there would be a formal “settlement surplus” arising from inter-temporal water trading. The market manager still faces significant risk, in an uncertain world, but at least the surplus can be formally analysed in terms of the shadow prices on storage bounds. This section will assume that kind of market arrangement, and discuss financial instruments that can be supported by the revenue collected by the market manager as settlement surplus, just as FTRs are supported in electricity markets. Suppose that  $\tilde{q}_{i,b}^t$  and  $\tilde{y}_{k,b}^t$  denote cleared consumptive/non-consumptive quantities. Then, the settlement surplus will be:

$$\sum_t \sum_i \sum_b \lambda_{i,t} \tilde{q}_{i,b}^t + \sum_t \sum_{(i,j) \in A} \sum_{k \in \Omega(i,j)} \sum_b (\lambda_{j,t} - \lambda_{i,t}) \tilde{y}_{k,b}^t$$

Here the amounts  $\sum_t \sum_i \sum_b \lambda_{i,t} \tilde{q}_{i,b}^t$  (LMP revenues) and  $\sum_t \sum_{(i,j) \in A} \sum_{k \in \Omega(i,j)} \sum_b (\lambda_{j,t} - \lambda_{i,t}) \tilde{y}_{k,b}^t$  (arc flow revenues) can be interpreted as the SO’s revenue (i.e., paid by the consumptive and non-consumptive users respectively), after clearing the market. Any settlement surpluses due to price differentials induced by the binding constraints will contribute positively or negatively to the above revenue depending on different ownership regimes. For example, in the real world, a participant in the market can also be an asset owner (e.g., hydropower generator owns long term storage and/or canals). In this particular case, though, it is assumed that the supply system (i.e., storage, tributaries and inflows) and transportation system (i.e., channels), are “owned” by the market manager, rather than by any party trading in the market. If the system “owns” the inflows/tributaries cost them at zero, the market is accumulating all these revenues, with an adjustment for the value of starting and closing stocks.

$S_0 \gamma_0$  in Equation [3.7] denotes the initial valuation of the storage at the beginning of the planning horizon, while  $s_0^T \zeta_{T+1}$  in Equation [3.7] denotes the final valuation of the storage at the end of the

planning horizon. The term  $\sum_m V_m^T s_m^T$  of the primal objective function expresses the piecewise linear end-of-horizon storage value function in the primal formulation together with the end-of-horizon storage block limits in constraint [3.3d].  $\theta_T^\pm \sum_m s_m^T$  is the end-of-horizon (piece-wise linear) constraint rent component for future/contingent bid tranches in the dual objective function. If there are real future bids for end-of-horizon water in storage, and the SO settles participant accounts at the end of each market horizon, there will be actual payments to/from participants contributing to the settlement surplus.<sup>56</sup> These will cancel out, though, if the SO merely clears the end-of-horizon market between participants, and takes no net position itself. More generally, the SO could be exposed to spot market risk in each period when there is a storage imbalance in the absence of hedging instruments. In particular, she will definitely face a loss when prices fall as no adequate storage capacity to carry water forward.

#### 4.2.4 Overview of Hedging Instruments

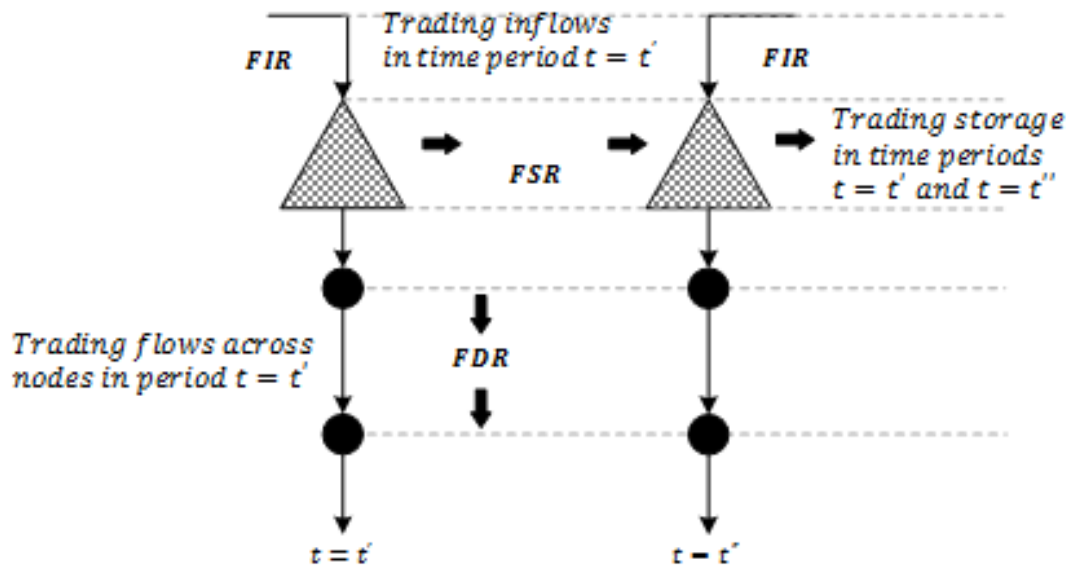
Section 4.2.2 above has already discussed the concept of FIRs, based on inflow rentals. Sections 4.3 and Section 4.4 below, complete the picture by describing hedging instruments designed to complement the FWR market by creating and trading property rights associated with storage capacity and network flow capacity, and investigate revenue adequacy issues associated with those products.

As shown in Figure 4-1, Financial Inflow Rights (FIRs), Financial Storage Rights (FSRs) and Financial Delivery Rights (FDRs) can be defined to hedge against risks associated with trading of inflows, water in storage, inter-temporal storage, and flow transportation across network, respectively.

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<sup>56</sup> With ‘indicative’ bidding (Starkey, 2014) for the end-of-horizon water, or SO-estimated penalty values, the SO would be facing a risk of collecting end-of-horizon settlement surpluses because pool account balances only will be accrued as expected future receipts. However, under steady state conditions, we could assume the closing stock to have approximately the same value and/or level as the opening stock. Hence, unless participants can bid for end-of-horizon stock, the initial and end-of-horizon stock valuations may only be used for internal purposes, and will cancel off each other in the long run, without implying any commercial risk for the SO.





**Figure 4-1.** Structure of water market hedging instruments

The following table presents spectrum of the hedging instruments discussed in various sections of this chapter, and their associations.

**Table 4-1.** Spectrum of water market hedging instruments

System Component	Hedging Instruments	Relevant Section
Inflow	Financial inflow rights	Section 4.2.2
Storage	Financial storage capacity rights	Section 4.3.1
	Financial storage capacity options	Section 4.3.3
Transportation network	Financial Delivery Rights	Section 4.4.

### 4.3 Supply System Rents and Rights

This section interprets the “supply system” rent components, associated with stored water (Section 4.3.1), inflows (Section 4.3.2) and storage capacity (Section 4.3.3) in the dual objective function, and then goes on to discuss the issue of corresponding property rights. These rents can be explicitly handled by defining separate accounts for each constraint, as in the case of financial flow gate rights. Settlement of rents in successive spot market clearings can be implicitly done using periodic accounting.

In principle, the users should be willing to pay the expected value of these supply system rents, plus some risk premium, for rights to receive water and/or use the system. And, in the absence of scale economies, the payments they are willing to make should be able to fund infrastructural upgrading in

the long-term under optimal expansion conditions. In the context of electricity networks, Read & Sell (1988) noted that rents actually collected on transmission capacity constraints, after expansion occurs, will only partially offset the costs of capacity expansion, under realistic scale economies. Read (1989, 1997) discusses an ex ante contracting regime designed to overcome this deficiency, and a similar approach would apply in water markets, but that is beyond our present scope. Here, though, this section will ignore the longer term implications, and merely consider the role of rents as support for hedging instruments.

#### 4.3.1 Financial Storage Rights

This section proposes financial storage rights to hedge against the inter-temporal storage price risks. The  $\bar{S}\gamma^+$  and  $\underline{S}\gamma^-$  terms in the dual objective function represent the rents associated with the storage constraints. The corresponding settlement surplus could be used to support an FTR-like instrument that we will refer to as a “Financial Storage Right” (FSR). These are similar to the gas market FTRs in Read et al., (2012)<sup>57</sup> but FSRs are less complicated, because “storing” and “transporting” can be more “cleanly” distinguished from each other for water than for gas, where both occur in the same pipeline.

These FSRs could be seen as representing a virtual pipe that enables flow transfers (within the storage) between two time periods  $t'$  and  $t''$ . For example, if the inflows are ignored, participants might own a share of the initial stock, and use FSRs to cover the risks involved in storing that stock until it is consumed. But the initial stock will be exhausted after some time, and uncertain inflows to the storage in the future time periods will create risk unless someone buys a contract to receive those inflows, as well as FSRs to then store them to be released in different periods. We could create index-based FIR rights corresponding to those uncertain and variable inflows as described in a previous section. The storage capacity itself is deterministic, though, and thus similar to the transmission line capacities in an electricity market. And, by analogy, we would like to be able to determine feasible FSR allocation

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<sup>57</sup> As discussed in Section 4.4.5 of Read et al, gas market FTR implementation could be relatively complex due to significant transit flow delays, with non-linear flow compressibility equations implying that the transportation network works as a temporary storage. Thus the FSR/FTR concepts discussed here are less easily distinguished, in that context.

patterns over the planning horizon. Then users may trade both water and FSRs separately, or perhaps in a combined market. But revenue adequacy needs careful consideration. There is a clear incompatibility here, between indexed FIRs and non-indexed FSRs, but that incompatibility merely reflects the physical incompatibility between uncertain inflows and certain storage capacity. Thus we will develop a fixed volume FSR concept here, discuss the implications of that incompatibility in Section 4.3.3 on “alterative concepts”.

Most FTRs are defined in the “obligation-inclusive” form (two-sided obligation), in which the “pay-out” is implicitly defined by the inter-nodal price difference, and can be positive or negative<sup>58</sup>. That form could be used for FSRs too. An FSR can be labelled as  $FSR(t', t'')$ , paying out at  $(\lambda_{0,t''} - \lambda_{0,t'})$  per unit of water, to hedge the risk implied by the inter-temporal price difference on a specified storage volume between  $t'$  and  $t''(> t')$  (e.g., from winter to summer). That is, it effectively hedges the transaction of selling water into the reservoir, then buying water out of the reservoir later.

As noted in Starkey (2014), water market clearing can range from simple clearing for a single period to a more complex clearing for the multiple stages in a scenario tree. Here, the latter is assumed, in which case rents are explicitly ‘generated’ by the market clearing solutions when the upper/lower bounds are binding at the end of any period, in any scenario, in the multi-period water market clearing, thus creating an (expected) marginal water value difference. To further simplify we will assume that the lower bound is constant, so that the problem can be transformed to make the lower storage bound zero, thus avoiding some secondary issues related to rent generated by changes in the non-zero lower bounds. Under these conditions we might expect that the SO can only sell a maximum FSR volume of  $(\bar{S})$ <sup>59</sup>.

Revenue adequacy is a tricky issue, though. In electricity markets, an FTR hedges a specific volume (in MW) transmitted between nodes, in a single period. There is no carryover of current flows from one period to another. The FTR settlement is simple and straightforward, as there is no uncertainty yet to

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<sup>58</sup> Often just referred to as an “obligation FTR”.

<sup>59</sup> More generally, the rent associated with the upper storage bound is  $\sum_t (\bar{S} - \underline{S}) \gamma_t^+$ , and the SO could issue an FSR volume of  $(\bar{S} - \underline{S})$ . Varying  $\underline{S}$  would mean that the SO would need to do an FIR adjustment every time the lower bound changes.

be resolved, at the time settlement is made. This means that the electricity market ISO is not exposed to any residual revenue adequacy risk. Similarly, there would be no risk in a deterministic situation, and hence no risk associated with a deterministic “FSR settlement”. But the situation is different under uncertainty.

Our inter-temporal FSRs will hedge a specific volume stored between time periods, at a single node (i.e., at the reservoir). Settlement on the inter-temporal nodal FSRs will occur based on market prices reflecting expected water values determined at the time when the FSR commences, and when it concludes. But conditional expected marginal water values change in every period, reflecting changes in the probability that the storage trajectory will reach the storage bounds, and this will create market price differences between periods, even when storage is well away from the bounds. In the dual, though, storage bound rents will only be generated when storage bounds are actually active, and carryover of storage means that the true marginal water values for each period will only be known in retrospect, when the full path is known as described in Section 4.3.2.1.

So the water market SO does need to be concerned about the risk when settling an inter-temporal FSR under uncertainty, for two reasons. First, as noted earlier, the fact that rents are being “generated” when storage bounds are active, does not mean that they are “collected” in those periods. So a market manager who settles an FSR using rents “generated” in a market clearing solution would be doing so on the assumption that those rents will eventually be collected, in the course of future water market transactions. In the real world, that means that the market manager may face a significant and ill-defined non-collection risk. But we can at least define and discuss that kind of risk if we assume (simplistically) that the uncertainty faced by the market is completely described by the scenario set in the event tree formulation. This means that stochastic market-clearing optimisation actually only needs to be solved once, in order to determine the quantities and prices applying in all possible future states. So, while the market manager does not know which scenario path will occur, it does at least know, from the beginning, what revenue it will receive in each future situation it could face, and can pre-calculate conditional expected values for each node. It will still be in the situation, though, of paying out a water

value difference when neither the initial nor final water values are actually known, only their (conditional) expected values.

This means that the notion of inter-temporal revenue adequacy for FSRs needs to address the issue of the residual risk faced by the SO when making FSR pay-outs. As we will see, revenue adequacy is only guaranteed in expectation. We will develop a set of propositions regarding revenue adequacy in a stochastic setting, but first discuss the issue for FSRs in a (hypothetical) deterministic environment:, where:

- If no storage limits are binding during the interval  $(t', t'')$ , then there is no related settlement surplus contributing to the support of  $FSR(t', t'')$ . But then there would be no price difference either, and hence no pay-out required. In a stochastic environment we might imagine FSR sales for that period generating positive revenue for the SO. But, if the future really were known to all parties, no-one would see any point in paying positive prices for FSRs whose value was known to be zero.<sup>60</sup>
- If the upper limit is binding at some point during that interval, it will create a positive rent, adding positive value to the FSR, reflecting the fact that water will be worth more after the constraining period than before. Similarly, if the lower limit binds at some point during the interval  $(t', t'')$ , it will add a negative value to the FSR. In that case, though, (with a zero lower bound) there will be no negative rent, since no water is carried over in an empty reservoir.
- So we can derive a multi-period (deterministic) relationship between FSR pay-out (per unit) and accumulated rents (per unit) by repeatedly applying constraint equation [3.12b] to the periods from  $t'$  to  $t''$ , we get:

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<sup>60</sup> Actually the whole concept of hedging, and FSR trading, breaks down under deterministic assumptions, because all parties value all instruments at exactly what they would have to pay for them. But this discussion is helpful in setting up our understanding of dual relationships in the real situation, where uncertainty is obviously the critical issue.

$$\begin{aligned}
& \left( (\lambda_{0,t'} + \gamma_{0,t'}^+ - \gamma_{0,t'}^-) - \lambda_{0,t'+1} \right) + \left( (\lambda_{0,t'+1} + \gamma_{0,t'+1}^+ - \gamma_{0,t'+1}^-) - \lambda_{0,t'+2} \right) + \\
& \left( (\lambda_{0,t'+2} + \gamma_{0,t'+2}^+ - \gamma_{0,t'+2}^-) - \lambda_{0,t'+3} \right) + \dots + \dots + \left( (\lambda_{0,t''-1} + \gamma_{0,t''-1}^+ - \gamma_{0,t''-1}^-) - \right. \\
& \left. \lambda_{0,h,t'',t''} \right) = 0
\end{aligned}$$

Re-arranging the above:  $\lambda_{0,t''} - \lambda_{0,t'} = \sum_{t=t'}^{t=t''-1} \gamma_{0,t}^+ - \sum_{t=t'}^{t=t''-1} \gamma_{0,t}^-$ .

Using this, in the deterministic case, if the SO has sold a total of  $Q \leq \bar{S}$  units of obligation-inclusive  $FSR(t', t'')$ , the pay-out is  $Q(\lambda_{0,t''} - \lambda_{0,t'})$ . Then, using the above we have:

$$Q(\lambda_{0,t''} - \lambda_{0,t'}) = Q \left( \sum_{t=t'}^{t=t''-1} \gamma_{0,t}^+ - \sum_{t=t'}^{t=t''-1} \gamma_{0,t}^- \right) \leq \bar{S} \sum_{t=t'}^{t=t''-1} \gamma_{0,t}^+$$

So, for the deterministic case, the FSR pay-outs are covered by the rents generated, and thus the SO will not face any revenue adequacy issues.

However, water is subject to uncertainty affecting both demand and supply. In principle, most participants prefer some form of certainty about their future water allocation, though. In particular, participants will, on average, want to increase storage draw-down to maintain adequate supply in dry conditions. However, future allocations cannot be made certain for all participants, in reality, and the centralised market clearing authority will look to decrease withdrawal in that situation. Revenue adequacy of FSRs under uncertainty could be a major issue, and in the next section, we show that it can only be guaranteed in expectation.

#### 4.3.2 Inter-temporal FSR Revenue Adequacy Problem

Now, in a stochastic environment, suppose  $\lambda_{0,h,t}$  denotes the marginal water value at the storage node as a function of the period and scenario node index. That is, it is the conditional expected marginal water value at node  $(t, h)$  in our event node formulation. Hereinafter this section omits network node indexing (e.g.,  $\lambda_{h,t}$  denotes  $\lambda_{0,h,t}$ ) to reduce notational complexity.

Consider an FSR contract from time period  $t'$  to  $t''$ . The pay-out occurs once we have observed market prices in both periods  $t'$  and  $t''$ . More specifically, a multi-stage FSR pays out the actual market price difference across the inter-period sub-scenario actually realised. However, the market price in each period is based on the expected value of the range of prices which could be seen over all possible

outcomes, from that period forward. And evolving expectations can make the market price differ between  $t'$  and  $t''$  even if the storage bounds remain slack throughout the sub-scenario, and so no rent is generated. However, on average, the price difference can be supported by the rents generated on the storage constraints that actually bind in intervening periods, as shown in the results below.

We could use unconditional expectations to describe the revenue adequacy problem, and show that the expected rent collected would cover the pay-outs required, on average, in the long run. That may be true, at a very high level, but hardly gives much assurance to an SO who could still be left carrying a huge risk exposure, perhaps over decades. So revenue adequacy results that only apply in expectation are of limited practical use. Ideally we would like to show that, provided the physical storage is available, revenue adequacy can be guaranteed, with certainty, with respect to each and every FSR, just as in the FTR case. Unfortunately, that proves to be impossible. What this section seeks is a revenue adequacy result that is at least as tight as possible, in the sense that the risk faced by the SO is minimised.

The SO is increasingly becoming exposed to the risk of revenue adequacy on FSR pay-outs as we traverse backwards through the scenario tree along a specific scenario path towards the root node from the final leaf event node at which all uncertainty is resolved. FSR pay-outs must be based on a conditional expected value in looking forward from the event tree node at which the FSR terminates, leaving perhaps significant uncertainty yet to be resolved. Thus any revenue adequacy proof needs to work with the conditional expectations, given all the information available at the time when settlement must be made.

The first proposition is about the revenue adequacy of a single period FSR under uncertainty. The second proposition then employs that result to prove an inter-temporal multi-stage FSR revenue adequacy result. Finally, we will consider multiple, multi-stage (overlapping) FSRs. It will be seen that these propositions will depend upon arguments derived using conditional expectations. Proposition [4-A] can be expressed as follows.

**Proposition 4A.** *Suppose that the lower bound is zero and the upper bound remains unchanged during the planning horizon. The SO issues a single period obligation–inclusive FSR, for*

quantity  $Q_s \leq \bar{S} - \underline{S} = \bar{S}$ , between time periods  $t'$  and  $t' + 1$ . Then the rents generated on the binding storage constraints, over the contract period from  $t'$  to  $t' + 1$ , will be sufficient to meet the conditional expected value of the FSR payments, when that expectation is taken at the beginning of period  $t'$ . [4-A]

**Proof.** The FSR holder has the right to collect (or obligation to pay) the actual market price difference, for the sub-scenario that actually occurs between the time periods  $t'$  and  $t' + 1$ , for any realisation of the inflow for period  $t' + 1$ , given the entire scenario leading up to node  $(h, t')$ . And the FSR pay-out must be known at the time it is made (end of period  $t' + 1$ ) based on events that have been realised at that time.

Recall the conditional expected dual constraint equation, [3.14a] in the previous chapter. Here  $(h, t')$  is the ancestor of event node  $(h', t' + 1)$ . We will re-arrange equation [3.14b] as follows:

$$\mathbb{E}[\lambda_{h',t'+1}|(h, t')] - \lambda_{h,t'} = (\gamma_{h,t'}^+ - \gamma_{h,t'}^-)$$

Taking expectations on both sides, conditional on node  $(h, t')$  eventuating, and noting that there is, in fact, only one end-of-period period storage shadow price pair for each event node, the expected per unit pay-out requirement is just the price differential, which is as follows:

$$\mathbb{E}[\lambda_{h',t'+1} - \lambda_{h,t'}|(h, t')] = \mathbb{E}[(\gamma_{h,t'}^+ - \gamma_{h,t'}^-)|(h, t')] = \gamma_{h,t'}^+ - \gamma_{h,t'}^-$$

But, since non-zero shadow prices only occur when the reservoir is either full or empty, the following relationship yields the expected FSR pay-out requirement for an FSR quantity of  $Q_s$ :

$$\mathbb{E}[\text{Payout}_{h',t'}(Q_s)] = Q_s (\mathbb{E}[\lambda_{h',t'+1}|(h, t')] - \lambda_{h,t'}) = \begin{cases} Q_s \gamma_{h,t'}^+ & \text{if } s = \bar{S} \\ 0 & \text{if } \underline{S} < s < \bar{S} \\ -Q_s \gamma_{h,t'}^- & \text{if } s = \underline{S} \end{cases}$$



Now we must compare this requirement with rent generated, using an inequality implied by the FSR volume limit which, with the lower bound to zero, we can write as  $Q_s \leq \bar{S}$ . The required result then follows using similar logic as for the deterministic dual constraint equation [3.12b]. Again, there is only one pair of end-of-period storage bounds, and hence storage prices, for each event node. The terms  $\bar{S}\gamma_{h,t'}^+$  and  $\underline{S}\gamma_{h,t'}^-$  in the equation below can be thought of as conditional expected rents associated with the upper and the lower storage bounds. The conditional expected rent is thus either:  $\text{Rent}_{h',t'}(\bar{S}) = \bar{S}\gamma_{h,t'}^+$ , if the reservoir is full, or  $\text{Rent}_{h',t'}(\underline{S}) = -\underline{S}\gamma_{h,t'}^- = 0$ , if the reservoir is empty<sup>61</sup>. So, comparing the conditional expected requirement with the rent generated, we have:

$$\mathbb{E}[Q_s(\lambda_{h',t'+1} - \lambda_{h,t'}) | (h, t')] = \begin{cases} Q_s\gamma_{h,t'}^+ (\leq \bar{S}\gamma_{h,t'}^+) & \text{if } s = \bar{S} \\ 0 & \text{if } \underline{S} < s < \bar{S} \\ -Q_s\gamma_{h,t'}^+ (\leq -\underline{S}\gamma_{h,t'}^- = 0) & \text{if } s = \underline{S} = 0 \end{cases}^{62}$$

Therefore, revenue sufficiency is guaranteed, because the conditional expectation, at the beginning-of-period  $t'$ , of the net surplus after settling the FSR, is positive, that is:

$$\text{Netsurplus}_{h',t'}(\bar{S}, Q_s) = \text{Rent}_{h',t'}(\bar{S}) - \mathbb{E}[\text{Payout}_{h',t'}(Q_s)] \geq 0. \blacksquare$$

The following proposition generalises the above result for the above single period FSR. When the conditional expectation is taken at any time period before commencement of the FSR

**Proposition 4B.** *Suppose that the assumptions above hold, and the SO issues a single period FSR which commences at time period  $t'$ . The conditional expected value, taken at any time period,*

$$t_0 < t' \text{ of } \text{Netsurplus}_{h',t'}(\bar{S}, Q_s) \text{ will be positive.} \quad \textbf{[4-B]}$$

<sup>61</sup> In section 4.3.1, we manipulated the canonical form of the lower bound to make the shadow price positive:  $\gamma_{h,t}^- \geq 0$ . That would still give us a negative rent when the lower storage constraint was binding, because that same manipulation makes the RHS =  $(-\underline{S})$ . But we have already set  $\underline{S} = 0$ . This means the rent is actually zero. By way of contrast, electricity transmission lines have a positive flow capacity in each direction, so electricity networks always generate positive rent when transmission capacity constraints are binding, regardless of the current flow direction.

<sup>62</sup> Based on personal communication with E.G. Read, (Read, 2016).

**Proof.** Proposition 4A guarantees revenue adequacy for a single period FSR issued and commencing in the same time period. But in this proposition we are focusing on the conditional expectation of the FSR pay-out taken in a time period  $(t_0)$ , prior to the FSR commencing time period  $t'$ . The proof is straightforward, by splitting the computation of the expectation for  $t' + 1$  taken in  $t_0$  into two stages (i.e.,  $(t_0, t')$  and  $(t', t' + 1)$ ). Applying the laws of iterated expectations, we state the following:

$$\mathbb{E}[\lambda_{h',t'+1}|(h, t_0)] = \sum_{h'} (\mathbb{P}[(h', t')|(h, t_0)] \cdot \mathbb{E}[\lambda_{h',t'+1}|(h', t')])$$

Similarly, we have the following expression for the conditional expected value of  $\text{Netsurplus}_{h',t'}(\bar{S}, Q_s)$ , taken at node  $(h, t_0)$ :

$$\begin{aligned} \mathbb{E}[\text{Netsurplus}_{h',t'}(\bar{S}, Q_s)|(h, t_0)] &= \\ \sum_{h'} (\mathbb{P}[(h', t')|(h, t_0)] \cdot \text{Netsurplus}_{h',t'}(\bar{S}, Q_s)) \end{aligned}$$

Therefore, we can safely state that the conditional expectation of the net surplus for period  $t'$ , taken at any time period  $t_0 \leq t'$  will always be positive, because it is a weighted sum of terms, one for each event node  $(t', h')$  that might occur at time  $t'$ , each of which is itself positive, by Proposition 4A. ■

The next proposition is about the inter-temporal revenue adequacy issue for a multi-stage FSR. The problem is separable in time, thus it decomposes into a collection of single period sub-problems.

**Proposition 4C.** *Suppose that the assumptions above hold, but the SO is now issuing a multi-stage obligation-inclusive FSR, for quantity  $Q_s \leq \bar{S}$ , between time periods  $t'$  and  $t''$ , where  $t'' > t'$ . The conditional expected value of the aggregate rents generated on the binding storage constraints, over the contract period from  $t'$  to  $t''$ , will be sufficient to meet the conditional expected FSR payments, where those expectations are taken at any time  $t_0 \leq t'$ .* [4-C]

**Proof.** We can decompose  $\text{FSR}(t', t'')$  into a set of single period FSRs, from  $t'$  to  $t' + 1$ ,  $t' + 1$  to  $t' + 2$  etc, assuming that the SO issues them all at some time period  $t \leq t'$ . The

pay-out required, from  $t'$  to  $t''$ , is just the sum of the pay-outs for these single period FSRs, because the end-of-period/beginning-of-period price terms cancel, as follows:

$$(\lambda_{h',t'+1} - \lambda_{h,t'}) + (\lambda_{h',t'+2} - \lambda_{h,t'+1}) + \dots + (\lambda_{h',t''} - \lambda_{h,t''-1}) = \lambda_{h',t''} - \lambda_{h,t'}.$$

So the expected pay-out requirement, conditioned on node  $(h, t_0)$  is as follows:

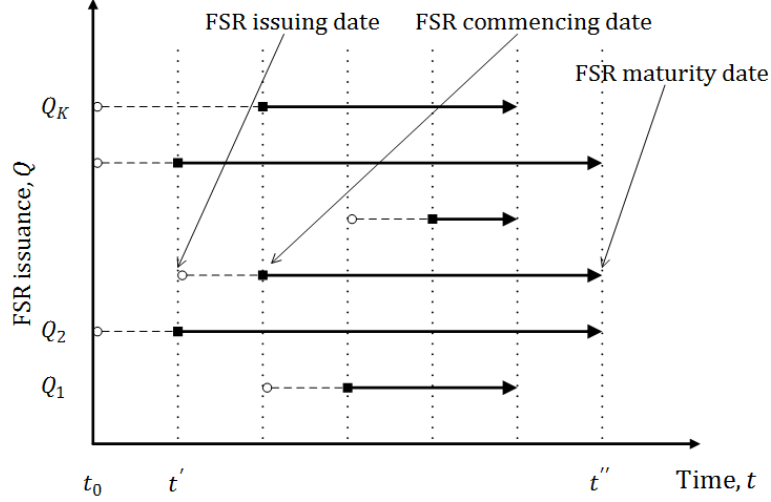
$$\mathbb{E}[\lambda_{h',t'+1} - \lambda_{h,t'} | (h, t_0)] + \mathbb{E}[\lambda_{h',t'+2} - \lambda_{h,t'+1} | (h, t_0)] + \dots + \mathbb{E}[\lambda_{h',t''} - \lambda_{h,t''-1} | (h, t_0)] = \mathbb{E}[\lambda_{h',t''} - \lambda_{h,t'} | (h, t_0)]$$

Here, each term represents the expected pay-out of a single period FSR conditioned on  $(h, t_0)$ , as in the previous proposition. By that proposition, then, each of these conditional expected pay-out requirements is covered by the conditional expected value of the rents generated in the period to which it applies, where those expectations are taken at any time  $t_0 \leq t'$ . So it follows immediately that the conditional expected sum of those pay-out requirements is covered by the conditional expected sum of rents, as stated in this proposition. ■

In reality, the SO will expect to issue multiple FSRs with different time periods. Proposition [4-D] is about revenue adequacy to support multiple FSRs covering different periods, in the same market horizon.

**Proposition 4D:** *Suppose that the assumptions above hold, and the SO issues multiple multi-period FSRs indexed by  $k = 1, 2, 3, \dots, K$ , commencing at various time periods, and sometimes overlapping each other. Define, the pay-out of the  $k^{\text{th}}$  FSR as  $Q_{S_k}(\lambda_{h'',t''} - \lambda_{h',t'_k})$  where  $Q_{S_k}$  is the contract quantity,  $t'' > t'$  and event nodes  $(h'', t'')$  and  $(h', t')$  eventuate. If the net sum of overlapping FSR allocations, for each period, is less than the storage capacity, then the conditional expected value of the aggregate net surpluses, taken at any time period,  $t_0 \leq \text{Min}\{t'\}$ , will be positive, for each period  $t \geq t_0$ , and in total over all such periods.* **[4-D]**

**Proof:** In general, we could have all FSRs issued at any time period on or after  $t_0$  and commencing in any period ( $t' \geq t_0$ ) as sketched in Figure 4-2, where the dashed/solid lines represent the periods from when an FSR is issued, and then when it is active, within the market horizon.



**Figure 4-2.** Example of multi-stage multiple FSRs issued and starting at different periods.

First, each multi-period FSR can be decomposed into a set of single period FSRs, each for quantity  $Q_{S_k}$ , as in proposition 4C. Second, for each time period,  $t$ , we can define the aggregate FSR quantity to be:  $Q^t = \sum_k Q_{S_k} : t_k'' > t \geq t_k'$ . Then the original FSR set is equivalent to a set of single period FSRs, one for each  $t$ , with quantity  $Q^t$ . Clearly, taking expectations at time  $t_0 \leq \min\{t_k'\}$ , proposition 4B applies with respect to each of these aggregate single period FSRs.

So the conditional expected net surplus  $E[\text{Net surplus}_{h',t'}(\bar{S}, Q_{S_k}) | (h, t_0)]^{63}$  will be positive for all FSR issuances, provided,  $Q_t \leq \bar{S}$ , for each  $t \geq t_0$ . Clearly, the same is true for the sum of the above conditional expected net surpluses, over the market horizon. ■

So we will get a positive conditional expected net surplus for the aggregate of any set of FSRs commencing in different periods, provided we take the expectations in any time period before any of them commences. In reality, though, the SO will be issuing new FSRs continuously during the market horizon, and will be interested in (re-)assessing revenue adequacy in any period, while earlier issuances are still functioning (as shown in Figure 4-2). The overall revenue adequacy situation at any period

<sup>63</sup>  $\text{Rent}_{h',t'}(\bar{S}) - \sum_k E[\text{Payout}_{h',t'_k}(Q_{S_k}) | (h, t_0)]$

during the market horizon will depend on the losses/gains already accumulated on the FSR portfolio already issued, and there is no way we can guarantee that a conditional expectation taken during the term of (some) FSRs will be positive. But we can still expect a positive conditional expected net surplus values, at any time in the horizon, provided the total issuance is less than storage capacity,  $\bar{S}$ ,<sup>64</sup> and the value of unrealised losses/gains on existing FSR issuances can be written-off (e.g., by “mark-to-market” valuation of tradable FSRs) and the assessment focused only on the conditional expected value of aggregate future risk exposure, across the whole portfolio. .

**Proposition4E:** *Suppose that the assumptions above hold, and the SO has issued multiple, multi-stage obligation-inclusive FSRs, with aggregate volume  $Q^t \leq \bar{S}$ , for each  $t$ , as in Proposition 4D. At any event node  $(h, t)$ , let the residual FSR pay-out for the  $k^{\text{th}}$  FSR be defined by:  $Q_{S_k}(\lambda_{h'', t_k''} - \lambda_{h, t})$ , if  $t \geq t_k'$ , and otherwise  $Q_{S_k}(\lambda_{h'', t_k''} - \lambda_{h, t_k'})$ . Then the conditional expected value of the aggregate rents generated on the binding storage constraints, over the future market horizon, will at least cover the conditional expected residual FSR pay-out requirement , where both conditional expectations are taken at event node  $(h, t)$ .*

[4-E]

**Proof:** Any FSR operating at time  $t$ , in the range  $(t' < t < t'')$ , can be decomposed into a series of single period FSRs, as in proposition 4C, and hence re-composed into two FSRs:  $FSR(t', t)$  and  $FSR(t, t'')$  The value of former is already known at time  $t$ , and that will imply unrealised losses/gains, which may be written-off as “sunk”, and irrelevant to this proposition. However, proposition 4D applies to the residual FSR values, with the conditional expectation being taken at time period  $t$ , which is less than or equal to the commencement date for each of these (residual) FSRs, as required by that proposition. Hence we have the result that above proposition holds. ■

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<sup>64</sup> That is, provided the new issuance is only up to the remaining unassigned storage capacity  $(\bar{S} - Q_t)$

In theory, then, proposition 4E implies that a rational forward-looking market manager who is willing to write-losses (or write-up gains) to date on existing FSRs (e.g., by “mark-to-market” valuation of tradable FSRs), could issue “fresh” FSRs, at any time, up to the remaining unassigned reservoir capacity, for any future period in the remaining market horizon. The market manager then has the same assurance of expected future revenue adequacy for its portfolio of new and residual FSRs, as if she was starting with a clean slate.<sup>65</sup>

### **4.3.3 Alternative Concepts**

While this thesis only sets out to provide a preliminary exploration of hedging concepts, it should be recognised that the FSR concept developed above is not entirely satisfactory, for two main reasons:

- Revenue adequacy only seems possible on an expected basis and
- It does not fully meet the participants’ likely requirement to manage an uncertain volume of inflow entitlements.

Thus, while the development of alternatives lies beyond the scope of this preliminary investigation, we conclude by discussing some of the issues, and suggesting some alternative concepts that might be considered.

#### **4.3.3.1 Revenue adequacy and Retrospective FSRs**

It may seem disappointing that we were only able to prove revenue adequacy “in expectation” for the FSRs described above. But Read 2016<sup>66</sup> suggests that “on reflection, it does not seem reasonable to expect any more than this. At best, we might expect a financial instrument like this to put the holder in the same position as if they owned, controlled and benefited from a corresponding slice of the physical system. And he observes that even a reservoir owner with full control over the system would not be able to know the actual (rather than expected) initial or final value of water at any time  $t < T$ . Typically, the reservoir owner will go through phases of conserving water because it has a high

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<sup>65</sup> In reality, decision-makers are seldom free to act in such a purely rational fashion, and we might expect issuance policy to be significantly impacted by the gains or losses accumulated to date. But we will not explore that here.

<sup>66</sup> Personal communication with E. G. Read.

expected value due to the likelihood of future shortage, only to find that the water so conserved is eventually spilled. Conversely, they will go through phases of releasing water freely because it has a low expected value due to the likelihood of future spill, only to find that the water so released might eventually have staved off a shortage. In other words, the true marginal value of water is only known to the reservoir owner, let alone the market, retrospectively. Neither the true initial nor true final water values can be known at the time an FSR of the type described above is settled.

In light of this, Read notes that, while the previous chapter models the whole range of future uncertainty using the event tree formulation method, the analogy between FSRs and FTRs, may be better understood in the context of the alternative “scenario path” formula, as described by Rockafellar & Wets (1991). In that formulation it becomes clear that<sup>67</sup> the “true” shadow prices (including marginal water values) for each scenario path behave in exactly the same way as for a deterministic formulation. But they are only slowly revealed to the decision-maker, who must base each real-time (event node) decision only on conditional expectations taken at that node. That requirement is expressed, in that formulation, by a “non-anticipativity restriction” which, mathematically, implies that the decision variables for all paths passing through an event node be identical, in all periods prior to that event node. For the reservoir management problem, Read (1979) shows that this implies that the “multiplier (shadow price) on the non-anticipativity restriction” for each path, at each event node is just the difference between the conditional expected marginal water value at the node, and the true marginal water value for that path, which will typically be known only at the end of the planning horizon, when the path has been completed.

Thus Read suggests that, at least within the simplified context of a finite and discrete scenario tree, the SO could avoid all risk if it were allowed to defer FSR valuation and settlement until the end of the horizon, when all uncertainty has been resolved. At that time the exact scenario path will have been revealed, and the actual water value at any node along that path will also be known, depending on whether storage actually did reach its upper /lower bounds, spill/shortage actually did occur, etc. And a “retrospective FSR” (RSR) could then be settled using the “true” retrospective marginal water values of the initial and final period, for the path that did occur. In reality, situations are constantly evolving, and

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<sup>67</sup> See discussion in Chapter 8 of Read (1979) and other working papers.

uncertainty is not fully described by a scenario tree. But there could be an interim payment made at some date, with subsequent adjustments occurring, perhaps until some definitive event, such as a reservoir actually spilling, reveals what the true marginal water value actually was.

Naturally, the higher level of security provided to the SO by an RSR kind of hedging instrument is reflected in a lower level of certainty given to participants, who would find the instrument less attractive as a result. But Read argues that the level of certainty provided is in fact realistic, and that participants could not do any better by building their own reservoirs. Thus he suggests that participants might eventually accept that this kind of instrument is better than having none at all, because the SO cannot face the risks of issuing firm FSRs of the type discussed above. So this alternative should not be dismissed out of hand. It is not unusual, after all, to buy a product the true value of which only becomes evident after some period of time.

#### **4.3.3.2 Volume Uncertainty and Financial Storage Options**

It should be recognised that hydrological uncertainty does not just create price uncertainty for participants in water markets, but volume uncertainty. But the FSRs discussed above, like FTRs in electricity markets, only hedge price uncertainty, not volume uncertainty. In the presence of non-seasonal inflow patterns, ideally, participants would want to hold fixed volume FSRs from one period to another together with indexed FIRs. But, the problem with this FSR-FIR regime is that the SO will be exposed if the “expected” inflows do not occur. The SO would have to buy back the FSRs from the market to support the current period consumption.

It should be recognised that all of the above propositions and discussion relate to obligation-inclusive FSRs, but it is not clear that those are actually the best hedging instruments to use in this context, even if they are traded dynamically to match inflows, as above. At least they are not the natural analogues of physical reservoir capacity.

As noted above, if the lower limit may be binding at some point during that interval, the marginal water values will typically be declining between the two periods, on average. However, since we assume ‘zero’ storage lower bounds, there is no associated (negative) rent component appearing in the dual objective function. This reflects the fact that the stochastic optimisation ensures that we would never



carry water forward in situations when its ‘market observed’ (conditional expected) value was expected to fall.<sup>68</sup>

The conditional expected dual prices (“rents”) associated with the storage bounds belonging to different scenarios (taken at the FSR starting period) will take sometimes positive and sometimes negative values, when those particular scenarios hit the upper/lower storage bounds over the market horizon as described in the above discussion. If the balance of the storage limit shadow price effects is positive over the interval, when assessed in the terminal period of the FSR, the obligation-inclusive FSR holder has the right to exercise the property right, and receive the value difference. If the balance of these effects is negative, the SO can be expected to “exercise the property right”, and the holder of an obligation-inclusive FSR has the obligation to pay the negative value difference. And we have shown that the net pay-out requirement can be supported by settlement surpluses, on average.

So an obligation inclusive FSR contract holder could be sometimes exposed to risks if future storage conditions turn out to be adverse, and will definitely face a loss, on average, if FSRs are held during that part of the annual cycle when prices always fall. Meanwhile, there will be a positive contribution to the settlement surplus, on average, from the periods in which prices can rise, because storage may be full, and the SO would collect these rents for every year. But, while the SO may face some negative rents, on average, from periods in which prices can fall, reservoirs will be much emptier than. In equilibrium, the average price rise over part of the annual cycle will be exactly offset by the average price fall over the rest of the cycle. But the differences in volume carried forward mean that the rental fund contribution will always be positive, over an annual cycle. But, if participants hold a constant volume of obligation inclusive FSRs over one or more annual cycles, the seasonal price fluctuations can be expected to cancel each other out over successive annual cycles, leaving a net pay-out requirement that may be positive, but may be negative, but does not grow, on average, over the years.

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<sup>68</sup> In reality, we will often observe, that, looking at past events, we have carried storage forward when its market value was, in fact, dropping. But that is because an adverse scenario actually occurred. We would not have (willingly) carried storage forward if we had thought that its value would drop, on average across all possible intra-period scenarios.

Thus obligation-inclusive FSRs would only seem appropriate as essentially seasonal instruments, allowing a participant to hedge a specific volume of storage from one season to another, within an annual cycle. Presumably participants would buy, and perhaps actively trade, such FSRs to hedge risks between times when they plan/expect to purchase water for storage (see below), and when they expect to withdraw it. In a seasonal system, participants could be expected to buy FSRs corresponding to moving water from wet(ish) seasons to dry(ish) seasons.

If a longer term instrument is desired, to avoid participants being exposed to the risks of falling water values, we could suggest that participants buy a strip of annual “wet to dry” obligation inclusive FSRs, thus effectively giving them the right (and requirement) to store a specified volume of water over the relevant months, for each future year covered. But they might also consider an FSR-option (FSO) contract. This would be similar to an FTR option<sup>69</sup> which allows its holder the exercise rights when the power price difference is positive (Hogan, 2001; Ritcher et al, 2001; Deng & Oren, 2006), but implies no obligation in the opposite case. This means that the FSO holder in a water market is holding a right to store water, but has no obligation to do so. Thus the FSO is really the natural representation, in financial terms, of the physical capability a reservoir provides, to store water if, but only if, the water values are expected to increase during the contract period, just as electricity market option contracts give the holder the financial equivalent of a right to operate a thermal plant, at an SRMC equivalent to the strike price. We note that such options are seem the natural form of contract to be sold by thermal generators, at least, in electricity markets, and so might be expected to form the main basis for trading in such markets. But we also note that participants in such markets actually seem to prefer trading “sculpted” portfolios of obligation inclusive instruments. Thus we would not be surprised if the same turned out to be true in water markets.

#### **4.4 Distribution System Rents and Rights**

The previous section described several financial instruments to hedge inter-temporal risks, based on supply system related settlement surpluses. But participants in a water market are also exposed to

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<sup>69</sup> An FTR option holder collects the congestion rents when the rents were positive, but there is no obligation to pay negative congestion rents.

locational price differences in each particular time period, and the uncertainty in these price differences could make some participants unwilling to engage in market activities unless insurance is available. In electricity markets, FTRs allow any user to hedge against locational price differences, and thus help to insulate her from flow transportation risks. In this section, we discuss how similar financial instruments are applicable to hedge against locational price risks in the water transportation network. It examines the transportation constraint rents in the dual objective function and then describes how these can be used to hedge against “transportation price risks”.

The term  $\sum_{(i,j) \in A} (\bar{X}_{(i,j)} \mu_{(i,j),t}^+ - \underline{X}_{ij} \mu_{ij,t}^-)$  of the dual objective function in Section 3.3.2 represents the rents implicitly collected from the consumptive/non-consumptive users, via the settlement surplus, when one or more arc capacity limits bind in period  $t$ . The value created by these rents should logically be credited to the transport system owners. However, this section initially avoids the problem of allocation to asset owners by assuming that all channels are owned by the market, or by an independent third party (e.g., government).

#### 4.4.1 Rents and the Settlement Surplus

Let us drop the time superscript,  $t$ , and let  $i$  and  $j$  denote two adjacent network nodes. The SO is paid the nodal price for consumption at each node and the users implicitly retain the consumer surpluses, determined by  $Q_{i,b} \sigma_{b,i}^+$ , as above. The multiple copies of the primal constraint equations [3.2a] and [3.2b] across the network imply that the net release  $\sum_i q_i$  is equal to the net inflows across the network, plus the change in storage volume. For this scenario, one could assume that spill  $z_i$  is a part of  $q_i$ . Using [3.2a], the term  $\sum_i \lambda_i q_i$  where  $i \neq 0$  indicates the revenue collected from consumptive users who consume water anywhere in the network, while the cost of withdrawing this quantity from the reservoir will be  $\lambda_0 \sum_i (q_i - f_i)$ .

If the flow on arc  $(i, j)$  is limited by the arc flow limits this implies an arc transport price (per unit), given by re-arranging equation [3.10]:  $\lambda_{i,t} = \sum_{k \in \Omega(i,j)} \omega_{k,t} - (\mu_{(i,j),t}^+ - \mu_{(i,j),t}^-) + \lambda_{j,t}$ . With no non-consumptive use,  $\sum_{k \in \Omega(i,j)} \omega_{k,t} = 0$ , and the above equation becomes:  $\lambda_{j,t} - \lambda_{i,t} = (\mu_{(i,j),t}^+ - \mu_{(i,j),t}^-)$ .

The RHS prices can be interpreted as the capacity constraint rent generated for a unit volume when

upper/lower arc limits are binding, while the locational marginal price difference (LHS) on arc  $(ij)$  shows how this rent is implicitly collected in the “settlement surplus” from market clearing for that particular arc, in period  $t$ .

So the RHS identifies the rents which could, in principle, be returned to particular asset owners, out of the settlement surplus the SO is collecting from the consumptive users on the LHS. The net rent  $(\sum_{(i,j) \in A} \bar{X}_{(i,j)} \mu_{(i,j),t}^+ - \sum_{(i,j) \in A} \underline{X}_{ij} \mu_{ij,t}^-)$  indicated in the dual objective function, is determined by taking the sum of the positive rent components for arcs on which flow is constrained in the normal stream flow direction and the negative rent components for arcs traversed in the reverse stream flow direction, over the network<sup>70</sup>. These revenue and rent components will be relevant in the following revenue adequacy proofs.

#### 4.4.2 Hedging Against Flow Transportation Price Risks

Physical delivery contracts could give the holder an opportunity to physically take a defined water volume through a specified arc, for an agreed price. But we propose financial delivery rights (FDRs) which take/pay the difference between a locational marginal price difference and an agreed “strike” price, for a defined volume of water. In real time, the optimal water allocation/market clearing problem is solved as if the FDRs do not exist. If the holder does not utilise the arc, its limits may not prove to be binding, and there may be no rents collected, or need to make hedging payments. If others use the arc to transport water, though, the upper flow limits may bind, and the FDR settlement ensures that the rents they implicitly pay into the settlement surplus are re-cycled to finance pay-outs to the rights owners. Thus there is strictly no need for a secondary rights market that allows participants to trade the initial issuance. But such a market may still be desirable, to improve liquidity, and to allow participants to adjust their positions over time. Authorities could let the water market FDRs all be traded in a combined market with simultaneous clearing to ensure feasibility of user trades in arc flow capacities.

These FDRs will be somewhat like electricity market FTRs (Hogan, 2002), or like the capacity rights for gas markets suggested in Read et al (2012) if we disregard any transportation flow delays. For the

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<sup>70</sup> Positive/negative rents will be generated when upper/lower limits are binding. No rents will be collected when the upper/lower arcs limits are not constrained.

purpose of completeness, the locational marginal price differential  $(\lambda_j - \lambda_i)$  between any two non-adjacent points in the network could be used to create an FTR type obligation-inclusive node-to-node:

$$FDR_{ij}(Q_{ij}, SP_{ij}) = (\lambda_j - \lambda_i - SP_{ij})Q_{ij}.$$

Here  $Q_{ij}$  denotes the contract quantity and  $SP_{ij}$  denotes the strike price. This effectively gives the holder a right, and obligation, to swap the actual price differential corresponding to pre-specified flows between any two locations within the network in a particular time period, for the agreed differential,  $SP_{ij}$ .<sup>71</sup>

Instead of using node-to-node FDRs similar to FTRs, this thesis will focus on arc rights that are more like a Flow Gate (FGR) transmission right (Chao & Peck, 1998)<sup>72</sup> returning the rents generated when specific transmission links become congested. Such rights appear both conceptually and practically appealing in a network where participants can reasonably identify the flow of “their water” across specific arcs at specific times. Note, though, that this study is dealing with a tree-structured network, so there is always a unique path between any pair of nodes. And, since there are no “loop-flow problems”, our “arc rights” are equivalent to “node-to-node” rights between adjacent nodes.<sup>73</sup> Thus it seems reasonable that participants could construct any “node-to-node” right as a series of “arc rights”. It will be seen that this is not true for options, as discussed in Section 4.4.4, but it is true for obligation inclusive rights. We refer to these arc node-to-node rights as Arc Capacity Rights (ACRs) and node-to-node options as Arc Capacity Options.

#### 4.4.2.1 Obligation-inclusive Arc Capacity Rights

An ACR compensates the right holder when the prices rise across the arc (i.e., positive price differential in the normal stream flow direction), and that will happen if those flows are constrained by the upper

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<sup>71</sup> By way of contrast,  $SP$  is normally assumed to be zero, in the FTR literature, corresponding naturally to a situation where there are no non-consumptive costs or benefits, and losses are small enough to be ignored.

<sup>72</sup> The volume of an FGR available depends only on the state/capability of the “gate” element itself, and not on network topology. It is the market value of the rights that varies over time, as flows vary, partly reflecting topology changes.

<sup>73</sup> This statement is not generally valid for electricity market FGRs and FTRs because in most cases there are non-tree node configurations with loop current flows.

arc capacity limit on the specified channel/flow gate in the specified direction. On the other hand, flows are sometimes constrained by the lower arc capacity limit on the specified channel/flow-gate in the normal stream flow direction, or perhaps by the upper arc capacity limit on the specified channel/flow-gate in the reverse stream flow direction. Then the right holder is obliged to pay into the settlement surplus, because prices fall across the arc (i.e., there is a negative price differential in the normal stream flow direction).

That is:

$$ACR_{ij}(Q_{ij}, SP) = (\lambda_j - \lambda_i - SP_{ij})Q_{ij}.$$

Here  $Q_{ij}$  denotes the contract quantity and  $SP_{ij}$  denotes the strike price. Following the structure of the FSR/FSO discussion in Section 4.3, we could define arc capacity flow gate options (ACO) as discussed in the next section.

#### 4.4.2.2 Arc Capacity Options

An ACO represents a non-obligatory type of contract, effectively entitling its holder to use the arc capacity ( $ij$ ) in the direction from  $i$  to  $j$ , at an agreed price, if that turns out to be profitable. Given the contract quantity,  $Q_{ij}$  and the strike price  $SP_{ij}$ , we can define an ACO as follows:

$$ACO_{ij}(Q_{ij}, SP_{ij}) = \max\{(\lambda_j - \lambda_i - SP_{ij})Q_{ij}, 0\}.$$

For simplicity, we will assume that  $SP_{ij} = 0$  in this case, with no non-consumptive, pumping or leakage cost/benefits in the network. Applying [3.10], yields the following relationship:

$$ACO_{ij}(Q_{ij}, 0) = \max\{(\lambda_j - \lambda_i - 0)Q_{ij}, 0\} = \mu_{ij}^+ Q_{ij}.$$

Here  $\mu_{ij}^+$ , the per unit rent generated for arc ( $ij$ ) is only non-zero when congestion occurs in the normal stream flow direction. The ACO gives the holder a right to receive the price differential when it is positive, that is when congestion occurs in the normal stream flow direction. And the above equation shows that the required pay-out is just a share of the settlement surplus,  $\bar{X}_{ij}\mu_{ij}^+$  corresponding to a pre-specified flow of  $Q_{ij}$  on that arc, in a particular time period<sup>74</sup>. That would hedge the risk associated with

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<sup>74</sup> Because we deal with single period cases, we can drop the time index in this discussion.

transportation of that quantity of water in that time period, in the normal stream flow direction. Clearly, we have revenue adequacy provided  $Q_{ij} \leq \bar{X}_{ij}$ .

So, an ACO gives the holder the right to use, or receive rents on, an amount of capacity in the normal stream flow direction, and that the rent is either positive or zero. An ACO issued for a non-congested direction will have no value, and the holder is not liable for any payment (Chao et al., 2000). ACO holders have no obligations when the price differential becomes negative because flows are so low that minimum flow requirements are only just met in the normal stream flow direction, or even reversed to use all arc flow capacity in the reverse stream flow direction. Because an ACO excludes the downside risk of the obligation, it does not fully hedge inter-locational risk, though.

As a result, these rights cannot be concatenated into node-to-node rights. Suppose the SO issues the following ACOs for the flow gates lying along the path  $(k, k')$ :  $\max(\lambda_{k+1} - \lambda_k, 0)$ ,  $\max(\lambda_{k+2} - \lambda_{k+1}, 0)$ , ...,  $\max(\lambda_{k'} - \lambda_{k'-1}, 0)$ . Their aggregate pay-out will only be the same as for a composite option for the path,  $FDO_{k,k'}$ , if the price differences for individual flow gates are non-negative, and so the corresponding ACOs are exercised. Then the pay-out  $(\lambda_{k'} - \lambda_k)$  for the path option will be matched, because the intermediate price terms do cancel off each other. But, otherwise the sum of ACO pay-outs will be higher than that for the path option.

The desired result does prove to be true for obligation-inclusive rights, though. Suppose that a path between the two non-adjacent nodes  $(k, k')$  is comprised of a series of arcs:  $\{(k, k+1), (k+1, k+2), \dots, (i, j), \dots, (k'-1, k')\}$ . Deploying the dual constraint equation [3.10] in the previous chapter, we can unbundle the shadow prices and the locational marginal prices (per unit quantity), for the entire path (i.e., for all flow gates  $(k' - k)$  in the selected path between the two nodes) as follows:

$$\begin{aligned} \lambda_{k'} - \lambda_k &= (\lambda_{k+1} - \lambda_k) + (\lambda_{k+2} - \lambda_{k+1}) + \dots (\lambda_{k'} - \lambda_{k'-1}) \\ &= (\mu_{(k,k+1)}^+ - \mu_{(k,k+1)}^-) + (\mu_{(k+1,k+2)}^+ - \mu_{(k+1,k+2)}^-) + \dots (\mu_{(k'-1,k')}^+ - \mu_{(k'-1,k')}^-) \end{aligned}$$

Since the intermediate price terms cancel out each other, the above relationship between the concatenated arc price differentials and the composite price differential ( $k' - k$ ) will easily allow us to form a node-to-node path right by combining the obligation-inclusive arc capacity rights<sup>75</sup>.

#### 4.4.2.3 Arc Flow Losses and Delays

Up to this point, this chapter has ignored losses (i.e., leakage, evaporation) in the transportation network, and supply system, and we ignore such for the later developments of this chapter. But this section briefly considers how ACRs might operate when flow losses create price differences between nodes, as in electricity networks<sup>76</sup>.

In electricity markets, the generators are paid for total generation, which implicitly includes losses. But the market collects the marginal cost of losses from the loads. Since losses are quadratic (DC load flow approximation), the marginal cost is twice the average. So this leads to a positive settlement surplus under normal conditions, even when line limits do not bind.

In a water market model, “losses” arise due to evaporation and/or leakage in the storage and/or transportation network, and can be represented by introducing a loss function in the flow balance constraints. We will assume a convex increasing transportation loss function, which would be analogous to the transmission loss function in the economic dispatch model of an electricity market. If the loss function is linearised in a piece-wise fashion, there will be loss rents corresponding to each flow segment in the linearized flow loss curve, as for electricity market transmission losses<sup>77</sup>.

FTR systems in many electricity markets hedge only against the congestion component of the locational marginal price differentials. However, Hogan & Harvey (2002) and Rudhevich et al (2005) have

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<sup>75</sup> This will hold true with non-zero strike prices, too, provided  $SP_{kk'}$ , the composite strike price of the FDR issued for path  $(k, k')$ , equals the sum of all  $SP$ s assigned for all ACRs lying along the path.

<sup>76</sup> In addition there are supply system (reservoir) losses due to seepage and evaporation.

<sup>77</sup> In reality, the loss function may have a fixed loss if a minimum stream flow is essential to keep the channel wet, in which case we should really use an integer variable to model wet or dry channel, plus a piece-wise linear incremental loss function for positive flows, as above. Non-convex losses may also arise if leakage occurs at particular levels, for example. In such cases we could approximate the function by inserting linear segments e.g. from the origin to some critical flow above which the loss/cost functions is convex (and monotonically increasing). Or we could use/successive linearization (van der Vlerk, 2005; Pepper et al., 2012). But these complications will be ignored.



illustrated two different proposals to redistribute the surplus arising from loss related revenues. More recently Read & Miller (2011) discussed the issue of revenue adequacy with loss contracts and or reserve contracts, for the HVDC link in New Zealand.

If FTRs do not include loss hedging, participants will implicitly be expected to pay for the losses in the spot market, and may wish to hedge the implied risk. Even though we do not wish to pursue the issue of hedging the loss price risk further in this thesis, we could imagine issuing a combination of balanced ACRs and loss hedges, or general unbalanced ACRs (Harvey & Hogan, 2002; Philpott & Pritchard, 2004; Rudkevich et al., 2005; Hogan et al., 2010; Read & Miller, 2011). Unlike a conventional obligation-inclusive ACR, the latter effectively gives the holder a right to receive less water at the receiving end of the arc than is injected at the sending end of the arc.

A general unbalanced ACR can be defined as:  $(Q_j \lambda_j - Q_i \lambda_i)$ .

This could be decomposed into the following form:  $(Q_i \lambda_j - Q_i \lambda_i) + (Q_j - Q_i) \lambda_j$ .

Hence, the term  $(Q_i \lambda_j - Q_i \lambda_i)$  defines the balanced obligation component, a balanced obligation-inclusive ACR provides a perfect hedge for transportation congestion, but not for flow losses (Hogan, 2002). Technically, the  $(Q_j - Q_i) \lambda_j$  term can itself be thought of as a type of unbalanced ACR, but it is effectively a loss hedge. That is, it is an agreement to buy a specified quantity of water, corresponding to the loss volume that needs to be made up at the receiving node, at the water price for that node.

If the marginal loss rate was constant then revenue adequacy would be provided if balanced ACR issuance is within arc capacity limits, and the constant marginal loss rate in the unbalanced ACR matches the real loss rate. While participants would prefer balanced ACRs, they are likely to accept some fair proportion of unbalanced ACRs if the contracts are priced attractively.

The situation is more complicated, though, if the marginal loss rate varies. Convex piece-wise linear loss curves would generate infra-marginal rents, to be collected in the settlement surplus. One cannot accurately determine the losses because the loss rate is not known when the ACR is issued. As a result, the level of revenue adequacy is uncertain, because it depends on both the loss rate, as determined by the actual flows on the day, and the cost of making up those losses, as determined by the actual bids.

This is similar to the pumping case discussed below. The situation is slightly different, though, because piece-wise linear losses are not quite equivalent to piece-wise linear costs. Unlike marginal pumping costs, which depend on factors such as fuel prices, the costs implied by losses depend on the marginal water values. That may make them more variable, but it also means that they can be hedged by loss contracts, within the water market, as above<sup>78</sup>.

Finally, inter-temporal effects will also arise within the water transport network, due to delays in arcs. One could create ACRs to cover both space and time price differentials when a single arc can transport water from one node and time to another and node and time. In this case, we could consider defining inter-temporal ACRs like those proposed for the gas market described by Read et al., (2012)<sup>79</sup>. However, hedging linked temporal and locational marginal price differences in this thesis.

#### 4.4.3 Implications of ACR Arrangements

With the above generic definition of an ACR, we now turn to describe the way in which those instruments might be employed, in increasingly complex system situations. The goal here is not to develop a concise or comprehensive mathematical theory of ACR behaviour, but to develop insight and understanding about the real world implications and potential applications of such instruments. So the sections below discuss the way in which transmission system rents may be implicitly collected from users in the settlement surplus, and used to provide locational hedging via ACRs. But they also discuss, at a common-sense level, the implications that creating and holding such rights would have on the balance of power and benefit between participants. Clearly, there is much more that could be done, in terms of formally analysing participant interactions, but the aim of this preliminary discussion is just to

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<sup>78</sup> However, note that estimating the loss coefficient is not straightforward, as it varies with the flow level and many other parameters.

<sup>79</sup> The inter- temporal-FSRs discussed in Section 4.4.2 enable hedging between the locational marginal prices at two different times, for the same location in the network (e.g.,  $\lambda_j^{t'} - \lambda_j^t$  where  $t' \neq t$ ). But an extra unit of flow, with a delay from  $(i, t)$  to  $(j, t')$  will often be feasible when flows from  $(i, t)$  to  $(j, t)$ , and from  $(i, t)$  to  $(i, t')$  or  $(j, t)$  to  $(j, t')$  are not. This might seem to imply the need to define combined inter-locational-temporal FDRs, instead of using separate inter-locational and inter--temporal instruments, as discussed for the gas market by Read et al (2012). They argue, though, that this is not actually necessary, because the feasibility, and sustainability of the individual instruments does not matter, provided the inter-locational and inter-temporal markets are always cleared jointly by an auction process that ensures “simultaneous feasibility”.

establish the plausibility of creating and operating a market in this way. These are the kind of issues that potential participants would need to be satisfied about before they would contemplate accepting such revolutionary concepts as hydro generators having to bid in a market to induce water flow through their facilities. And we see little point in pursuing more esoteric conceptualisations until at least the possibility of such acceptance has been established. We discuss the issues in the context of three scenarios, in the following order:

- Scenario A: Consumptive benefits only in Section 4.4.1,
- Scenario B: Consumptive benefits and non-consumptive costs in Section 4.4.2, and
- Scenario C: Consumptive benefits and non-consumptive benefits in Section 4.4.3.

#### 4.4.3.1 ACR Issues under Scenario A: Consumptive Benefits Only

This section first discusses the collection of rents in a simple generic network architecture with no non-consumptive arc flow users, assuming that the initial rights are allocated to consumptive users. Then we discuss the issue of the revenue adequacy and then focus on some practical implications, in this Scenario context. We restate the ACR definition for the current scenario for the sake of completeness. An obligation-inclusive ACR is:

$$ACR_{ij}(Q_{ij}, SP) = (\lambda_j - \lambda_i - SP_{ij})Q_{ij}.$$

Here  $Q_{ij}$  denotes the contract quantity, and  $SP_{ij}$  denotes the strike price. The locational marginal price differential is  $(\lambda_j - \lambda_i)$ .

##### 4.4.3.1.1 Revenue Adequacy

In this section, we investigate revenue adequacy for an ACR under Scenario A. Suppose the SO has issued an obligation-inclusive ACR on arc  $(ij)$  for a quantity of  $Q_{ij}$  with  $SP_{ij} = 0$ . In order to set up a hedge against arc price differences, the users should be buying rights ahead of time. Using equation [3.10], we can write the pay-out required as:

$$ACR_{ij}(Q_{ij}, 0) = (\lambda_j - \lambda_i)Q_{ij} = \mu_{(i,j)}^+ Q_{ij} - \mu_{(i,j)}^- Q_{ij}.$$

The rents collected for the actual arc flow quantity,  $q_{ij}$  are  $\mu_{(i,j)}^+ q_{ij}$  and  $\mu_{(i,j)}^- q_{ij}$ . The arc flow capacity congestion (net) rent associated with the binding upper and lower arc flow constraints is:  $(\mu_{ij}^+ \bar{X}_{ij} - \mu_{ij}^- \underline{X}_{ij})$ .

The terms  $\mu_{ij}^+ \bar{X}_{ij}$  and  $\mu_{ij}^- \underline{X}_{ij}$  are the rents associated with the upper and lower arc capacity bounds. The rent is thus either:  $\mu_{ij}^+ \bar{X}_{ij}$ , if the upper bound is constrained, or  $-\mu_{ij}^- \underline{X}_{ij}$ , if the lower bound is constrained (in the normal stream flow direction). Comparing this rent with the ACR pay-out requirement, we get:

$$(\lambda_j - \lambda_i) Q_{ij} = \begin{cases} \mu_{ij}^+ Q_{ij} (\leq \mu_{ij}^+ \bar{X}_{ij}) & \text{if } q_{ij} = \bar{X}_{ij} \\ 0 & \text{if } \underline{X}_{ij} < q_{ij} < \bar{X}_{ij} \\ -\mu_{ij}^- Q_{ij} (\leq -\mu_{ij}^- \underline{X}_{ij}) & \text{if } q_{ij} = \underline{X}_{ij}^{80} \end{cases} \quad [4.2]$$

The term  $\mu_{ij}^+ \bar{X}_{ij}$  is the positive rent generated if those flows are constrained by the upper arc capacity limit on the specified channel/flow gate in the specified direction (i.e.  $(ij)$ ). So the first line in the above equation implies that the ACR pay-out indicated by  $(\lambda_i - \lambda_j) Q_{ij}$  on the LHS, is sufficiently funded by the arc capacity rent generated on the RHS], provided all ACR issuances  $Q_{ij} \leq \bar{X}_{ij}$ .

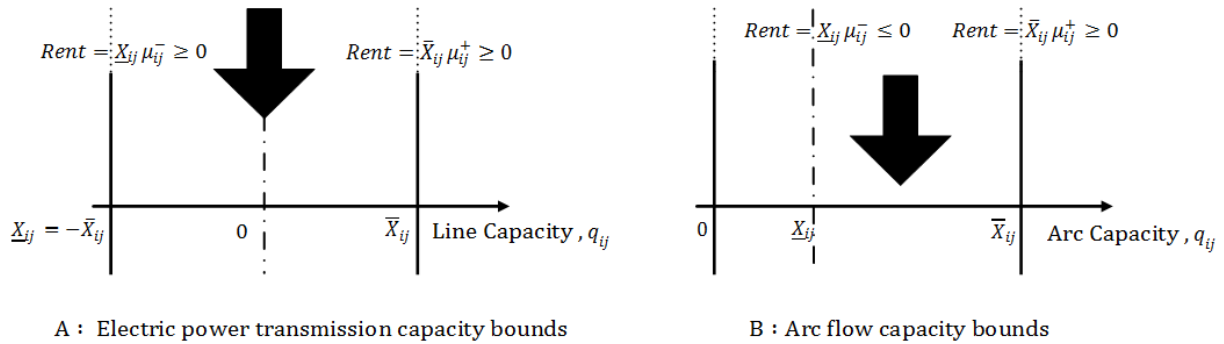
#### 4.4.3.1.2 Implications of Sign Conventions

The previous dual price analysis manipulates the canonical form of the lower bound to make the arc capacity shadow price positive:  $\mu_{ij}^- \geq 0$ . However, that same manipulation will create a negative LHS  $(-\underline{X}_{ij})$ , for a constraint expressing a positive lower bound on flows in the normal flow direction. So we still get a negative rent  $(-\mu_{ij}^- \underline{X}_{ij})$  when the lower arc capacity constraint is binding (see Diagram B of Figure 4-3). Thus we typically have both positive and negative rent components  $(\mu_{ij}^+ \bar{X}_{ij}$  and  $-\mu_{ij}^- \underline{X}_{ij})$ , unlike in electricity networks where flows are normally reversible, thus generating positive

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<sup>80</sup> This last expression is true, provided  $\underline{X}_{ij} \geq 0$ . See next section for a discussion of the alternative case.

rent when transmission capacity constraints are binding, regardless of the current flow direction (see Diagram A of Figure 4-3).<sup>81</sup>



**Figure 4-3.** Rents associated with electricity line capacity bounds and arc flow capacity bounds. Arrow indicates the water flow direction.

With regard to revenue adequacy, then, it was noted in the above that the ACR pay-out requirement is itself negative when the flows are constrained by the lower arc capacity limit on the specified channel/flow-gate in the normal stream flow direction, creating a negative price differential across the specified arc. And the final inequality above,  $-\mu_{ij}^- Q_{ij} \leq -\mu_{ij}^- \underline{X}_{ij}$  (or  $\mu_{ij}^- Q_{ij} \geq \mu_{ij}^- \underline{X}_{ij}$ ), implies that what the (obligation-inclusive) ACR holder is obliged to pay into the market settlement fund will at least cover the negative rents collected by the SO, provided  $Q_{ij} \geq \underline{X}_{ij}$ .

Thus, while ACRs are generally issued in the normal stream flow direction, they will have positive/negative value depending on whether flows are against an upper arc flow limit or against a lower arc flow limit. On the other hand, a participant who wishes to transport water against the normal flow direction along some arcs could be offered ACRs in that direction, with the value of those being equal, but opposite, to the value of ACRs in the normal stream flow direction. But, unless reverse flow is physically possible, such reverse rights could only be issued as an offset to rights issued in the forward direction. In fact, rational market-clearing implies that issuing a forward right to party 1, and a

<sup>81</sup> If arc flow is specified with a net flow variable, it could have positive ( $\bar{X}_{ij}$ ) and negative ( $\underline{X}_{ij}$ ) bounds. A negative lower net arc flow limit indicates that one could send a flow up to  $(-\underline{X}_{ij})$  units in the reverse stream flow direction. This could be achieved using a pump, but a syphon or common pond can also accommodate bi-directional flows. In such cases we will have a negative bound quantity, and a negative price, generating positive rents, as in the electricity transmission system.

reverse right to party 2, is equivalent to having to party 2 sell the forward right to party 1, at the market clearing price.

This means that the SO could issue obligation-inclusive ACR rights exceeding the actual physical capacity in one direction, provided she issues an offsetting amount in the reverse direction. The extra pay-outs required in one direction would be covered by payments made by those holding ACRs in the opposite direction. Note, though, that the net issuance of ACRs on any arc is still constrained to lie between the minimum and maximum arc capacity, in that direction. And that means that the settlement surplus can only support node-to-node ACRs up to the minimum of the arc capacities along the path between the nodes, in the relevant direction. In particular a “blocked arc”, with no flow possible in the desired direction (e.g., because it is uphill) means that the path actually has zero capacity. So the net issuance, in that direction, must not exceed zero, if revenue adequacy is to be maintained.

The possibility of negative rents is a different issue though. They are not associated with the existence of reverse flow capacity (which would actually create positive rents), but with minimum flow requirements in the forward direction. That may seem surprising, and we should consider what it means, and what would happen if participants did not hold ACRs corresponding to at least the minimum flow requirement.

Recall that we are assuming that the SO is strictly following the market clearing solutions with regard to dispatching of water flows, and consider what would happen if the SO were made responsible to keep flows within bounds, and no arc flow capacity rights had been issued. The minimum in-stream flow requirements will not be met if the downstream participants do not bid to consume sufficient water. In trying to meet these in-stream minimum flow requirements, the market clearing solution must lower the downstream delivery prices to encourage downstream consumptive demands. So water will have to flow from a higher to a lower priced node, implying a loss in the settlement surplus collected by the SO. If the SO were to face a penalty for not meeting the in-stream minimum flow demands, and lowering the downstream marginal water value to the point where the upstream-downstream marginal

water value difference is less than the penalty value produced insufficient response, the SO will prefer to pay the penalty, rather than lower downstream prices any further.<sup>82</sup>

#### 4.4.3.1.3 Implications of ACR Allocations

From the above, we see that, if the SO starts out without any (financial) rights protecting its position, it can incur a loss in the settlement surplus, whenever she has to meet the minimum in-stream flow requirements. The SO could be protected against that risk, though, if downstream users take up ACRs for (at least) the minimum flow quantity, thus committing them to pay into the settlement surplus, to cover the deficit, when it occurs. That negative value may be more than offset by the payments made to holders when the maximum flow constraint binds, thus giving the ACR a positive value overall. However the SO might have to compensate participants, up-front, in order to get them to accept ACRs for the minimum flow quantity, if they turn out to have a negative value, overall. Alternatively, in a regime where ACO call options are issued, we could think of creating a financial “put” option, under which the SO would receive:

$$ACP_{ij}(Q_{ij}, SP_{ij}) = \max\{-(\lambda_j - \lambda_i) - SP_{ij}, 0\} \text{ where } Q_{ij} = \underline{X}_{ij}$$

Here participants have the obligation to accept water flow (or to pay as if they were making water flow) if the SO exercises the option, which it will when  $\lambda_i - \lambda_j \geq SP_{ij}$ .<sup>83</sup> In this case, with no non-consumptive costs or benefits, the SO faces no risk, if  $SP_{ij} = 0$ . However, such put options obviously

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<sup>82</sup> If we assume free disposal of downstream water, it will always be possible to get sufficient response, when the downstream marginal water value falls to zero. In principle, though, negative prices must be allowed for when downstream flows cannot be disposed of costlessly, e.g., if downstream agriculturalists have the right to refuse the water during floods.

<sup>83</sup> Alternatively, participants could have the obligation to take water at the downstream node (or to pay as if they were taking water) when the price there,  $\lambda_j \leq \lambda_i - SP_{ij}$ . But note that the upstream price in that expression is unknown. In order to guarantee incentives to maintain flow, such a put option would have to be accompanied by a contract to buy the water at the upstream node. Otherwise, incentives to maintain minimum flows would prove insufficient whenever the upstream price exceeded whatever upstream price was assumed in setting the strike price for the downstream put option. So a put option on the water price differential is really more effective.

have negative value for participants and, if they were not pre-allocated, the SO would definitely need to pay participants to accept them.<sup>84</sup>

In reality, an SO is unlikely to accept a position where they had the responsibility of maintaining minimum flows, without also having the right to over-ride market solutions in order to do so, or having commercial protection when they did. “Put option” obligations corresponding to minimum in-stream flows, perhaps packaged into obligation-inclusive ACR rights, would probably have to be established prior to market start.

Allocation of initial property rights prior to market start is important for both water and water distribution capacity. In general, participants may be contested for such rights. The form of such rights may be implicit and sometimes unclear<sup>85</sup>. Clearly the issue of allocating these initial “rights” to participants needs further attention. Similar situations will also arise in contexts involving non-consumptive uses, and Section 4.4.2 discusses the case where non-consumptive pumping costs are present in the network. The purpose of the above discussion is just to point out the need for re-casting such traditional rights into financial instruments, in order to remain effective in a market context.

#### **4.4.3.2 ACR Issues under Scenario B: Consumptive Use and Non-consumptive (Pumping)**

##### **Costs**

So far this chapter has briefly discussed flow losses on arcs, but ignored any arc flow costs such as pumping costs (or usage fees). Scenario (B) involves pumping of flows from one point to the other in the network. Assuming no other non-consumptive uses on the arc, the  $\sum_{k \in \Omega(i,j)} \sum_b P_{k,b}^t y_{k,b}^t$  component of the primal objective function (see Equation 3.1 in Section 3.3.1) would be just the pumping cost function. The pumping cost function is assumed to be a convex and piecewise linear, implying a stepped non-decreasing marginal pumping cost function. Note that the primal formulation in Section 4.4.1 involves non-consumptive participants each offering a “demand curve” defining benefits produced by “forward” flows on arcs, which we can generally assume will be in a downstream

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<sup>84</sup> The issue of “gaming incentives” is beyond the scope of this thesis, but its importance cannot be ignored because it significantly impacts on participant bidding strategies.

<sup>85</sup> They are determined the factors such as physical configuration and history of the system, legal aspects, and economics.



direction. Let us now introduce a new class of non-consumptive pump participants, each offering a ‘supply’ curve for (pumped) “forward” flows on arcs, which we would assume to be in an upstream direction. Instead, we let the arc flow be specified by a net downstream flow variable, and allow a negative bound on  $x_{ij}$ , to indicate that pumping upstream, from  $j(k)$  to  $i(k)$  at the maximum rate of  $\underline{X}_{ij}$  would send a negative net flow of  $\underline{Y}_k = -\bar{X}_{j,i}$  units in the forward stream flow direction, i.e., downstream in this case. This gives us a modified version of primal constraint equation [3.5a] for pumped arcs:

$$\underline{X}_k + \sum_b y_{b,k}^t \leq x_{i(k),j(k)}^t \quad \text{where} \quad \underline{Y}_k = -\bar{X}_{j,i}.$$

The constant appearing in the above primal constraint implies an additional “negative benefit term”  $(-\sum_b P_{k,b}^t \bar{Y}_{k,b}^t)$  in the primal objective function. Because it is constant, this is irrelevant in LP terms, but it represents the cost of maximum pumping, at the maximum upstream flow rate. Above that level, though, we now have a “demand curve” with increasing benefit as  $x_{i(k),j(k)}^t$  increases with each successive positive  $y_{b,k}^t$  variable, representing a reduction in pumping, with a positive benefit coefficient  $P_{k,b}^t$ .<sup>86</sup> The above complies with the notion of net demand curves of the CDDP models discussed in other chapters.

In the interests of simplicity, this section assumes a constant marginal pumping cost function, so that there is only one step in the arc pumping cost offer curve, with offered marginal cost  $P_{ji}^t$ . Assuming the upper and lower bid limits are not binding<sup>87</sup>, the shadow price on the arc flow constraint must be:  $\omega_{ij} =$

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<sup>86</sup> Starting the demand curve from a firm lower bound makes sense in this case, because it is physically impossible for upstream flows to exceed the pumping limit. For normal arcs, though, we defined our demand curve to start from 0, rather than  $\underline{Y}_k$ . On those arcs it often will be physically possible for downstream flow to be less than a non-zero lower bound of  $\underline{Y}_k$ , even if non-csv users only bid for incremental release, above that level, as discussed in Scenario C. This raises the question as to who might be responsible for setting flows at least that high. If the SO is made responsible it may need a put-option to protect its position, as discussed in Section 4.4.6.3.

<sup>87</sup> That is, the block bounds are wide enough to ensure that, if any limit binds it will be an arc flow capacity bound. So we only need to consider the shadow prices on one set of bounds, not a combination of shadow prices on one set of bounds and/or the other.

$P_{ij} = -P_{ji}$ <sup>88</sup>. So, using [3.10], the relationship between the nodal price difference (per unit) and the marginal cost of pumping (per unit), is:

$$\lambda_j - \lambda_i = P_{ij} + (\mu_{(i,j)}^+ - \mu_{(i,j)}^-).$$

In the case of pumping, the downstream-upstream price differential  $(\lambda_j - \lambda_i)$  will be negative, unless flow is at a lower bound. Considering the  $j \rightarrow i$  direction, the above per unit rent expression can be transformed into the following form:

$$(\lambda_i - \lambda_j) = P_{ji} + (\mu_{(j,i)}^+ - \mu_{(j,i)}^-)^{89}$$

Suppose that the SO issues an obligation-inclusive ACR for arc  $(ij)$  to move  $Q_{ji}$  from  $j$  to  $i$ , with a strike price,  $SP_{ji}$ . We have:

$$ACR_{ji}(Q_{ji}, SP_{ji}) = (\lambda_i - \lambda_j - SP_{ji})Q_{ji}$$

Because the pay-out is directionless, the following will be true:

$$ACR_{ji}(Q_{ji}, SP_{ji}) = ACR_{ij}(Q_{ij}, SP_{ij}).$$

Hence, we can formulate the pay-out of an ACR issued in the reverse stream flow direction similarly to an ACR issued in the normal stream flow direction with appropriate sign transformations, and then we can proceed to explain the revenue adequacy problem.

#### 4.4.3.2.1 Revenue Adequacy with Arc Costs

One could think of unpacking an ACR into two separate CfD contracts in which the first contract (i.e., locational marginal price differential) hedges against the capacity constraint rent risk and the other

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<sup>88</sup> The coefficients for the  $x$  variables would actually be  $P_{ij} = P_{ji}$ , because positive  $x$  represents decreasing pumping in the reverse stream flow direction, which increases benefit by saving money. In this simplified context, we have  $P_{ij} = -P_{ji}$ , though, because we are applying the coefficient directly to the arc flow variable, which is negative, in the downstream flow direction.

<sup>89</sup> Let  $P_{ij} = -P_{ji}$ ,  $SP_{ij} = -SP_{ji}$ ,  $Q_{ij} = -Q_{ji}$ . Using this transformation, we can write the per unit rents as follows:

$$\begin{aligned} (\lambda_j - \lambda_i - P_{ij})(+1) &= (\mu_{(i,j)}^+ - \mu_{(i,j)}^-)(+1) \rightarrow (\lambda_j - \lambda_i - (-P_{ji}))(-1) = ((-\mu_{(ji)}^+) - (-\mu_{(ji)}^-))(-1) = \\ (\lambda_i - \lambda_j - P_{ij}) &= (\mu_{(ji)}^+ - \mu_{(ji)}^-). \end{aligned}$$

The following relationship can be formed for the pay-outs of the ACRs issued in  $i \rightarrow j$  and  $j \rightarrow i$  directions:

$$\begin{aligned} ACR_{ij}(Q_{ij}, SP_{ij}) &= (\lambda_j - \lambda_i - SP_{ij})Q_{ij} = (\lambda_j - \lambda_i - (-SP_{ji}))(-Q_{ji}) = (-\lambda_j + \lambda_i - SP_{ji})Q_{ji} = \\ ACR_{ji}(Q_{ji}, SP_{ji}). \end{aligned}$$

contract hedges against variation in the marginal non-consumptive cost (benefit)<sup>90</sup>. In that form, a CfD with a non-zero strike price subtracts a constant  $Q_{ij}SP_{ij}$  from the pay-out received by the participant, on a contract of volume  $Q_{ij}$ . Under this contract, the participant is unconditionally obliged to pay an agreed amount,  $Q_{ij}SP_{ij}$  in the specified time period. So this strike price really has no impact on the risk reducing effect of a CfD<sup>91</sup>, and is just subtracted from the total rent received by the participant. So the difference between adjusting the strike price in the CfD and adjusting the purchase price of the CfD, really only relates to the timing of payments, and even that will depend on the detail of market arrangements. Thus, when we say that we have revenue adequacy for a CfD, provided the strike price is above level  $P_{ij}$ , we could equally well say that selling that same CfD, but with a zero strike price, will make a net positive contribution to the settlement surplus, provided the buyer buys the CfD for at least  $P_{ij}$ , discounted to the time when the CfD is paid for.

Still, we will persist with the CfD form used above, because it allows us to easily state revenue adequacy results. Clearly, the value of this CfD can be re-expressed, using a composite of the shadow price on the arc flow capacity limits and the contract quantity  $Q_{ij} = (-Q_{ji})$  as follows:

$$ACR_{ji}(Q_{ji}, SP_{ji}) = (\lambda_j - \lambda_i - SP_{ij})Q_{ij} = (P_{ij} - SP_{ij})Q_{ij} + (\mu_{ij}^+ - \mu_{ij}^-)Q_{ij}$$

Considering the re-expressed upper and lower arc capacity bounds (i.e.,  $\bar{X}_{ij} = -\underline{X}_{ji}$  and  $\underline{X}_{ij} = -\bar{X}_{ji}$ ), and noting that the  $P_{ij}$  component in the price differential must be paid to the pumper, on the volume pumped, the following shows the net rent components associated with the arc:

$$\text{Rent}_{ji}(q_{ji}, Q_{ji}) = \text{Rent}_{ij}(q_{ij}, Q_{ij}) = \begin{cases} \mu_{ij}^+ q_{ij} & \text{if } q_{ij} = \bar{X}_{ij} = -\underline{X}_{ji} \\ 0 & \text{if } \underline{X}_{ij} < q_{ij} < \bar{X}_{ij} \\ -\mu_{ij}^- q_{ij} & \text{if } q_{ij} = \underline{X}_{ij} = -\bar{X}_{ji} \end{cases}$$

Taking the difference between the rent and the ACR pay-out, the net surplus equation can be expressed in a general form as follows:

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<sup>90</sup> A similar approach can be found in Harvey et al (1996).

<sup>91</sup> The situation for put or call options is very different, because there the strike price determines whether a payment will be made, or not.

$$\text{Netsurplus}_{ji}(q_{ji}, Q_{ji}) = \text{Netsurplus}_{ij}(q_{ij}, Q_{ij}) = \begin{cases} \mu_{ij}^+ q_{ij} - (P_{ij} - SP_{ij})Q_{ij} - \mu_{ij}^+ Q_{ij} & \text{if } q_{ij} = \bar{X}_{ij} = -\underline{X}_{ji} \\ -(P_{ij} - SP_{ij})Q_{ij} & \text{if } \underline{X}_{ij} < q_{ji} < \bar{X}_{ij} \\ -\mu_{ij}^- q_{ij} - (P_{ij} - SP_{ij})Q_{ij} + \mu_{ij}^- Q_{ij} & \text{if } q_{ij} = \underline{X}_{ij} = -\bar{X}_{ji} \end{cases} \quad [4.3a]$$

Thus revenue sufficiency is guaranteed, if the net surplus, after settling the ACR, is positive. With the strike price ( $SP_{ij}$ ) set to equal  $P_{ij}$ , the middle term  $(P_{ij} - SP_{ij})Q_{ij}$  drops out, and we can form the following inequalities as in the previous scenario:

$$ACR_{ji}(Q_{ji}, SP_{ji}) = ACR_{ij}(Q_{ij}, SP_{ij}) = \begin{cases} \mu_{ij}^+ Q_{ij} (\leq \mu_{ij}^+ q_{ij}) & \text{if } q_{ij} = \bar{X}_{ij} = -\underline{X}_{ji} \\ 0 & \text{if } \underline{X}_{ij} < q_{ji} < \bar{X}_{ij} \\ -\mu_{ij}^- Q_{ij} (\leq -\mu_{ij}^- q_{ij}) & \text{if } q_{ij} = \underline{X}_{ij} = -\bar{X}_{ji} \end{cases} \quad [4.3b]$$

The net-surplus for this case is as follows:

$$\text{Netsurplus}_{ij}(q_{ij}, Q_{ij}) = \begin{cases} \mu_{ij}^+ (\bar{X}_{ij} - Q_{ij}) & \text{if } q_{ij} = \bar{X}_{ij} \\ 0 & \text{if } \underline{X}_{ij} < q_{ji} < \bar{X}_{ij} \\ \mu_{ij}^- (\underline{X}_{ij} - Q_{ij}) & \text{if } q_{ij} = \underline{X}_{ij} \end{cases}$$

So the top and bottom expressions in [4.3b] yield a tight revenue adequacy result, based solely on arc capacity rents, as above, provided the net ACR volume (issuances) lies between  $\underline{X}_{ij}$  and  $\bar{X}_{ij}$ . But the strike price on all ACRs issued can only be set equal to  $P_{ij}$  if we know the marginal pumping cost, at the time the ACRs are issued. In general there will be a net surplus or deficit corresponding to the  $(P_{ij} - SP_{ij})Q_{ij}$  term in [4.3a]. Note that this term only depends on the contract quantity  $Q_{ij}$ , and not physical flow,  $q_{ij}$ . This payment is made to or from the participant, irrespective of the actual flow level, if any. This is because an obligation inclusive financial right has the same financial effect as a physical agreement to actually move the agreed amount of water. Thus, by accepting such a right, the participant is agreeing to make, or receive, payments as if that amount of water was moved, possibly at a profit ( $P_{ij} < SP_{ij}$ ), or possibly at a loss ( $P_{ij} > SP_{ij}$ ). This complicates the overall revenue sufficiency situation.

#### 4.4.3.2.2 Profit/Loss Trade-offs on ACRs with Arc Costs

First, Figure 4-4 describes the revenue adequacy situation for the top relationship in [4-3a].<sup>92</sup> If  $P_{ij} < SP_{ij}$  (i.e.,  $P_{ji} > SP_{ji}$ ) the ACR issuer will have to pay a positive base payment (shaded area  $C$  in the LHS diagram in Figure 4-4) of  $(P_{ji} - SP_{ji})Q_{ji}$ .<sup>93</sup> An additional payment, shown as area 'A', represents the term  $(\mu_{ij}^+ Q_{ji})$ , which will only be required if the arc capacity limit binds, as assumed in Figure 4-4. In that figure, area 'A + C' represents the ACR pay-out if  $P_{ji} > SP_{ji}$ . Area 'A + B' indicates the surplus collected by the SO, on the actual flow,  $q_{ji}$ , which in this case is at the lower bound in the upstream direction. The common area 'A' cancels out (i.e.,  $(A + B) - (A + C)$ ), and revenue adequacy will be met if  $B > C$ . In other words, we could have revenue adequacy, provided the SO has not issued too many ACRs. But it is not easy to guarantee revenue adequacy because the extent to which issuances need to be scaled back depends on the degree to which  $P_{ji}$  exceeds  $SP_{ji}$ . As a result the risk being taken on by the issuer depends on how accurately one can estimate the future pumping offer price. Obviously that risk may be significantly reduced if forward contracts can be obtained for pumping.<sup>94</sup>

In the case of  $SP_{ji} > P_{ji}$ , the rent difference  $(SP_{ji} - P_{ji})Q_{ij}$ , indicated by area  $C$  in the RHS diagram in Figure 4-4, will be added to the settlement surplus, representing a profit<sup>95</sup>. Area 'A - C' thus indicates the net payment to be made to the ACR holder, if the upper arc capacity constraint in the upstream direction binds. In this case it is positive, but it could easily be negative, and always will be if neither flow limit binds. In any case, the net payment required is always smaller than the rental component collected in A, even before accounting for the additional rental component in B. So revenue adequacy is

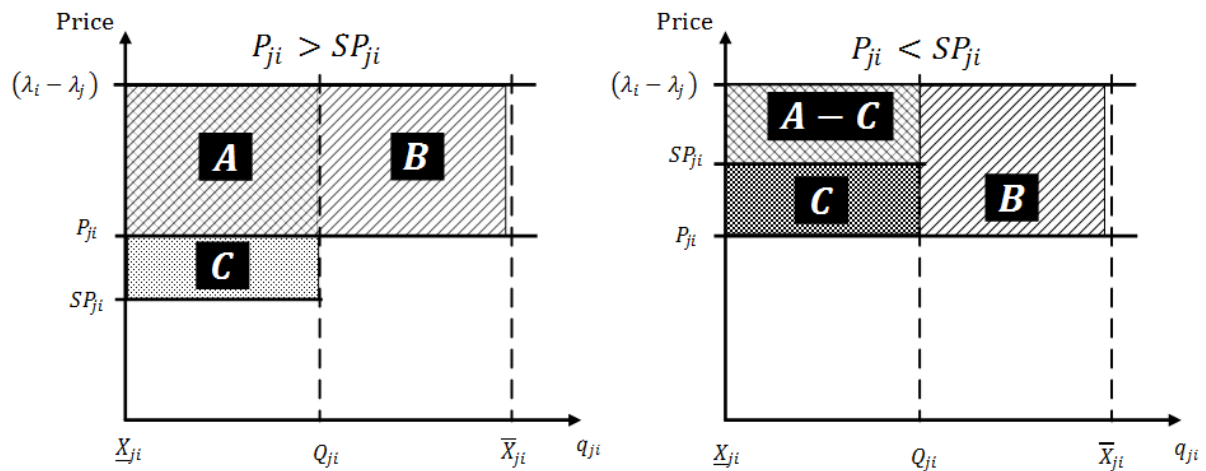
<sup>92</sup> This figure has been drawn with all flows in the upstream ( $ji$ ) direction, which is more intuitive for a pump.

<sup>93</sup> Deduction of area  $C$  from the settlement surplus arises because the ACR holder agreed to pay the SO a pumping cost of  $SP_{ji}$ , but the SO needs to pay the pump  $P_{ji}$  (the offered marginal cost), per unit, in the spot market.

<sup>94</sup> This situation is very like that discussed by Miller and Read (2011) with respect to covering ancillary service support costs for inter-regional transmission in New Zealand.

<sup>95</sup> Addition of area  $C$  to the settlement surplus arises because the ACR holder agreed to pay the SO a pumping cost of  $SP_{ji}$ , but the SO only needs to pay the pump  $P_{ji}$ , per unit, in the spot market.

guaranteed because there is always a positive net surplus, as indicated by area  $B + C > 0$ , in the RHS diagram in Figure 4-4<sup>96</sup>.



**Figure 4-4.** Pumping costs, strike prices and revenue adequacy

Suppose, instead, that arc flow level lies within the arc flow capacity bounds. The net surplus is then equal to  $(SP_{ji} - P_{ji})Q_{ji}$  in the middle relationship of [4.3a], as represented by the shaded area  $C$  in Figure 4-4. Participants holding ACRs are effectively agreeing to pay (only) the projected pumping cost,  $SP_{ji}$ , when they settle up in the spot market, if no capacity constraints are binding. In that case, revenue adequacy will only be met if the pumping cost,  $P_{ji}$ , turns out to be less than the strike price  $SP_{ji}$ .

The first relationship in [4.3a], arises because an ACR of this form also obligates the holder to use those facilities (or at least to pay or be paid as if it used them), even if the price differential is less than the strike price ( $\lambda_i - \lambda_j < SP_{ji}$ ), and possibly negative. At that point, net downstream flow must equal its upper bound,  $\bar{X}_{ij} = -\underline{X}_{ji}$ . Often a pumped channel is an artificial path, which is not likely to have any “environmental” flow requirement.<sup>97</sup> So we can assume that  $\bar{X}_{ij} = 0$  and thus the rent generated when

<sup>96</sup> In fact there may be a significant surplus, which suggests the possibility of issuing ACRs beyond the arc flow limit. As above, though the number of issuances would depend on the degree to which  $P_{ji}$  turned out to be less than  $SP_{ji}$ , and there is generally no guarantee that it will be less than  $SP_{ji}$  at all. Thus the overall effect of uncertainty with respect to  $P_{ji}$  is always to reduce ACR issuances, as above.

<sup>97</sup> A channel whose flow can be reversed by pumping might have minimum flow requirements in the downstream direction, although these would have to apply in different periods from those in which pumping occurred. For the sake of completeness, though, there is a theoretical possibility of negative rent associated with a positive

the “upper” downstream flow limit (corresponding to the lower upstream limit in Figure 4-4) is binding will be zero, too. We will have revenue adequacy, provided:

$$\text{Netsurplus}_{ji}(\bar{X}_{ij}, Q_{ji}) = \mu_{ij}^+ \bar{X}_{ij} - (P_{ij} - SP_{ij})Q_{ij} - \mu_{ij}^+ Q_{ij} = (SP_{ji} - P_{ji})Q_{ji} + \mu_{ij}^+ Q_{ji} \geq 0$$

$$\text{i.e., provided: } (SP_{ji} - P_{ji} + \mu_{ij}^+) > 0$$

Since  $\mu_{ij}^+$  is never negative, revenue adequacy will always be met if  $SP_{ji} \geq P_{ji}$ , as for the previous case, where flow is not against either limit. In this case revenue adequacy may be boosted by rent from the lower upstream limit, if that limit does bind, and that may sometimes provide revenue adequacy even when  $SP_{ji} < P_{ji}$ . But there is no certainty of occurring  $(SP_{ji} - P_{ji} + \mu_{ij}^+) > 0$  with  $SP_{ji} - P_{ji} < 0$ . So, as in the above, ACR volume cannot really be increased on that basis, without risk to the issuer.

#### 4.4.3.2.3 Arc Capacity Options with Arc Costs

Our discussion has focussed on obligation-inclusive ACRs, because these can be concatenated into general point-to-point FDRs. However, it seems unlikely that participants will want to bind themselves to pay a penalty in situations where they do not actually want pumping to occur. They will most likely prefer ACOs, which do not imply such obligations:

$$\begin{aligned} ACO_{ij}(Q_{ij}, \pi_{ij}) &= \max\{(\lambda_j - \lambda_i - SP_{ij}), 0\}Q_{ij} = \max\{(\lambda_i - \lambda_j - SP_{ji}), 0\}Q_{ji} = \\ &ACO_{ji}(Q_{ji}, SP_{ji}) \end{aligned}$$

Suppose that the strike price can be set to  $SP_{ji} = P_{ji}$  in advance. Then ACO holder will exercise the option when locational marginal price differential is greater than the marginal cost of pumping ( $\lambda_i - \lambda_j < SP_{ji} = P_{ji}$ ). And those are precisely the situations in which pumping will be optimal for the system, and scheduled by the market. Clearly, the pay-out required by such an ACO will either be zero, as in the middle relationship of [4.3b], or positive, as in the last relationship. Thus the (zero) upper limit

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minimum flow requirement in the reverse direction that is upstream, in this case. Obviously, the negative rents plus the pumping costs will lead the SO to incur a loss in the settlement surplus if she directs pumping to meet these minimum in-stream flow requirements, in that direction, when the participants do not bid to consume sufficient water. We could again think of assigning a “put option” obligation to participants corresponding to minimum flows with a strike price of  $SP_{ij} \geq P_{ij}$ , under which participants accept to pay the arc capacity contract price as in the previous scenario on aggregate quantity of  $\underline{X}_{ij}$ , when the SO exercises the option, i.e., when  $(\lambda_i - \lambda_j) < SP_{ij}$ . Similarly to the previous case, these could be implicit in obligation-inclusive ACR rights.

on nett downstream flow, and its shadow price, becomes irrelevant. However, if pump owners were committed to honouring such contracts, it is likely that they would submit their offers close to the strike price, because the contract gives strong incentives to operate the pumps if  $\lambda_i - \lambda_j < SP_{ji}$ .

The situation would be more complex if flows were against their lower bound, downstream, i.e., pumping is at a maximum, there would be a positive shadow price on that bound, creating a price difference implying an excess rental component corresponding to the difference between the pump capacity and the ACR volume issued<sup>98</sup>.

But note that setting the strike price  $SP$  equal to  $P_{ji}$ , would require us to know the “offered” marginal price in advance. If the SO owns the pump, we can at least assume that the “offer cost” is the true marginal pumping cost, but  $P_{ji}$  may vary with pumping rate, in a piece-wise linear fashion. That creates uncertainty, and jeopardises revenue adequacy. But revenue adequacy will be improved because the infra-marginal pumping rents implied by a piece-wise linear curve create an additional participant/producer surplus (similar to the transmission line loss rents) that will be collected in the settlement surplus<sup>99</sup>. Just as for transmission losses, the actual cost of pumping would not be hedgeable, but the infra-marginal pumping rent would form part of the settlement surplus, from which the SO could fund the issuance of ACRs etc.

#### 4.4.3.2.4 Implications of ACR/O Allocation with Arc Costs

If an asset is separately owned, e.g., by participants or external service providers, the SO may have limited control, and will not normally collect the additional rent. We have assumed that pump owners

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<sup>98</sup> Numerical example: To give an example, suppose the upstream price  $\lambda_i = 18$ , the downstream price  $\lambda_j = 14$ , marginal cost of pumping  $P_{ji} = 6$ , and the strike price  $SP_{ji} = 5$ . Now the locational price differential in  $j \rightarrow i$  direction is:  $\lambda_i - \lambda_j = 18 - 14 = 4$ . So pumping is undesirable, at a cost of 6, and the lower pumping bound will be active in the LP. Here  $\lambda_i - \lambda_j < SP_{ji}$  implies that, if aggregate ACR holdings are positive, ACR holders will be paying into the settlement surplus (i.e., receiving  $(\lambda_i - \lambda_j - SP_{ji}) = 18 - 14 - 5 = -1 < 0$  per unit), reflecting an implied commitment to support pumping at a positive rate, even when it is not economic. A similar issue will be described in Section 4.4.3 (Scenario C) with the non-consumptive arc flow benefit use.

<sup>99</sup> With the existence of block limits, the following relationship for the marginal pumping cost is formed by combining [3.10] and [3.11]:  $P_{k,b}^t = (\mu_{(i,j),t}^+ - \mu_{(i,j),t}^-) - (\rho_{b,k,t}^+ - \rho_{b,k,t}^-) + \lambda_{i,t} - \lambda_{j,t}$ . This creates participant surpluses (e.g.,  $\rho_{b,k,t}^+ \bar{X}_{b,k}$ ) which could be explained similarly to the consumer surpluses created when the consumptive bid constraints are binding, as shown in Figure 4-4.



bid into the market, possibly over and above the marginal cost of pumping. So they would be paid their share of the clearing price difference, which is equal to the arc price differential minus the arc capacity rents, if we assume that they do not also own the arc capacity. Or they may own both assets, and collect the entire inter-nodal marginal price difference. Either way, the SO would not collect that part of the settlement surplus, and ACRs or ACOs funded by that portion would have to be issued by the arc capacity/pump owner. That owner may well also collect additional rent corresponding to the discrepancy between its offered pumping prices and the true marginal cost of pumping, but presumably it will want to set strike prices to at least cover the marginal cost of pumping, and reserve prices to try and cover the cost of providing and maintaining the pump facility. So participants might have to buy separate rights for arc flows and pump usage, as discussed above and, in theory, these ACRs and ACOs issued by different parties could all trade in the same market.

Or the SO could issue composite rights, provided it contracts with the asset owners in a way that gives it the rents required to support the rights it issues. One way to do that would be to use either physical or financial contracts of the same general form as discussed above. This may be preferred from the market design perspective, because participants, who may be competitors, prefer to deal with pump owners through the SO. Also, once such contracts are in place, or hedges issued, market efficiency will increase because the incentives of asset owners to increase profits by “gaming” offer prices will be greatly reduced. But these topics lie beyond this thesis. We now turn to discuss ACRs for situations where non-consumptive use has positive benefits in the forward stream flow direction.

#### **4.4.3.3 ACR Issues under Scenario C: Consumptive and Non-consumptive Benefits**

Suppose that the network now has both consumptive and non-consumptive use (e.g. hydropower), with a known positive marginal benefit,  $P_{ij}$ . In the interests of simplicity, we will not consider pumps in this scenario.<sup>100</sup> We assume no parallel arcs and a single non-consumptive use on arc  $(ij)$ . Assuming upper

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<sup>100</sup> In reality, some systems (e.g., a pumped storage power generator) provide both non-consumptive benefits (in the forward stream flow direction) and pumping (in the reverse stream flow direction). But we can refer to the previous section when a participant wishes to send flows against the normal flow direction provided that it is physically supported.

and lower limits on the (single) non-consumptive bid block are not binding, as in the previous scenario, [3.11] reduces to:  $\omega_{ij} = P_{ij}$ .

Using [3.10], the relationship between the nodal price difference (per unit) and the shadow prices (per unit), including the arc flow benefits is:

$$\lambda_j - \lambda_i = -(P_{ij}) + (\mu_{(i,j)}^+ - \mu_{(i,j)}^-).$$

Note that, unless the upper arc flow limit binds, the downstream marginal water value is less than the upstream marginal water value. So we are moving water from a higher valued node to a lower valued node. That will be optimal provided the value drop is less than or equal to the marginal benefit from non-consumptive use. But the market will release water as determined by the bid curve specified by the non-consumptive user, who desires water to flow between nodes, and the participants who wish to consume it downstream.

Ignoring other participants, the generator will always bid so that the release decision is driven by the electricity market prices. In fact, it will want flow to occur whenever the electricity market price, times the conversion efficiency for the relevant block, is greater than the price differential across the respective arc. But downstream consumptive users can also induce the market to release more water if they submit attractive bids for downstream water. This will raise  $\lambda_j$  and so lower the net cost of flow movement across arc  $(i, j)$ . That could make it worthwhile to move water in higher non-consumptive use blocks, with lower marginal benefits (e.g., because of lower efficiency) and perhaps even when no more non-consumptive use is possible, so that the market must spill water past the generation facility.

As above, the value of an obligation-inclusive ACR (now in the downstream direction) is defined by price differences as follows:

$$ACR_{ij}(Q_{ij}, SP_{ij}) = (\lambda_j - \lambda_i - SP_{ij})Q_{ij}.$$

Combining the above two equations, the value of  $ACR_{ij}(Q_{ij}, SP_{ij})$  can be re-expressed, using the shadow price differential on the arc flow capacity limits and the contract quantity  $Q_{ij}$ , as follows:

$$ACR_{ij}(Q_{ij}, SP_{ij}) = (\lambda_j - \lambda_i - SP_{ij})Q_{ij} = (-P_{ij} - SP_{ij})Q_{ij} + \mu_{(i,j)}^+ Q_{ij} - \mu_{(i,j)}^- Q_{ij}$$

The following section discusses the issue of revenue adequacy using the above.

#### 4.4.3.3.1 Revenue Adequacy and Participant Interactions with Arc Benefits

Considering the arc capacity bounds, we can write the rents generated as follows:

$$\text{Rent}_{ij}(q_{ij}, Q_{ij}) = \begin{cases} \mu_{ij}^+ q_{ij} & \text{if } q_{ij} = \bar{X}_{ij} \\ 0 & \text{if } \underline{X}_{ij} < q_{ij} < \bar{X}_{ij} \\ -\mu_{ij}^- q_{ij} & \text{if } q_{ij} = \underline{X}_{ij} \end{cases}$$

In the following, the above CfD pay-outs are subtracted from the rental receipts to form a general expression for the net surpluses:

$$\text{Netsurplus}_{ij}(q_{ij}, Q_{ij}) = \begin{cases} \mu_{ij}^+ q_{ij} - ((-P_{ij} - SP_{ij})Q_{ij} + \mu_{ij}^+ Q_{ij}) & \text{if } q_{ij} = \bar{X}_{ij} \\ -(-P_{ij} - SP_{ij})q_{ij} & \text{if } \underline{X}_{ij} < q_{ij} < \bar{X}_{ij} \\ -\mu_{ij}^+ q_{ij} - ((-P_{ij} - SP_{ij})Q_{ij} - Q_{ij}\mu_{ij}^-) & \text{if } q_{ij} = \underline{X}_{ij} \end{cases} \quad [4.4a]$$

Revenue sufficiency is guaranteed, if the net surplus, after settling the ACR, shortfall is positive. If the strike price ( $SP_{ij}$ ) can be set to equal  $(-P_{ij})$ , the  $(-(-P_{ij} - SP_{ij})q_{ij})$  term becomes zero and the following inequalities can be formed as in Scenario A:

$$\text{ACR}_{ij}(Q_{ij}, SP_{ij}) = \begin{cases} \mu_{ij}^+ Q_{ij} (\leq \mu_{ij}^+ q_{ij}) & \text{if } q_{ij} = \bar{X}_{ij} \\ 0 & \text{if } \underline{X}_{ij} < q_{ij} < \bar{X}_{ij} \\ -\mu_{ij}^- Q_{ij} (\leq -\mu_{ij}^- q_{ij}) & \text{if } q_{ij} = \underline{X}_{ij} \end{cases} \quad [4.4b]$$

We could think of a trading scenario in which an agriculturalist at the downstream node  $j$ , who already has purchased (relatively expensive) upstream water, enters into a hedging contract for an arc flow quantity of  $Q_{ij}$  in a particular time period, with an upstream hydropower generator who has the right to determine what flows on that arc. But it is more natural to think of a scenario evolving from a situation where a generator has traditionally bought water at its marginal downstream value, which was  $\lambda_j = 14$  in our example in the previous section, and it has been agreed that a generator buys upstream water, provided the downstream users' rights are protected. We will again let  $P_{ij} = 6$ , but this time it denotes the marginal benefit implied by the generator's bid, rather than the marginal cost implied by the pump's offer. The presence of the generator now raises the value of upstream water, so that  $\lambda_i = 20$ , unless a capacity limit binds. Then the position of the downstream consumer can be protected by a CfD of the

form, discussed above, with a negative strike price,  $SP_{ij} = -6$ , reflecting the benefits derived from water movement.

In the case of the middle equality in [4.4a], the non-consumptive rents (in the forward stream flow direction) are paid into the settlement surplus<sup>101</sup>. The CfD pay-out is:

$$ACP_{ij}(Q_{ij}, SP_{ij}) = (\lambda_j - \lambda_i - SP_{ij})Q_{ij} = (-P_{ij} - SP_{ij})Q_{ij}$$

Again, if  $SP_{ij}$  can be set equal to  $(-P_{ij})$  and if no arc capacity constraints are binding, the net surplus is given by the following:

$$\text{Netsurplus}(q_{ij}, Q_{ij}) = (\mu_{ij}^+ - \mu_{ij}^-)(q_{ij} - Q_{ij}) = 0.$$

This means that the holder would neither pay nor receive anything from the CfD contract. This leaves them in the same situation as a participant holding a zero-strike price CfD on an arc with no non-consumptive use, as in Scenario A. So the contract holder hedges against the locational marginal price difference. She is depending on the flows driven by the non-consumptive arc flow demand curve. Again, the non-consumptive marginal benefit,  $P_{ij}$  will be added to the spot settlement surplus by the generator, but most likely returned to the generator under another contract between the two parties.

In our numerical example, with  $SP_{ij} = 0$ , the per unit net surplus is:

$$\text{Netsurplus}(q_{ij}, Q_{ij}) = (-P_{ij} - SP_{ij})Q_{ij} = (-6 - 0) \cdot 1 = -6.$$

The contract holder would “receive” the marginal water value difference:  $\lambda_j - \lambda_i = -6$ . This implies that the holder would be required to pay a value of 6, per unit, into the settlement surplus account. This reflects the fact that, moving a unit of water, of itself, actually reduces overall system value by 6. This loss will be accepted by the agriculturalist, who requires the water to move from the upstream to the downstream, with no allowance being made, under this CfD arrangement, for any non-consumptive benefits to another party. In addition, the settlement surplus would also receive the non-consumptive marginal benefit of  $P_{ij} = 6$  per unit from the generator.

Suppose that we set the strike price to  $SP_{ij} = -8$ , with no change in the non-consumptive value. Per unit net surplus is:

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<sup>101</sup> There is no infra-marginal rent in any of these cases because  $P_{ij}$  is constant.

$$\text{Netsurplus}(q_{ij}, Q_{ij}) = (-P_{ij} - SP_{ij})Q_{ij} = (-6 - (-8)) \cdot 1 = 2.$$

Here the CfD holder agrees to pay the SO a value of 8 per unit. But the non-consumptive user only has to pay a value of 6 per unit for the movement of water in the spot market. So this positive differential contributes to the settlement surplus.

In the case of the first equality in [4.18a], the settlement surplus collects positive rents  $\bar{X}_{ij}\mu_{ij}^+$ , from the upper arc flow capacity binding in the forward stream flow direction. So ACR pay-outs are adequately supported provided  $Q_{ij} \leq \bar{X}_{ij}$  and  $SP_{ij} \geq -P_{ij}$ . If we can set  $SP_{ij} = -P_{ij}$ , only the constraint rent  $\bar{X}_{ij}\mu_{ij}^+$  contributes to the settlement surpluses. As in the previous scenario, this situation leaves the downstream participant to hedge against the upper arc flow congestion rents. We will have revenue adequacy, provided:

$$\text{Netsurplus}(\bar{X}_{ij}, Q_{ij}) = \mu_{ij}^+(\bar{X}_{ij} - Q_{ij}) - (-P_{ij} - SP_{ij})Q_{ij} \geq 0$$

The second term in the above expression disappears if  $SP_{ij} = -P_{ij}$ . There will (then?) be a positive net surplus if  $Q_{ij} < \bar{X}_{ij}$ .

Recall the following expression:

$$ACP_{ij}(Q_{ij}, SP_{ij}) = (-P_{ij} - SP_{ij})Q_{ij} + Q_{ij}\mu_{ij}^+.$$

We could imagine a scenario where the flows will be released to the downstream node because  $\lambda_i$  is too low to make it attractive for the generator to hold upstream water. Let the upstream marginal water value be set to  $\lambda_i = 18$ . So the CfD pay-out is:

$$(\lambda_j - \lambda_i - SP_{ij}) = 14 - 18 - (-6) = 2.$$

In our example, referring to the above equation, the upper limit ( $\bar{X}_{ij}$ ) will bind if  $\lambda_i$  lowers below 20, which it can if the generator would really like to “leak” the maximum through the arc. Changing the prices at the injection end implies a positive rent of  $\bar{X}_{ij}\mu_{(i,j)}^+ = 2$  per unit contributing to the settlement surplus. This rent belongs to the arc capacity owner, who could be either the hydro generator, or the SO. But that rent may be assigned to a downstream participant via a CfD with a strike price of  $SP_{ij} = -6$ . As noted in the previous section, the CfD gives the holder the right to use the arc to transport water.

If, on the other hand, flow rights were initially held by downstream agriculturalists, the generator would prefer to secure the right to control the water flow through the arc via some form of a contract with the incumbent rights holder. In that case, the downstream agriculturalists would be accepting the flows released by the generator.

In summary, at the natural negative strike price (i.e.,  $SP_{ij} = -P_{ij}$ ), the non-consumptive marginal benefit will be contributed to the spot settlement surplus. As in all other cases, the contract holder is accepting the non-consumptive offers in the spot market to move water on the arc. With the term  $(-P_{ij} - SP_{ij})Q_{ij}$  disappearing, the above equation gives us a strict revenue adequacy result. Even ignoring risk reduction, these contracts will have a positive expected value to the holder. Setting the strike price higher improves revenue adequacy, but it reduces the net value to the holder, thus reducing what participants are prepared to pay. CfD value will easily become negative if the strike price is too high. If the strike price is set equal to zero, the CfD holder is paying  $(\lambda_j - \lambda_i) = 14 - 20 = -6 < 0$  into the settlement surplus. The marginal price differential will be equal to  $(-P_{ij} + \mu_{ij}^+)$ , and the CfD holder will receive this differential as per the spot prices<sup>102</sup>. A higher strike price may provide revenue adequacy, even when excess rights are issued, but the exact volume of excess rights that can be supported is unclear, in advance, because it depends on prices, and hence on offers, and not just on arc capacity.<sup>103</sup>

In the case of the last equality in [4.4a], the settlement surplus collects  $(-\mu_{ij}^- \underline{X}_{ij})$ , the negative rent generated by flows being constrained by the lower arc capacity limit on the specified channel/flow gate in the specified direction. For revenue adequacy:

$$\begin{aligned} \text{Netsurplus}(\underline{X}_{ij}, Q_{ij}) &= -\mu_{ij}^- \underline{X}_{ij} - ((-P_{ij} - SP_{ij})Q_{ij} - Q_{ij}\mu_{ij}^-) \\ &= \mu_{ij}^-(Q_{ij} - \underline{X}_{ij}) + (P_{ij} + SP_{ij})Q_{ij} \geq 0 \\ \text{i.e., provided: } Q_{ij} &\geq \underline{X}_{ij} \text{ and } SP_{ij} \geq -P_{ij} \end{aligned}$$

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<sup>102</sup> In the long run, any spill at the upstream leads to  $(\lambda_j - \lambda_i) > 0$ . As a result, the shadow price differential will be greater than the marginal non-consumptive benefits if the arc capacity constraints are not binding.

<sup>103</sup> This situation is similar to that for a lossy electricity transmission line, as discussed in a section 4.4.2.3 above.

The lower arc capacity limit will bind if the upstream marginal water value,  $\lambda_i > 20$ , because the upstream generator assigns a higher value on the water that they would prefer to hold upstream water, even intending to release less than the minimum through the arc. In our numerical example,  $\lambda_i = 22$  implies a negative rent,  $(-\mu_{ij}^- \underline{X}_{ij} = -2)$  per unit in the settlement surplus. Unless the minimum capacity is zero, this will be incurred by the asset owner (or SO), who presumably has an obligation to meet the minimum flow requirement as already discussed in the previous sections. But responsibility for covering that negative rent would be assigned to a participant via a CfD with a strike price  $SP_{ij} = -6$ , the payout for which will be:  $(\lambda_j - \lambda_i - SP_{ij}) = 14 - 22 - (-6) = -2$ , per unit. This means that the CfD gives the holder the obligation to use the arc capacity to transport water, but without any of the non-consumptive arc flow benefits.

So CfD pay-outs are adequately supported provided  $Q_{ij} \geq \underline{X}_{ij}$  and  $SP_{ij} \geq -P_{ij}$ . If the strike price ( $SP_{ij}$ ) is set to equal the non-consumptive marginal benefit ( $-P_{ij}$ ), we get a strict revenue adequacy result, under which the non-consumptive arc user pays the rights holder, via the settlement surplus, for the movement of water, as above. Again, this means that one can improve revenue adequacy by setting the strike price to a higher value but, as in the previous discussion, we cannot guarantee revenue adequacy for CfD volumes outside the physical flow range without knowing the offer prices.

#### **4.4.3.3.2 Implications of ACR/ACO Allocation with Arc Benefits**

As we have already noted in Scenario A, it seems unlikely that parties will willingly enter into financial contracts to make up the negative rents implied by lower flow bounds, unless paid to do so. As in the previous scenario, the SO may have to make generators accept those option contracts, at market start.

The generator may decide to sell contracts directly or sometime via the SO, only if it increase her overall welfare, in expectation. An agriculturalist will be motivated to buy these contracts at that price only if it increases her overall welfare, in expectation. The agriculturalist will get guaranteed access to reasonably priced water supply. The generator could lower her risks, at least for volumes up to their minimum flow level. However, the risk reducing effect of the contracts lowers, as volumes increase. Under such contracts, the generators are obliged maintain sometimes flow levels unattractive given electricity prices at the time. So they might prefer an option contract implying no such obligations.

Sometimes, the generators may prefer a put option contract implying no such binding commitment and giving control over the exercise<sup>104</sup>. Ideally, the generators may prefer non-standard exotic type options, linked to both electricity and water prices.

Note that in the absence of any prior property rights, a non-consumptive arc flow user might be expected to pay for the point-to-point movement of the flows. But the real-world commercial outcome depends crucially on who “owns” the flow channel, or has the right to control it. For example, if the “arc owner” is a hydropower generator, that generator will generally have had the right, prior to market establishment, to make water flow through their facility, and to decide how much should flow, perhaps subject to restrictions contained in water rights etc. Thus they should not be expected to pay (again) for that right. Instead, we would expect arrangements to be made whereby the rents arising from their bid curve, and/or because the capacity constraint is binding, would be returned to them, and/ not retained in the settlement surplus<sup>105</sup>. Then, as above, it is that participant, and not the SO who would be in a position to issue ACRs with respect to the movement of water through that particular arc. The issues such as how to allocate initial property rights not just for water but for water distribution capacity, prior to market start and how to maintain minimum in-stream flows in the system require further attention. The generator (agriculturalist) will be incentivised to sell (buy) any contract at a price that increases their overall welfare, in expectation. A hydropower generator (arc owner) would be given the right, prior to market establishment, to make water flow through their facility, and to decide how much should flow, etc. The agriculturalist can reduce risks by acquiring guaranteed access to a reasonably priced water supply. The generator might prefer an option contract implying no commitment to flow levels that may be unattractive given electricity prices at the time. Ideally the generator prefers put options or non-standard exotic type options, linked to both electricity and water prices that gives greater control over the exercise of instruments. A “put option” obligation corresponding to minimum in-stream flows, could be packaged into obligation-inclusive ACR rights, would probably have to be established prior to market start.

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<sup>104</sup> Selling a call option might make the generator worse off as it allows the buyer to determine when to exercise.

<sup>105</sup> This could be achieved by issuing them with a long term ACR for the full arc capacity.



#### **4.5 Virtual Models to Define System Rights**

Bringing together the discussions in the previous sections, we conclude this chapter by considering an alternative regime under which participants make their own supply system and network trade-offs, rather than relying on a centralised market clearing process. Rather than having participants trade FSRs and FIRs, we could assign them long-term “slice of system” rights that give proportional shares of inflow, storage, and release capacity. Such arrangements may be thought of as creating, and selling shares in, a “virtual model” of the system. Discussion of the organisational arrangements and other issues relating to this concept can be found in Starkey (2014), Hughes et al., (2013) and Barroso et al., (2012). The latter discusses several actual or proposed “virtual models” for reservoir management in the US, South America, Canada, Tasmania, and New Zealand.

A “slice of system mechanism” has been functioning for some time in the Columbia River Catchment (Barroso et al., 2012). In this style of regime, a participant’s stake includes a proportion of each and every system resource (i.e., constraint RHS in canonical form) such as net inflows, transportation network capacity and storage. Unlike traditional centrally coordinated schemes, the users determine their own physical production strategies, for their share of the system, based on their own requirements, judgments with regard to the value of water, and projections of inflow uncertainty. And, in a market environment, they receive their share of the rents/surpluses arising from spot market electricity sales. But the Hydro System Operator (HSO) aggregates all the physical production schedules, handles the physical management and coordination of the system, and seeks efficiencies where it can. Thus, this regime still manages to preserve some positive attributes of the centralised planning systems such as handling externalities and environmental minimum flow.

This approach will not provide perfect hedging, because nothing can insulate the holder from inflow risks. But Barroso et al suggest that the hedging concept can be taken further, and largely divorced from the physical management of the system. A “virtual model” of the system can be created, entirely from financial contracts, so that the holder of the financial contract is facing a share of the risks and incentives that the HSO faces, while the HSO is free to manage the system for maximum gain, given the aggregate contract position implied by all the holders exercising their rights.

For a thermal power station (with an unlimited fuel supply) the natural “virtual model” financial contract is just a (European) energy call option, which specifies quantity, delivery time, location and a strike price, based on the station’s capacity, location, and marginal production cost. These call options are normally traded as strips (see Oren, 2005), thus giving the holder a virtual share of the station capacity, for the duration of the strip.

For a hydro system, though, we could employ the “swing option” based virtual reservoir management approach described by Read et al., (2012). Swing options were developed initially for coal contracts (Joskow, 1985; 1987), and later applied in other energy markets (Kaminski & Gibner, 1995; Kaminski, 1997; Garman & Barbieri, 1997; Jaillet et al., 2004). The main feature of a swing option is that it may be exercised in multiple time intervals during the period in which exercise is allowed, but the quantity (e.g., MW) exercised must lie within a prescribed range in each time interval, while the total quantity taken during the lifespan of the swing option must also be less than the maximum limit. The holder may exercise their rights at any time based on the opportunity cost of the contract to them. Thus they are effectively managing a “reservoir” of opportunities<sup>106</sup>. Unlike electricity swing options, though, a water swing option has to cover initial storage, inflows, storage capacity and release. And, given the uncertainty in inflows, we need to slice both firm and non-firm system resources, such as inflows. The next section formulates the swing option pay-offs for a virtual share of a supply system as follows:

#### 4.5.1 Mathematical Description of Swing Option Parameters for Trading Water

- Time at which the swing option is written:  $t = 0$
- Possible exercise time intervals:  $t \in [0, T]$
- Let  $\theta_a$  be the share of the system contracted by user, with  $\sum \theta_a = 1$ . The system components can be expressed as follows<sup>107</sup>:

$$\text{Virtual storage capacity: } \bar{S}_{av} = \theta_a \bar{S}$$

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<sup>106</sup> Unlike normal option contracts, which are automatically exercised whenever the strike price is exceeded. Parties may determine a “strike price” for their own internal purposes, but this will have to be adjusted, over time, so that the cumulative exercise, over any period of time, matches the volume actually available under the contract, over that period.

<sup>107</sup> Each system resource could be apportioned asymmetrically among different parties instead of equally shared, as proposed in this formulation.

Virtual initial storage:  $S_{a_v}^0 = \theta_a S^0$ <sup>108</sup>

Virtual inflows<sup>109</sup> :  $f_{a,t_v} = \theta_a f_t$

Virtual release capacity:  $Z_{a_v} = \theta_a \bar{X}$

- Options called during period  $t_m$  by the participant  $a$  at the reservoir:  $z_{a,t}$
- System slice continuity equation:  $s_{a,t} = s_{a,t-1} + f_{a,t} - z_{a,t} \quad \forall a, t$
- System (slice) constraints:<sup>110</sup>

Storage slice limits:  $0 \leq s_{a,t} \leq \bar{s}_{a_v}$

Release slice limits:  $0 \leq z_{a,t} \leq Z_{a_v}$

- Strike price = 0, so given the spot price  $\lambda_t$ , the pay-off is:  $\lambda_{t,t} z_{a,t}$ .

Any surpluses (e.g., spill) or deficits of firm components could be sliced accordingly. A user slice entitlement account tracks the accumulated difference between the exercised resource and the slice entitlement over time, as per the continuity equation above. Under a dynamic storage leasing arrangement, as opposed to the fixed storage capacity regime described so far, the users could be given the opportunity to *trade* their storage slices over the planning horizon. But the system might also have a fall-back spill-over rule directing any “virtual spills” to be transferred to a common stock, or to other storage slice owners.

As in a centralised system, the HSO carries out all physical coordination and operation of the system (including forecasting inflows, and providing information to right holders). In the regime discussed by Read et al, the hydro system was assumed to be operating in an electricity market context, and the HSO was to be free to determine the rate of physical delivery of electricity, and where it was to be produced, based on physical and market conditions, irrespective of whether users exercised their rights or not. It is argued, though, that the HSO would have strong incentives not to deviate too far from participants’

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<sup>108</sup> Closing storage value can be ignored if the contracts are perpetual.

<sup>109</sup> This approach can easily be generalised, but here we ignore the nodal network, and related factors like nodal tributary flows, to maintain the simplicity of formulation.

<sup>110</sup> The holder of the option has to comply with all slice limits during every exercise time period. But, in practice, the right to exercise these capacity rights might be traded between participants, on a short term basis, as need change over time.

aggregate “call”, so as to minimise their own risk, and avoid moving market prices to their own detriment. As explained by Read et al, the participants also have incentives to manage their “virtual reservoirs” of callable contract capacity in much the same manner as they would manage physical reservoirs.

We conclude this discussion by noting the following. Under this regime, participants own a virtual share of each component of the reservoir system. The swing option developed here allows participants to act on their own judgement by calling the swing option in multiple time intervals during the period in which exercise is allowed, without any intervention of other participants. The central coordination of the resource is unchanged. The issues such as how to ensure sufficient incentives for participants and, how to resolve possible gaming situations (e.g., monopolizing the HSO) are beyond the scope of this study.

#### **4.6 Chapter Conclusions**

Financial property rights are often useful to develop commercial and organizational arrangements to improve liquidity and ultimately increase efficient resource use in a water market. We assume the existence of a market in “Financial Water Rights” FWRs, at multiple nodes, cleared by solving a deterministic/stochastic formulation, as discussed by Starkey (2014). In this chapter, we did not discuss or analyse the workings of an FWR market, but explored a range of financial property rights that could be developed, and may be necessary, to complement such a market, drawing on the Financial Transmission Right (FTR) concepts developed for electricity markets. These financial hedging instruments could be used to deploy the settlement surplus arising from market clearing to hedge against trading risks, across both space and time.

First, we discuss “Financial Inflow Rights” (FIRs). An FIR would be priced as a special form of FWR giving rights to water expected to flow into a storage reservoir. With fixed volume FIRs, the SO would be exposed if the expected inflows do not realise. Instead, an index-based FIR provides the holder the right to receive a volume of FWRs, with that volume being proportional to the inflow received, so the holder faces the volume risks herself, and the SO does not. The participants would be required to hold FIRs together with the other system capacity rights such as FSRs and ACRs to meet their trading requirements.

Section 4.3 then develops “Financial Storage Rights” (FSRs). Fixed-volume, obligation inclusive FSRs will hedge a specific volume stored between time periods, at the reservoir. Rents arising in the supply system could be used to support obligation inclusive FSRs, which would protect participants against exposure to supply system rent uncertainty. Deterministic FSRs could be funded by storage capacity congestion rents, but we were only able to show revenue adequacy in expectation, in a stochastic environment. We also note that fixed volume FSRs are fundamentally incompatible with indexed FIRs. So a combination of the two will not fully insulate holders from the impact of inflow uncertainties.

While this may seem disappointing, Section 4.3.3 quotes Read (2016), who suggests that this is only what we should expect, because the owner of physical storage capacity could not expect any greater certainty than that provided by these hedging instruments. He argues that the “true” marginal water value is not actually known at the time these FSRs are settled, and suggests the possibility of “retrospective” storage rights (RSRs) to cover the implications of this incompatibility. With the RSRs, the SO would not be exposed to risks if the RSR valuation and settlement can be deferred until the end of the horizon, when all uncertainty has been resolved. This implies that the actual water value at any node along that path will also be known, depending on whether storage actually did reach its bounds. That section also explores the concept of FSR-option contracts, representing a right to store water, with no obligation to do so. Again, this is more akin to the rights implied by ownership of physical storage capacity. But, detailed investigation of such concepts lies outside the scope of this thesis.

As well as supply system rents, the SO also collects transmission/distribution system rents associated with arc capacity limits, in the settlement surplus. Thus Section 4.4 moves on to discuss on distribution system rents and rights. Instead of using node-to-node FDRs similar to FTRs, this section focuses on Arc Capacity Rights (ACRs) that are more like a Flow Gate Right (FGR). ACRs could be applied to hedge against the flow congestion price risks, for an arc in the network. In the absence of loop flow problems, ACRs could be equivalent to node-to-node rights between adjacent nodes. As a result, a series of ACRs could construct a “node-to-node” right. We discussed the implications of ACRs in the context of three scenarios. For the case with consumptive benefits only, the settlement surplus can only fund node-to-node ACRs up to the minimum of the arc capacities, in the relevant direction. The negative rents are associated with minimum flow requirements in the forward direction. For the case

with non-consumptive costs and both consumptive and non-consumptive benefits, the ACR hedges against both the capacity constraint rent risk and the variation in the marginal non-consumptive cost. In general, the revenue adequacy is met because it is unlikely to have any “environmental” flow requirement in the pumped channel. For the case with consumptive benefits, the revenue adequacy is met provided that the SO has not issued too many ACRs. In both cases, the revenue adequacy may be improved by setting the strike price to a higher value but, it cannot be guaranteed for contract volumes outside the physical flow range without knowing the offer prices. In Section 4.4.3, we recommend the possibility of using put/non-standard exotic option ACRs to handle situations such as minimum in-stream flows, linked to both electricity and water prices, perhaps packaged into obligation-inclusive ACR rights.

Although none of the instruments developed in this chapter to hedge against storage capacity rents and arc flow congestion system rents have actually been implemented, they show considerable promise in terms of increasing market liquidity and improving competition. But our discussion suggests that their most important role may actually be to reduce risks of agriculturalists by acquiring guaranteed access to a reasonably priced water supply and to provide option contracts to generators implying no commitment to flow levels that may be unattractive.

Finally, section 4.5 explains the concept of creating, and allocating shares in, a swing-option-based virtual model of the system, giving proportional shares of inflow, storage, and release capacity to participants. This method expands the range of choices available for consideration by participants and others. Also, it allows some of the benefits of competition to remain, even when the system cannot be feasibly disaggregated. This concept deserves investigation, as discussed by Barroso et al (2012).

Many issues relating to the allocation, support or trading of hedging instruments, and their value as risk management instruments, have significant potential for further research, as does their potential impact on gaming behaviour. We will not pursue any of these developments further in this thesis, though. Instead, the remaining chapters focus on developing an intra-period pre-computation routine, using the concept of Constructive Dual Dynamic Programming (CDDP), as originally developed by Scott & Read (1996).

## 5.

### **A Multi-nodal Intra-period Model for a Single Reservoir Mixed-use Catchment**

#### **5.1 Introduction**

This chapter is motivated by the intra-period reservoir optimization sub problem in a single reservoir mixed-use nodal catchment. Our work contributes to the CDDP literature by adding a new CDDP intra-period model which pre-computes intra-period demand functions, for a multi-use reservoir system. This intra-period model provides a CDDP model with spatial rather than temporal stages. The intra-period nodal CDDP developed in this chapter could be applied to clear a single period spot market for water with any means of determining the storage decision. Chapters 6 through 8 then undertake a series of intra-period CDDP developments to investigate issues such as environmental service flow requirements, flow transportation delays, and two-reservoir modelling. More specifically, this chapter addresses the question of how to clear a water market to meet intra-period consumptive and non-consumptive water demands in a river catchment with a single reservoir. Here we develop the deterministic multi-nodal version of the CDDP to form a series of demand curves for release. This problem is known as the intra-period problem to be used in the first stage of the two-stage stochastic CDDP.

This chapter is structured as follows. Section 5.3 provides an overview of CDDP models. Section 5.4 describes the implicit intra-period optimisation model and the CDDP algorithm used to construct it. Proof that the algorithm constructs the desired demand curve for release is given in Section 5.5. Application of the algorithm is illustrated in Section 5.5, while Section 5.6 describes the use of the

demand curve for release constructed here to optimise the full inter-temporal model. The next section is about return flow benefits. Section 5.8 provides conclusions and suggests future directions.

## 5.2 Intra-period versus Inter-period Optimization Problems

This section is about the intra-period reservoir optimization sub-problem and the inter-period reservoir optimization problem.

Here, a simple deterministic version of the inter-period model that uses the benefit function from a nodal optimization is illustrated. For the interest of simplicity, let us assume an operating policy where the market aims to maximise the benefit of allocating water in each period over the planning horizon based on the available inflows to the reservoir. Here,  $T$  is the number of periods,  $s_t$  is the reservoir storage for period  $t$ ,  $\bar{s}$  the storage capacity,  $r_t$  is the net release (accounting for inflow) which implicitly represents the met demand in a particular time period, and  $s_0$  represents the initial and target storage level.  $z_t$  is the spill (unusable overflow) during the period. The bounds on  $r_t$  could be implicit in the definition of  $NB_t$  or they can be explicitly defined as follows.

$$\max \sum_{t=1}^T NB_t(r_t) \quad [5-1]$$

Subject to

$$s_{t+1} = s_t - r_t - z_t, \quad \forall t = 1, \dots, T \quad [5-2]$$

$$\underline{R} \leq r_t \leq \bar{R}, \quad \forall t = 1, \dots, T \quad [5-3]$$

$$s_1 = s_0, s_{T+1} = s_0 \quad [5-4]$$

$$0 \leq s_t \leq \bar{s}, \quad \forall t = 1, \dots, T \quad [5-5]$$

The objective set by the equation [5-1] seeks to maximize the net benefit for a nodal network catchment deterministically for the current and future periods similar to traditional reservoir optimization models. The concave intra-period benefit function,  $NB(r)$  gives the welfare (e.g. revenue) earned from release variable. Here we could think that  $NB(r)$  is representing the first term of the primal objective function [3-1] in Chapter 3:  $(\sum_t \sum_i \sum_b P_{i,b}^t q_{i,b}^t + \sum_t \sum_{(i,j) \in A} \sum_{k \in \Omega(i,j)} \sum_b P_{k,b}^t x_{k,b}^t)$  where  $r = x_{0,1}^t$ . For the interest of simplicity we assume that there is only a single arc connecting the downstream nodes to the



reservoir node<sup>111</sup>. A general inter-period benefit maximising objective can be written by considering both intra-period benefit function,  $NB(r)$  and the future benefit function,  $V_{T+1}(s_{T+1})$  in line with Chapter 3 formulations as follows:

$$\max_{r,s} \left( \sum_{t=1}^T NB_t(r_t) + V_{T+1}(s_{T+1}) \right)$$

Constraint [5-2] ensures continuity of storage balance across time<sup>112</sup>. In this model  $r$  can take negative values (net flow injections) as the net release includes both inflows and releases at the reservoir node. The constraint [5-3] shows feasibility bounds for the net release. Chapter 3 explains spills in detail, but here for the interest of simplicity we include a spill variable  $z_t$ , with relevant bounds. Instead a separate spill node could be included in the intra-period model to take care any spill from the reservoir.  $z_t$  is non-zero only when the end-of-period storage is at reservoir capacity or the target storage in the last period. Constraint [5-4] maintains initial and end of the final horizon (target) storage value. The restriction,  $s_{T+1} = s_0$  means that we seek a steady state condition. Instead, the end-of-horizon storage could be left free by including an end-of-horizon benefit function, for case of a single year operation. Constraint [5-5] provides the lower and upper storage bounds.

In the previous SLP, the intra-period optimization sub-problem is embedded within the inter-period optimization problem and hence it is solved using the same optimization technique as the inter-period optimization problem. On the other hand, SDP can accommodate a benefit function generated using any type of intra-period pre-commutation method. This chapter attempts to solve the intra-period single reservoir mixed-use network optimization problem of the standard dynamic programming problem, which amounts to the term  $NB_t(r)$  of the convex programming problem,  $\max_{r,s} \left( \sum_{t=1}^T NB_t(r_t) + V_{T+1}(s_{T+1}) \right)$ , for some particular period,  $t$ . Here  $NB_t(r)$  denotes this intra-period benefit function which includes both demanders and suppliers in a nodal catchment.

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<sup>111</sup> However,  $r$  becomes net flow out in general, if there are multiple arcs connected to the reservoir with some flowing into the reservoir and some flowing out.

<sup>112</sup> When the net increase in storage,  $(s_{t+1} - s_t)$ , is less than the net inflow,  $-r_t$ , the system operator has been forced to spill some inflow coming in the time period  $t$  as the intra-period model is unable to make use of it all.

This chapter seeks to develop an intra-period pre-computation routine using mix of LP and DP approaches. Here we employ a CDDP intra-period model which pre-computes intra-period net nodal demand functions, for a multi-use reservoir catchment. The intra-period nodal CDDP computational routine proposed in this chapter focuses on a recursive technique to compute the convex intra-period nodal benefit function. This approach exploits the tree structure of the river system, and builds up  $NB(r)$  by building up its derivative (i.e., net marginal benefit for release or the net demand curve for release) in terms of component curves (e.g., marginal consumptive benefit function and marginal non-consumptive benefit function) that are added inversely and normal point-wise.

This procedure constructs a ‘demand curve for release’ at the reservoir; this demand curve gives the optimal marginal value of any reservoir release. This method is quite general, and can be used for any river catchment that can be represented as a tree radiating upstream and downstream from one reservoir. It has been designed to clear markets in which water is traded between water users of various types such as consumptive users and non-consumptive users, but can also be employed to optimise integrated operations by a catchment manager. The benefits these users receive can be modelled directly or as market bids where the river catchment forms part of a market clearing model. A market bid reflects user willingness to pay as a measure of her benefit from water trading.

With a relatively little effort, the CDDP could provide price information at node/arc levels, which is useful to explain pricing differences to water users and to understand the economic drivers behind nodal pricing. In addition, CDDP in particular, efficiently solve a single reservoir optimization problem over an SP with a similarly detailed scenario tree.

Unlike the previous CDDP intra-period models (e.g., Cournot gaming intra-period model proposed by Scott & Read (1996)), the multi-nodal CDDP procedure described here applies across space not time (providing locational marginal prices across the network). The spatial CDDP may be embedded in a more traditional temporal CDDP. A participant can backtrack to explore the spatial trajectory of both successful and rejected bids from the point of market clearing to her original position. Hence a user can be better equipped to decide her bidding strategies in subsequent market rounds.

### 5.3 Constructive Dual Dynamic Programming (CDDP)

As already noted in Chapter 2, CDDP can help the reservoir manager to select competing water demands for release and storage. As a part of the reverse recursive CDDP procedure, the marginal value curve for releasing water,  $mvr_t(r_t)$  is given or constructed by the intra-period problem (similar to the sub-problem  $NB_t$  in the standard DP formulation). This is referred to as intra-period pre-computations in the CDDP. Scott & Read (1996) noted that the “pre-computations” could be thought of as defining a “demand curve for release” ( $dcr$ ) within each period.

Similar to a standard DP formulation, the intra-period CDDP pre-computation model can be performed independently from the main procedure, employing any solution technique that produces a monotone non-increasing demand curve (or surface) for release. For example, Read (1989) used merit order filling of load duration curves, but Read & George (1990) used parametric LP, while Scott & Read (1996) and Batstone & Scott (1998) used Cournot gaming models. Even a heuristic approach can be used to construct monotone non-increasing  $dcr$ .

This chapter proposes a similar pre-computation method to construct the  $dcr$  for a single reservoir network represented by a set of nodes in a tree that enables off-take and in-flow water for different water uses. The nodal network and network structure-based graph representations are widely used to model the interconnected system in commoditised utility optimization models such as water, electricity and natural gas. For example Wang et al., (2007), Barros et al., (2008) and Cheng et al., (2009), explicitly represent water buyer-seller delivery relationship through a physical network of water distribution system in the optimization model.

Given those pre-computations, the CDDP algorithm can be described as equivalent to ‘horizontal addition’ or ‘augmenting’ of demand curves (Read & Hindsberger, 2010). Horizontal (demand curve) addition accumulates the demand at each price point, effectively adding the demand curve inverses (Scott & Read, 1996). The concept is common in the construction of industry supply curves in microeconomics. See, for example, Mankiw & Taylor (2006). To date CDDP implementations have all involved adding curves backwards, through time, to solve an inter-period CDDP problem by

recursively forming a new “demand curve for storage” (*dcs*) for the beginning of each period by adding *dcr* to the *dcs* for the beginning of the next period.

The next section presents a generic intra-period optimization model that could be used to optimise the nodal network. Then, we illustrate how the intra-period CDDP model could efficiently construct the demand curve for release for the reservoir.

#### **5.4 Constructing the Intra-period Demand Curve for Release to Clear a Water Market**

The term demand curve is used to denote the relationship between price and the demand quantity. It could, in general, refer to a price-quantity relationship without limiting to a strict functional form. Thus, ‘demand curve’ refers to both forms (price as a function of quantity or quantity as a function of price). Unless otherwise noted, in this thesis the term demand curve is used to mean price as a function of the release volume (quantity). Where the demand curve is piecewise constant, it may be expressed as a merit ordered set of prices which water users (e.g., hydropower producers) are willing to pay for water released to meet their current requirements (Read & Hindsberger, 2010). The inverse net demand function is defined in terms of the net quantity of water demanded at the node as a function of the nodal price<sup>113</sup>.

As we noted earlier, the inter-period CDDP optimisation does not depend on any particular solution technique being used for intra-period optimization. So here we propose to use CDDP at this intra-period level, too, developing an efficient algorithm to clear a multi-node market operating across the catchment, within each period. Although the method is also valid for centralised water management with no market, it is best described as constructing the *dcr* for the reservoir by recursively combining all (net) nodal demand curves for water at the nodes radiating from the reservoir as a tree.

Unlike previous CDDP implementations, though, the recursion in this algorithm operates “inwards” towards the main reservoir, rather than “backwards” over time. Each *dcr* is constructed solely as a function of long-term reservoir release for a fixed inflow and tributary flow realisation.

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<sup>113</sup> Merit ordered net demand curves representing efficient allocating water at a particular node in the river network will only need a slight change, without a full scale re-optimizing. An LP method (e.g., parametric LP method described in Read & George (1990)) could be used to optimise a complex network model and then analyse the dual price relationships.

### 5.4.1 Intra-period Tree Optimization Model Formulation

This section sets up a context for the nodal intra-period pre-computation sub-problem. Hence, we present underlying assumptions of the single reservoir constrained network optimization model and the single reservoir nodal CDDP procedure.

#### 5.4.1.1 Assumptions

We assume the catchment network has a single long-term storage reservoir (at the root) and a tree structure. We relax the single reservoir assumption in a later chapter and consider ways of generalising the tree requirement at the end of this chapter. Due to the tree-structure, each non-root node has a parent node, being the neighbouring node on the unique path towards the reservoir. Even though this chapter assumes a set of interconnected nodes radiating in a tree structure from the reservoir, we illustrate some limited applications of this procedure for non-tree structures within a nodal network in Chapter 6. A node's parent may be either upstream or downstream, depending on how it is situated with respect to the reservoir.

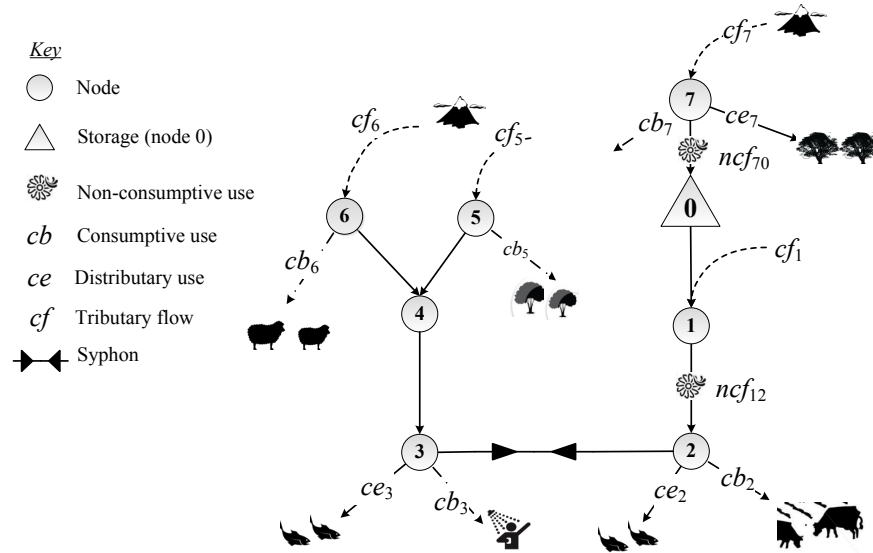
The costs and benefits of consumptive and non-consumptive use at different locations are assumed to be independent of one another. They are also independent of supplies and demands met in previous periods<sup>114</sup>. Any costs are convex and benefits are concave functions of water use. Non-consumptive use (such as a hydro power station or pumping) does not use up water, but transfers water from one location to another, at some cost or benefit. Further, the optimal power generation or pumping level is not determined by the price of water at a single node, but by trading off the difference between upstream and downstream prices against the cost or benefit associated with the flow use. This can be modelled by adjusting the marginal water value curve passed on from one node to the next to account for this external cost/benefit.

We first present a non-linear programming (NLP) nodal tree model that seeks to optimise the total benefits/costs of the accumulated water flows through the network. The CDDP procedure presented subsequently determines the optimal solution to the NLP model by re-ordering and/or adjusting marginal values at every node and arc. This moves the demand curves towards the reservoir. Hence, the CDDP operates on the dual by adjusting marginal water value curves at a node rather than specifying

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<sup>114</sup> This assumption can be relaxed by including supply/demand information in a DP state.

flows. We use the following single reservoir river network with a set of hypothetical consumptive, non-consumptive and distributary demands to illustrate the single reservoir nodal CDDP procedure later in this chapter. Figure 5-1 illustrates a single reservoir catchment, based on the Rangitata Diversion Race (RDR) hydro scheme in the south island, New Zealand.



**Figure 5-1.** A single reservoir multi node catchment topology styled on the RDR network in New Zealand. Consumptive, distributary and non-consumptive arc flow demands shown in the network are added for illustrative purposes of the algorithm.

A reservoir node may have multiple tributary and non-tributary inflows and a set of outflows. We assume no significant delays regarding the flow of water across the network. First the (primal) model is described in terms of demands, supplies and water flows. Then the algorithm is described, which generates a net demand curve covering all release decisions. For this purpose, all nodal data is re-cast as net demands and arcs are re-oriented towards the reservoir.

#### 5.4.1.2 Nodal Catchment Optimisation Sub-problem

In this section we define the catchment optimisation model used to determine the optimal net benefit function,  $NB(r)$ , as a function of net reservoir release,  $r$ . The  $dcr$  is derived from  $NB(r)$  as  $dcr(r) = NB'(r)$ . Here  $r$  represents the net decrease in storage level and thus it can take negative or positive values.

Following the notations declared in Chapter 3, let  $T = (N = \{0, 1, \dots, n\}, A)$  be a tree with the reservoir at the root node 0, and nodes,  $i \in N$  indexed so that child nodes (further from the reservoir) have a larger index than their parents. Arc  $(i, j) \in A$  has the usual water flow direction from node  $i$  to node  $j$ .

The amount of inflow captured at node  $i$  is denoted,  $\bar{F}_i \leq f_i \leq \underline{F}_i$ . However, in most general cases the lower bound of the tributary flows can be zero. A non-zero lower bound may be enforced in order to maintain a minimum in-stream flow requirement in a tributary flow path (arc) (see Chapter 3 and Chapter 4). Tributary inflows not captured (if any) are assumed as a lost to the system. For full generality,  $CF_i(f_i)$  denotes the convex cost function associated with the capture of this flow. This generality allows for supply participants who incur costs or who charge for the flow injection. This could also be water purchased from a different catchment (e.g., traded between water authorities) or a desalination plant (e.g., urban reservoir system). For a typical tributary flow  $CF_i(f_i) = 0$ . We could model inflows which include upstream non-consumptive uses by allowing  $CF_i$  to be negative. For example,  $CF_0 < 0$  reflects non-consumptive benefits of upstream flow users directly above the reservoir.

Each non-root node  $i$  has  $C_i > 0$  units of (potential) consumptive demand, with an associated concave benefit function  $CB_i(c_i)$ . Variable  $0 \leq c_i \leq C_i$ , represents the consumptive demand met at node  $i$ .

Variable  $0 \leq d_i \leq E_i$  represents distributary flow from non-root node  $i$  out of the system. The target flow is  $E_i (\geq 0)$ , with a convex cost function  $CE_i(E_i - d_i)$  associated with missing the target. These distributary flows may be used to apply desired environmental flow requirements.

Nodes without distributary flow, tributary flow, and/or consumptive demand would have the corresponding bounds set to zero. We do not account for consumptive and distributary return flows and assume these flows are completely lost to the system (discussed later).

Each arc  $(i, j) \in A$  has an associated variable  $x_{ij}$  representing downstream flow, within flow limits:  $\underline{X}_{ij} \leq x_{ij} \leq \bar{X}_{ij}$ . The lower bound may be negative for a syphon (see Figure 5-1) or if pumping or some other upstream transfer were available.

We refer to non-consumptive flow as flow on an arc which creates a benefit or cost to the system, such as hydropower production or pumping. The net benefit received from a non-consumptive arc flow of  $x$

over arc  $(i, j)$  is denoted by  $NCF_{ij}(x)$  (This is negative for costs incurred, for example, from pumping). We ignore issues such as power conversion functions and reservoir head effects in hydroelectric production.

As noted in the above section, variable  $r$ , here, represents the net release at the reservoir node. We adjust  $r$  to include the net flow injections at the reservoir allowing  $r \leq 0$ . That is,  $r$  is the difference between all downstream releases and any controlled or uncontrolled inflows into the reservoir. For a given release  $r$ , we seek to optimise the following:

$$NB(r) = \max_{c,d,f,x} \{ \sum_{i=1}^n CB_i(c_i) - CE_i(E_i - d_i) - CF_i(f_i) + \sum_{(i,j) \in A} NCF_{ij}(x_{ij}) \} \quad [5-6]$$

Subject to:

$$f_i + \sum_{(j,i) \in A} x_{ji} = c_i + d_i + \sum_{(i,j) \in A} x_{ij}, \quad \forall i = 1, 2, \dots, n \quad [5-7]$$

$$-f_0 - \sum_{(j,0) \in A} x_{j0} + \sum_{(0,j) \in A} x_{0j} = r \quad [5-8]$$

$$\underline{X}_{ij} \leq x_{ij} \leq \bar{X}_{ij}; 0 \leq c_i \leq C_i; 0 \leq d_i \leq E_i, \bar{F}_i \leq f_i \leq \underline{F}_i, \forall i \in N, (i, j) \in A \quad [5-9]$$

The objective is to maximize the net benefit across all nodes accounting nodal tributary, distributary, consumptive and arc non-consumptive flow demands. Constraint [5-7] represents the nodal flow balance for any upstream or downstream node. Constraint [5-8] ensures upstream and/or downstream flows from the reservoir meet the required net release (which may be negative for a net injection). Finally, constraint [5-9] enforces flow capacities and demand limits.

We avoid explicitly solving the nodal tree optimization problem as an NLP. Instead the next section proposes a multi-nodal CDDP procedure that could efficiently solve the intra-period problem for a single or a range of  $r$  values simultaneously. We use this nodal tree formulation later to prove the multi-nodal CDDP procedure.

#### 5.4.2 The Multi-nodal CDDP Algorithm

The previous model deals directly with cost and benefit functions for the various uses and supplies of water. The CDDP algorithm described here, on the other hand is applied to the related marginal cost and benefit functions, giving the marginal value or price as a function of water flow. To clearly link the two, we use capital letters for cost and benefit functions, with lowercase for the corresponding marginal



value function. For example,  $CB(c)$  is the net benefit from meeting consumptive demand and  $cb(c) = CB'(c)$  is the *marginal* net benefit associated with this.

Rather than solving NLP formulation [5-6] - [5-9] that seeks to optimize the water flow (quantity) explicitly for all possible release values across the network, the CDDP algorithm constructs  $dcr$  directly, in a single pass in the dual problem. The  $dcr$  is constructed by sequentially constructing marginal water value curves for each node, transferring the curves towards the reservoir.

Arc flow bounds restrict the opportunities which may be exploited at nodes. The cumulative effect of these restrictions is represented by a domain associated with each marginal value curve. We use the notation  $f: [a, b]$  to denote function  $f$  restricted to domain  $[a, b]$ .

The model data is pre-processed to simplify the algorithm description. All cost and benefit functions at nodes are converted to net demand functions, that is, *marginal* net benefit as a function of the net water supplied<sup>115</sup>. Specifically, for each node  $i$ , the pre-processing calculates the following net demand functions.

- Consumptive demands form net demand  $cb_i: [0, C_i]$  where  $cb_i(c) = \frac{dCB_i}{dq}(c)$ .
- Distributary flows form net demand  $ce_i: [0, E_i]$  where  $ce_i(d) = \frac{dCE_i}{dq}(E_i - d)$ . Distributary net demand function represents the penalty for not meeting the ecological requirements. Where  $d = E_i$  corresponds to fully meeting the distributary requirement. In general,  $d$  represents missing the target  $E_i$  by  $d$  units, that is, a flow of  $E_i - d$ .
- Tributary flows form net demand  $cf_i: [-\bar{F}_i, -\underline{F}_i]$  where  $cf_i(f) = \frac{dCF_i}{dq}(-f)$ . The negative flow bounds here represent the amount of additional inflow to node  $i$  not needed to meet demands from here, due to the amount of this supply that has been accepted. As the amount of supply accepted is reduced, the most expensive are discarded first.

By convention, arcs are directed away from the reservoir (regardless of flow direction). The flow bounds for arc  $(i, j)$  are adjusted to account for this, setting  $\underline{X}'_{ij} = \underline{X}_{ij}$  and  $\bar{X}'_{ij} = \bar{X}_{ij}$  if water flow is

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<sup>115</sup> Water supplied by a user (e.g., tributaries, desalination plant) at a node can take negative values as incoming flow supplies are converted to demands.

away from the reservoir, and  $\underline{X}_{ij}' = -\bar{X}_{ij}$  and  $\bar{X}_{ij}' = -\underline{X}_{ij}$  if water flow is directed towards the reservoir. Non-consumptive uses are similarly affected, forming net demand function  $ncf_{ij}: [\underline{X}_{ij}', \bar{X}_{ij}']$  where  $ncf_{ij}(x) = \frac{dNCF_{ij}}{dx}(x)$  for arcs with water flow away from the reservoir and  $ncf_{ij}(x) = -\frac{dNCF_{ij}}{dx}(-x)$  otherwise.

The algorithm can be described as alternating between applying horizontal curve addition at a node and vertical curve addition on an arc. That is, the quantities at nodes corresponding to each price are added, while on the arcs the prices corresponding to each quantity are added. Further details on these operations are provided below.

The algorithm starts from the node with the largest index (which must be a leaf node). A nodal net demand curve ( $nb: [a, b]$ ) is formed by “horizontally adding” all tributary flows, distributary, and consumptive demands ( $cf, ce, cb$ ) at the node with any previously transferred demands (e.g., demand curve of a child node). This horizontal adding corresponds to ordering the net demands from all of these sources, from the most to the least valuable. The bounds  $[a, b]$  give the feasible range of net supply that could be used to meet some or all of these net demands. Details on horizontal adding are given below. For the purpose of future illustrations (see Chapter 7), we use the term “net nodal consumptive demand curve”,  $ncb$  to represent the horizontal addition of just  $cf, ce$  and  $cb$  at a node. This gives a notational simplicity to express the local water off-take at a particular node.

The net demand curve, ( $nb: [a, b]$ ), is then transferred to the parent node by first truncating the domain using arc flow bounds  $[\underline{X}, \bar{X}]$ <sup>116</sup>; then “vertically adding” any non-consumptive flow demand components  $ncf: [\underline{X}, \bar{X}]$  to form  $(\widetilde{nb}: \tilde{S})$ . The vertical adding corresponds to adjusting prices due to additional benefits or costs arising from the non-consumptive uses. This defines a transferred net demand curve at the parent node. Thus, the parent node will have a net nodal consumptive demand curve and  $(\widetilde{nb}: \tilde{S})$  coming from this child node. The process is repeated until we reach the reservoir (node 0).

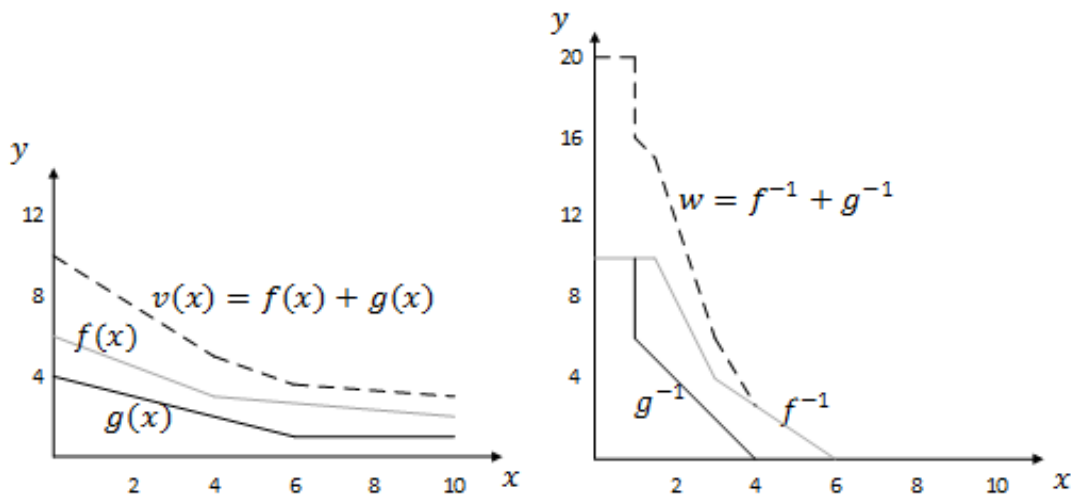
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<sup>116</sup> The truncated nodal demand curve is  $\widetilde{nb}: [\underline{X}, \bar{X}] \leftarrow nb: [a, b]$

### 5.4.2.1 Description of Horizontal, $+^h$ Addition and Vertical, $+^v$ Addition

A non-consumptive user does not consume the water. She transfers water from one node to another at some cost (e.g., for a pump), or to gain some benefit (e.g., for generation). The benefit or cost is in addition to benefits from other uses for the same unit of water. Vertical addition adjusts the value of each unit of water between the two nodes to account for this extra benefit or cost. For a water channel with a non-consumptive use, the water value is different at each end. For example, water above a hydro generator includes the marginal returns that can be gained from generation, while water below does not. This means that the hydropower benefit will add to the incentive to release water to the downstream node. As each subsequent unit of water is released down the channel it will be used to meet the next most valuable non-consumptive demand and, in addition, provide the next most valuable return from the lower node. The corresponding net demand functions are combined using vertical addition.

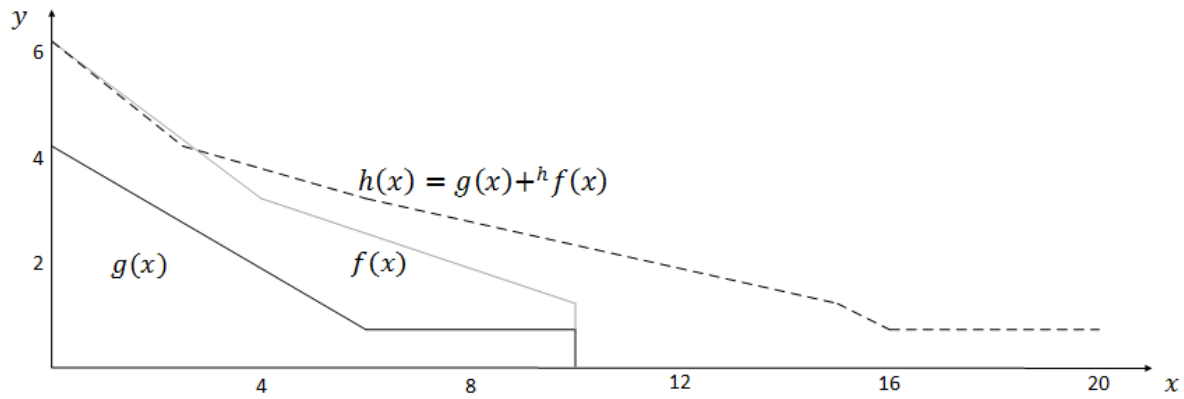
Vertical addition is the usual point-wise addition with respect to the *horizontal* axis over the coincident *domain*. That is, prices are added for each water flow quantity so when  $v: [a, b] = f: [a, b] +^v g: [a, b]$  then  $v(x) = f(x) + g(x)$ . Vertical addition is applied here only, when the domains match (see the left-hand graph of Figure 5-2).



**Figure 5-2.** Vertical addition (left) and inverse addition (right) of the same marginal water value curves,  $f(x)$  and  $g(x)$ .

Consumptive users extract water from the system, so each unit of water flow can be used only for a single consumptive use. Horizontal addition allows the different competing consumptive uses to be

allocated in order from highest to lowest marginal value (merit ordered marginal values of water of consumptive use).



**Figure 5-3.** Horizontal marginal water value curve adding

Horizontal addition is point-wise addition with respect to the *vertical* axis over the combined *range*. The concept is used to construct market supply curves in standard microeconomics, see for example Mankiw and Taylor (2006) . It corresponds to adding the water flow demand quantities from the horizontal axis for each fixed price value on the vertical axis. When  $h: [s, t] = f: [a, b] +^h g: [c, d]$ , we have  $h(x + y) = f(x) = g(y)$  for appropriate  $x$  and  $y$ . We can write  $h(x)$  as  $h(x) = (f^{-1} + g^{-1})^{-1}(x)$  where this is well defined. Note the relationship between the horizontal addition result in Figure 5.3 and the inverse addition result in the right-hand graph of Figure 5.2, namely  $h(x) = w^{-1}(x)$ . We apply horizontal addition for decreasing functions only.

Formally, we define vertical addition as follows.  $v: [a, b] = f: [a, b] +^v g: [a, b]$  where  $v(x) = f(x) + g(x)$  for  $x \in [a, b]$ . For horizontal addition,  $h: [s, t] = f: [a, b] +^h g: [c, d]$  has  $s = a + c$ ,  $t = b + d$ , and

$$h(x) = \begin{cases} f(x - d) & \text{where } f(x - d) \leq g(d) \\ g(x - b) & \text{where } g(x - b) \leq f(b) \\ \min \{ \lambda | \exists y \in [a, b], \lambda \geq f(y), \lambda \geq g(x - y) \} & \text{otherwise} \end{cases}$$

The vertical and horizontal adding processes are applied below in the proof. Piecewise constant demand curves are not a mandatory requirements to execute the CDDP procedure but efficient demand curve adding sub-procedures for these help to increase the efficiency of the algorithm.

#### 5.4.2.2 Multi-nodal Single Reservoir CDDP Algorithm

The following procedure constructs the  $dcr$  corresponding to  $NB(r)$  from the previous model. In the algorithm below,  $par(i)$  denotes the parent of node  $i$ .

**Procedure** Demand curve for release ( $dcr$ ) for single tree reservoir network

$nb_i: [a_i, b_i]$  is the  $n$ -nodal demand curve for flow,  $nb_i = 0, a_i = 0, b_i = 0 \forall i = 0, \dots, n$

For  $i = n$  to 1 Step  $-1$  do

$$\left\{ \begin{array}{l} nb_i: [a_i, b_i] \leftarrow [nb_i: [a_i, b_i] + {}^h cb_i: [0, C_i] + {}^h ce_i: [0, E_i] + {}^h cf_i: [-\underline{F}_i, -\bar{F}_i]] \\ \text{set } (j, k) = (i, par(i)) \text{ or } (j, k) = (par(i), i), \text{ whichever exists} \\ \overline{nb}_i: [\underline{X}'_{jk}, \bar{X}'_{jk}] = \text{truncate} \left( nb_i: [a_i, b_i], [\underline{X}'_{jk}, \bar{X}'_{jk}] \right) // \text{Truncated to arc limits} \\ \widetilde{nb}_i: [\underline{X}'_{jk}, \bar{X}'_{jk}] = [\overline{nb}_i: [\underline{X}'_{jk}, \bar{X}'_{jk}] + {}^v ncf_i: [\underline{X}'_{jk}, \bar{X}'_{jk}]] \\ nb_{par(i)}: [a_{par(i)}, b_{par(i)}] \leftarrow [nb_{par(i)}: [a_{par(i)}, b_{par(i)}] + {}^h \widetilde{nb}_i: [\underline{X}'_{jk}, \bar{X}'_{jk}]] \\ \text{Next } i // \text{Repeat process until reach node 0.} \end{array} \right.$$

Output:  $dcr \leftarrow [nb_0: [a_0, b_0] + {}^h cf_0: [-\underline{F}_0, -\bar{F}_0]]$

To ensure feasibility the *truncate* procedure pads  $nb_i: [a_i, b_i]$  if  $[a_i, b_i]$  does not cover  $[\underline{X}'_{jk}, \bar{X}'_{jk}]$ . In particular, quantities higher than  $b_i$  are given price zero while those below  $a_i$  are given a high penalty value. These adjustments can be avoided by careful construction of the data. Next, we provide proof the above algorithm constructs the required  $dcr$ .

#### Proof of the Multi-nodal CDDP Algorithm

Here we show the algorithm constructs the  $dcr$  corresponding to the solution of model [5-6]-[5-9] for all release values  $r$ . Specifically, we inductively reduce the problem to a single node, the reservoir. It is then simple to show that  $dcr(r) = \frac{d}{dr} NB(r)$  where  $NB(r)$  is given by model [5-9]-[5-12].

We next adjust and augment the model [5-6]-[5-9] to a form that better matches the algorithm and allows an inductive approach.

We apply the following transformations to construct an equivalent model. For nodes:  $\tilde{f}_i = -f_i$ ,  $\tilde{CF}_i(\tilde{f}_i) = -CF_i(-f_i)$ , and  $\tilde{CE}_i(d_i) = -CE_i(E_i - d_i)$ .

Arc flow is adjusted to represent net flow away from the reservoir. For any arc with stream flow directed away from the reservoir arc direction and limits are maintained:  $\forall(i, j): i < j, \tilde{x}_{ij} = x_{ij}, \underline{\tilde{X}}_{ij} = \underline{X}_{ij}$  and  $\widetilde{\tilde{X}}_{ij} = \overline{X}_{ij}$ . While for any arc with stream flow directed towards the reservoir arc direction and labelling are reversed:  $\forall(i, j): i > j, \tilde{x}_{ji} = -x_{ij}, \underline{\tilde{X}}_{ji} = -\overline{X}_{ij}$  and  $\widetilde{\tilde{X}}_{ji} = -\underline{X}_{ij}$ . The new arc set is labelled  $A'$ . Let  $\mathcal{W}(i)$  be the set of previously processed nodes with node  $i$  as parent, and  $BN_j$  denote the derived benefit function from net water flow towards previously processed node  $j$  from its parent. The first derivative of  $BN_j$  is denoted by  $\widetilde{nb}_j$  in the CDDP procedure. Initially define  $\mathcal{W}(i) = \emptyset$  for all nodes  $i$ . We use  $w_i$  for each node  $i$  to represent the water flowing from node  $i$  toward all previously processed nodes for which it is the parent, and we use  $q_j$  for the water flowing towards previously processed node  $j$  from its parent. Also for the moment let us assume that all users are consumptive,  $NCB_{ij}(x_{ij}) = 0, \forall(i, j) \in A$ .

#### Augmented model

$$NB(r) = \max \sum_{i=1}^n [CB_i(c_i) + \widetilde{CE}_i(\tilde{d}_i) + \widetilde{CF}_i(\tilde{f}_i) + \sum_{j \in \mathcal{W}(i)} BN_j(q_j)] + \sum_{j \in \mathcal{W}(0)} BN_j(q_j) \quad [5-10]$$

Subject to

$$-\tilde{f}_i - c_i - d_i + \sum_{(j,i) \in A'} \tilde{x}_{ji} - \sum_{(i,j) \in A'} \tilde{x}_{ij} - w_i = 0, \quad \forall i = 1, \dots, n \quad [5-11]$$

$$\sum_{(0,j) \in A'} \tilde{x}_{0j} + w_0 = r \quad [5-12]$$

$$w_i = \sum_{j \in \mathcal{W}(i)} q_j \quad \forall i = 0, \dots, n \quad [5-13]$$

$$-\overline{F}_i \leq \tilde{f}_i \leq -\underline{F}_i; \tilde{L}_{ij} \leq \tilde{x}_{ij} \leq \widetilde{\tilde{U}}_{ij}; 0 \leq c_i \leq C_i; 0 \leq d_i \leq E_i \quad \forall i \in N, (i, j) \in A' \quad [5-14]$$

$$\tilde{L}_j \leq q_j \leq \widetilde{\tilde{U}}_j, \quad \forall j \in \mathcal{W}(i), i = 0, \dots, n \quad [5-15]$$

We form an equivalent decomposed model by processing the last node  $n$  and reducing the size of the augmented model to  $n - 1$  unprocessed nodes. The variable  $\tilde{x}_{in}$  is replaced by  $q_n$  with bounds  $\underline{\tilde{X}}_n = \underline{\tilde{X}}_{in}$  and  $\widetilde{\tilde{X}}_n = \widetilde{\tilde{X}}_{in}$ .

#### Decomposed model

$$NB(r) = \max \sum_{i=1}^{n-1} [CB_i(c_i) + \widetilde{CE}_i(\tilde{d}_i) + \widetilde{CF}_i(\tilde{f}_i) + \sum_{j \in \mathcal{W}(i)} BN_j(q_j)] + BN_n(q_n) \quad [5-16]$$

Subject to

$$-\tilde{f}_i - c_i - d_i + \sum_{(j,i) \in A'} \tilde{x}_{ji} - \sum_{(i,j) \in A'} \tilde{x}_{ij} - w_i - \sum_{(i,n) \in A'} q_n = 0, \quad \forall i = 1, \dots, n - 1 \quad [5-17]$$

$$\sum_{(0,j) \in A'} \tilde{x}_{0j} + w_0 + \sum_{(0,n) \in A'} q_n = r \quad [5-18]$$

$$w_i = \sum_{j \in W(i)} q_j \quad \forall i = 0 \dots, n-1 \quad [5-19]$$

$$-\bar{F}_i \leq \tilde{f}_i \leq -\underline{F}_i; \tilde{L}_{ij} \leq \tilde{x}_{ij} \leq \tilde{U}_{ij}; 0 \leq c_i \leq C_i; 0 \leq d_i \leq E_i \quad [5-20]$$

$$\forall i, j = 1 \dots, n-1, (i, j) \in A' \quad [5-21]$$

$$\underline{\tilde{X}}_j \leq q_j \leq \tilde{\bar{X}}_j, \forall j \in \mathcal{W}(i), i = 0, \dots, n-1 \quad [5-22]$$

Where  $BN_n(q_n)$  is determined by subproblem  $n$  as

$$BN_n(q_n) = \max\{CB_n(c_n) + \widetilde{CE}_n(\tilde{d}_n) + \widetilde{CF}_n(\tilde{f}_n) + \sum_{j \in W(n)} BN_j(q_j)\} \quad [5-23]$$

Subject to

$$\tilde{f}_n + c_n + d_n + w_n = q_n \quad [5-24]$$

$$w_n = \sum_{j \in W(n)} q_j \quad [5-25]$$

$$\underline{\tilde{X}}_n \leq q_n \leq \tilde{\bar{X}}_n; -\bar{F}_n \leq \tilde{f}_n \leq -\underline{F}_n \quad [5-26]$$

$$0 \leq c_n \leq C_n; 0 \leq d_n \leq E_n \quad [5-27]$$

$$\underline{\tilde{X}}_j \leq q_j \leq \tilde{\bar{X}}_j \quad \forall j \in \mathcal{W}(n) \quad [5-28]$$

Adding  $n$  to  $W(par(n))$ , where  $par(n)$  is the parent node of  $n$ , the model can be rewritten in the same form as the augmented model but with only  $n-1$  as the highest index node with flows modelled. Applying this iteratively to the highest index node follows the order in which nodes are visited in the CDDP algorithm. Eventually only node 0 remains.

Sub-problem  $n$  is easily converted to the following general form where all  $F_i$  are concave.

$$G(q) = \max \sum_{i=1}^k F_i(y_i) \quad [5-29]$$

Subject to

$$\sum_{i=1}^k y_i = q \quad [5-30]$$

$$\underline{Y}_i \leq y_i \leq \bar{Y}_i \quad [5-31]$$

For  $k=2$  it is straightforward to show that  $G': [\underline{Y}_1 + \underline{Y}_2, \bar{Y}_1 + \bar{Y}_2] = F'_1: [\underline{Y}_1, \bar{Y}_1] + {}^h F'_2: [\underline{Y}_2, \bar{Y}_2]$  by considering the optimality conditions. This can be done, for example, by enumerating all complementary slackness cases. The result may be extended to arbitrary  $k$  by induction. Applying the above result to sub-problem  $n$  gives:

$$BN'_n: [a, b] = CB'_n: [0, C_n] + {}^h\widetilde{CE}'_n: [0, E_n] + {}^h\widetilde{CF}'_n: [-\overline{F}_n, -\underline{F}_n] + \sum_{j \in W(n)} BN'_j: [\underline{X}_j, \overline{X}_j] \quad [5-32]$$

where the last summation is applied as horizontal addition. It follows that  $nb_i = BN'_n$  and the algorithm indeed constructs  $dcr$ .

#### Incorporating non-consumptive flow demands

Including non-consumptive flows changes the construction of the sub problem, and the computation of  $BN_n$ . Only the objective function is changed. It is now written as:  $\widetilde{BN}_n(q_n) = \max\{CB_n(c_n) + \widetilde{CE}_n(\tilde{d}_n) + \widetilde{CF}_n(\tilde{f}_n) + \sum_{j \in W(n)} \widetilde{BN}_j(q_j) + NCF_{par(n),n}(q_n)\}$  [5-33]

The vertical addition in the algorithm follows since effectively:

$$\widetilde{BN}_n(q_n) = BN_n(q_n) + NCF_{par(n),n}(q_n) \quad [5-34]$$

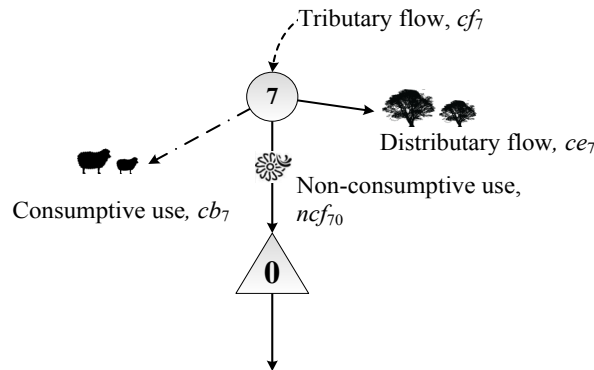
The next section applies the multi-nodal CDDP procedure to the nodal network shown in the Figure 5-1 as an illustration of the algorithm. We assume a set of hypothetical consumptive, non-consumptive and distributary demand curves exist at selected locations in the network.

### 5.5 Illustration of multi-nodal CDDP algorithm.

We illustrate the CDDP algorithm to construct the  $dcr$  in the catchment shown in Figure 5-1. We assume that the catchment has no suppliers, but only consumptive users requiring water extracted from the catchment, and non-consumptive users gaining benefit from flow within the catchment. For ease of explanation, all marginal value curves are piecewise constant over a regular grid of breakpoints.

#### 5.5.1 Processing the First Node

The algorithm begins at node 7 which (coincidentally) is the only node above the reservoir. The relevant part of the network is shown in Figure 5-4.

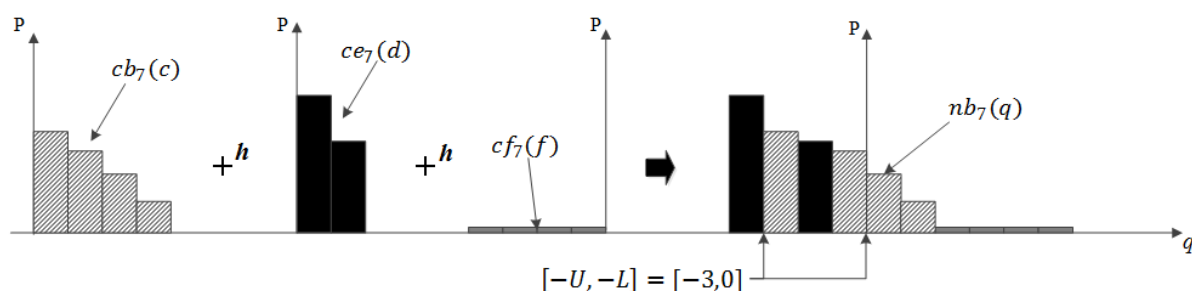


**Figure 5-4.** A node located above the reservoir (detail from Figure 5-1)



Up to four units of consumptive demand for water form  $cb_7(c)$ <sup>117</sup>. A distributary flow with at least one unit is required. A high penalty applies if this is not met. An additional unit of distributary flow is desirable, with a lower penalty for not meeting this. These two penalties form  $ce_7(d)$ . Up to four units of inflow may be captured at node 7. No costs or penalties are associated with this inflow, so  $cf_7(f) = 0$ .

Figure 5-5 shows construction of the nodal demand curve for water  $nb_7(q)$  by horizontal addition:  $nb_7: [-4,6] \leftarrow cb_7: [0,4] +^h ce_7: [0,2] +^h cf_7: [-4,0]$ . This is equivalent to sorting all increments of consumptive, distributary and (net) tributary demand by decreasing incremental value. The resulting net demand curve represents the marginal benefit from upstream participants if we limit flows to the next downstream node. Arc flow limits will restrict the ability of the downstream node to take advantage of the opportunity this represents.



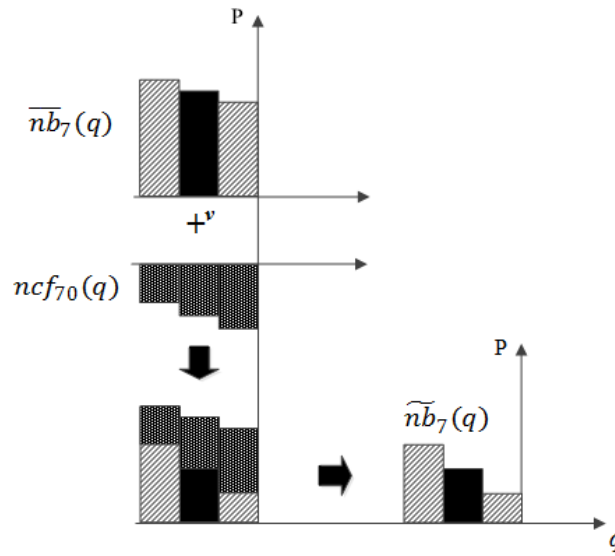
**Figure 5-5.** Horizontal addition of consumptive  $cb_7$ , distributary flow  $ce_7$ , and tributary flow  $cf_7$ , net demand curves to form a nodal demand curve  $nb_7$

Arc (7,0) has a flow capacity of 3 units. As the flow on this arc is towards the reservoir, we set the arc flow limits to become  $[-3, 0]$ . These arc bounds truncate the nodal demand curve, as shown in Figure 5-5. But here there is also a non-consumptive use to take into account.

The benefit of non-consumptive flow is accounted for by the vertical addition:  $\widetilde{nb}_7: [-3,0] = [\overline{nb}_7: [-3,0] +^v ncf_{70}: [-3,0]]$  as shown in Figure 5-6. Note that, since flow from node 7 to the reservoir gains additional non-consumptive benefit, the net benefit gained by using water at node 7, rather than letting it flow through to the reservoir, is reduced. The prices in the net demand curve

<sup>117</sup> Suppose there are  $k$  consumptive users at node 7. Each user will express her own bid set  $cb_k(c_k)$ . The nodal consumptive demand curve is formed by aggregating individual demand bid sets,  $\bigcup_{k \in K} cb_k$  and then the final bid set  $\bigcup_{k \in K} cb_k$  is stacked in monotone, non-increasing price order to construct the consumptive demand curve  $cb_7$ .

passed through to the reservoir are also reduced. This effect is achieved by the pre-processing, which makes  $ncf_{70}$  negative, implying vertical ‘subtraction’.



**Figure 5-6.** Vertical addition with non-consumptive flow demands  $ncf_{70}$  and truncating with arc flow bounds  $[L, U] = [-3, 0]$ .

### 5.5.2 Constructing Demand Curves for Node 4

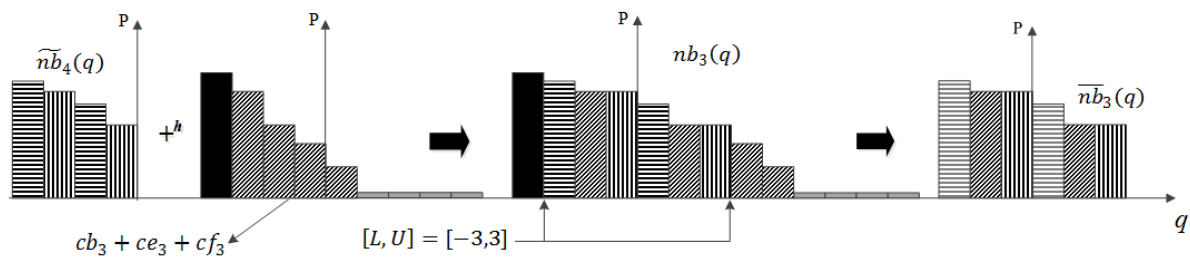
The next nodes processed are nodes 6 and 5. Both might be considered “downstream” from the reservoir. But the processing of  $nb_6$  and  $nb_5$ , the net demand curves for these nodes, follows the process for node 7, because the flow direction from both is “towards” the reservoir. In this case, arcs (5,4) and (6,4) have no non-consumptive uses, so the net demand curves constructed at nodes 6 and 5 can be transferred directly to node 4 after truncation for flow limits on the corresponding arcs. In this case,  $(\widetilde{nb}_6: [\underline{X}'_{64}, \overline{X}'_{64}])$  and  $(\widetilde{nb}_5: [\underline{X}'_{54}, \overline{X}'_{54}])$  represent the willingness, and ability, of participants at nodes 6 and 5, respectively, to pay to limit flows to node 4. These curves are treated as further consumptive net demand curves at node 4, in addition to the local demands there, when determining  $nb_4$ .

### 5.5.3 Truncating for Siphon Flow Limits

We follow an identical procedure for node 4 and forming  $(\widetilde{nb}_4: [\underline{X}'_{43}, \overline{X}'_{43}])$  for input into the formation of  $nb_3$ . Figure 5-7 illustrates processing node 3 and, in particular, accounting for arc (3, 2). This arc represents a siphon which allows flow in either direction. To account for the unknown flow direction,

the processing truncates  $nb_3$  by using a domain interval with a negative lower bound and a positive upper bound.

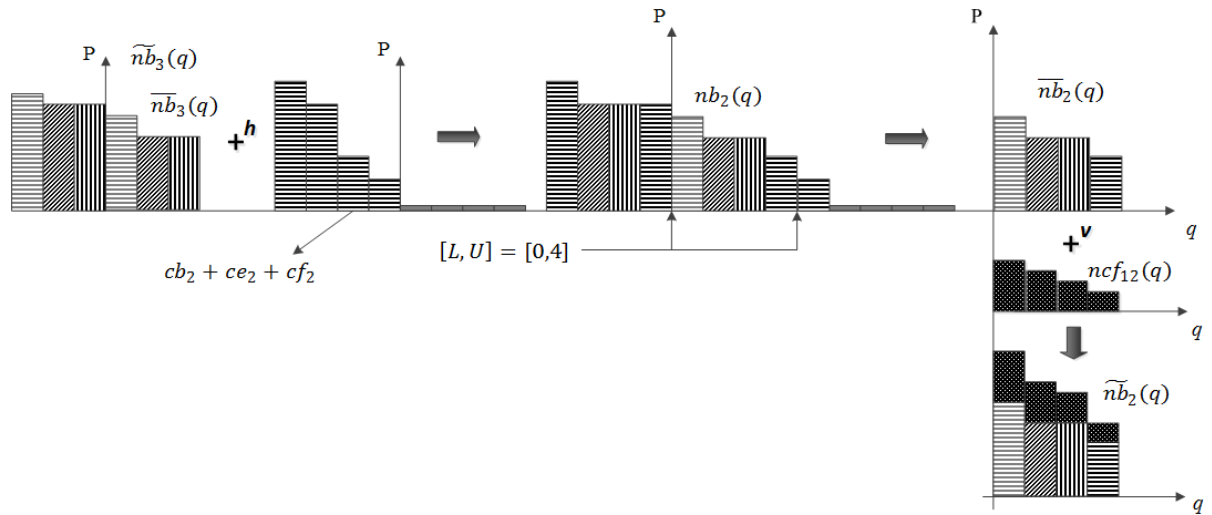
The resulting  $(\widetilde{nb}_3: [-3, 3])$  represents the willingness of participants to provide flow from node 3 to node 2 for quantities in the negative part of the interval, and the desire of participants for flow from node 2 to node 3 for quantities in the positive part of the interval. To put this another way, if the price of water were low enough, then node 3 would like to ‘buy’ water from node 2 (at prices to the right of the vertical axis). However, if the price were high enough, node 3 would be willing to ‘sell’ water to node 2 (at the prices to the left of the vertical axis).



**Figure 5-7.** Processing node 3 and truncating across syphon arc (3, 2).

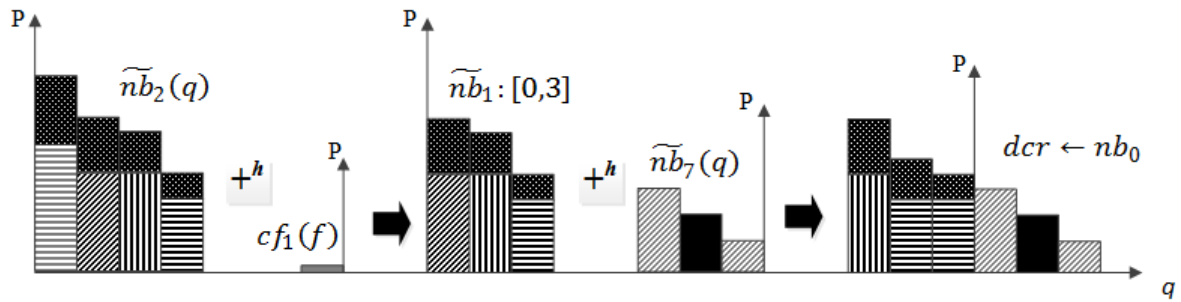
#### 5.5.4 Processing Upstream

The syphon can flow either way, while the other arcs processed so far have had water flow ‘towards’ the reservoir. But the water flow direction on arc (1, 2) is away from the reservoir. The flow limits on such arcs will imply non-negative upper/lower limits on the net demand curve passed back through the tree, towards the reservoir. And non-consumptive uses on such arcs will add positive increments to the prices in those demand curves, representing the additional non-consumptive benefit from flow away from the reservoir, in this case. Figure 5-8 illustrates the formation of  $\widetilde{nb}_2: [0, 4]$ .



**Figure 5-8.** Processing node 2 via horizontal addition (across), then arc (1, 2) via vertical addition (down)

Node 1 is processed in a similar manner. Finally, we reach node 0, where we obtain  $dcr$  for the reservoir (see Figure 5-9) itself by horizontal addition of the net demand curves for node 1 ( $\tilde{nb}_1$ ) and node 7 ( $\tilde{nb}_7$ ).



**Figure 5-9.** Processing nodal demand curves of node 1 and 7 to construct  $dcr$ .

We can ‘unpack’ the  $dcr$  using the ‘style coded’ demand stack labels to find out the intra-period optimal release schedule for the reservoir. This style code implicitly represents a set of indices such as individual bid tranche index, participant index, and node/arc index. The above release schedule indicates the demands that are met throughout the river network. Hence, each style is a local nodal and/or arc release decision which can be easily identified at the reservoir level. For example, the second left tower in the  $dcr$  pack gives the locational marginal price at node 2. This means that nodal CDDP can be implemented to not only produce net demand curves but also help us to easily unpack final bid stack and read nodal release decisions.

The nodal CDDP procedure can be used in each period of a ‘higher level’ CDDP (e.g. deterministic CDDP) to form a set of *dcrs* over the planning horizon. The next section describes the methods employed within this high level CDDP. The stochastic version the spatial CDDP is applied to form a *dcr* for each hydrology state in each stage. The inter-temporal CDDP simply works in reverse recursive fashion to combine these resulting *dcr* in the above.

## 5.6 Using the Multi-nodal Demand Curve for Release in Stochastic CDDP Model

This section explains how to integrate intra-period nodal CDDP procedure with the single reservoir stochastic CDDP model. The deterministic intra-period multi-nodal CDDP described above is used to construct a monotone decreasing *dcr* for each period (week, month) assuming a particular pattern of tributary/distributary flows, and set of bids. In this setting, CDDP could be described as a two-stage process. First stage employs a deterministic multi-node version of the CDDP procedure constructed in the above. This constructs a series of marginal net benefit curves for release  $NB_t$ s under various catchment inflow scenarios which is described as the intra period pre-computation step. This is analogous to the intra-period benefit function in the classical SDP problem as described in Scott & Read (1996). Then the higher level stochastic version of the CDDP uses the above constructed *dcr*’s to optimize the system, or clear the market over multiple periods, under uncertainty (Starkey, 2014).

The most general form of the stochastic inter-period CDDP described by Starkey et al., (2012) allows for “state-dependent (case-dependent)” net *dcrs* in each period. In order to provide a complete set of inputs to such a model, our deterministic intra-period CDDP would have to be run several times, producing a *dcr* for each possible hydrology state or the Markov state in the intra-period problem as shown later in this section, that is for each tributary inflow pattern and associated bid set, for each period. This set of *dcrs* would then define the optimal release, and intra-period market-clearing, as a function of the intra-period state, and the reservoir’s marginal water value. The *dcrs* have the same functionality as the value derived  $NB_{h,t}(r_{h,t}(s_{h,t}))$  in period  $t_{h,t}$ , for a release  $r_{h,t}$ , in the primal version of the problem (i.e., stochastic DP). However, the dual solution of the SDP may require less computational effort.

The stochastic, *inter*-period CDDP will construct a beginning-of-period demand curve for storage (*dcs*), defining the marginal water value of stored water as a function of storage level, for each inflow state,  $h$  in each period. The hydrology state variable  $h$  is defined later in this section. This works in a similar analogy to the SDP future value function  $V_{t+1}(s_{h,t+1})$  over the remainder of the planning horizon. The stochastic CDDP uses an approximated *dcs* for the last period (end-of-period marginal values for stored water as an input), and works backwards to construct the *dcs* for the beginning of each period from the *dcs* at the end of that period, and the intra-period *dcr* for that period, as determined by the nodal CDDP procedure above (or any other intra-period pre-computation technique). Interested readers are directed to Starkey et al., (2012) for more detail on this approach. The *dcr*'s could also be used (in the form of net benefit functions) as part of a convex programming approach.

The intra-period CDDP procedure constructs *dcr*'s which give the marginal return for all possible releases. Having run the inter-period stochastic CDDP, we do not know what the release should actually be, and have not “cleared the multi-period market”, until we know the reservoir’s marginal water value. Similar the *dcs*'s constructed give the marginal return for all possible storage levels. As a result the final step in the process must be to identify a particular solution (at least for the first market period), starting from a particular storage level. So the intra-period model looks at the ‘current’ storage level and hydrology state to read a price for the reservoir in the first stage. The procedure reads the release and determines the other arc flows from the CDDP output. The stochastic CDDP can be used to find the multi-period solution which determines the net release for each subsequent possible storage-hydrology scenario for each period forward (see Dye et al., 2012; Starkey et al., 2012). This means, it finds an optimal release policy, over time and for each state.

The *dcr*'s can be generated by the multi-nodal CDDP described above, assuming the tributary flows for the given hydrology state. Regardless of whether hydrology state ( $h$ ) dependent bids are used in the multi-nodal market, the “informed” release policy described in Dye et al (2012) will be assumed here. Under the informed policy assumption, the reservoir manager has prior information at the beginning of the period regarding exactly how much water will arrive during the period so that she could plan to allocate it all. The reason for this choice is because the informed policy allows releases and demand

requirements to be directly related to the tributary stream flows observed. Under a conservative kind of policy assumption, user decision-making is based only on information from previous periods<sup>118</sup>. As a result, the end-of-period marginal value of storage function will be conditioned upon the hydrology conditions. Since we propose trading at the nodes making use of the current period's tributary flows sometimes, the conservative release policy (Dye et al., 2012) is a less suitable candidate for our purpose. Moreover, the reservoir manager may apply other intermediate release policy assumptions but this thesis does not pursue those.

We assume that the hydrology sequence follows a first order (or lag-one) Markov Chain<sup>119</sup> with discrete state space  $\{1,2,3, \dots, H\}$  and transition matrix  $P = [\rho_{hh'}]$  where  $\rho_{hh'} = \text{Prob}(f_{h,t+1} = h' | f_{h,t} = h)$  is the transition probability of moving to state  $h'$  from state  $h$  ( $h', h \in \{1,2, \dots, H\}$ ). Following Section 5.4.5 of Starkey (2015), we can formulate the end of the period demand curve for storage (ignoring any storage bounds):

$$edcs_{h',t}(s_{h,t}) = \mathbb{E}_{f_{h,t}|f_{h,t}} \left[ \max_r \left( cdc_{h,t}(r_t, f_{h,t}) + \rho_{hh'} cdc_{h,t+1}(s_{h,t+1}) \right) \right] \quad [5-35]$$

The inter-period CDDP has three parts: intra-period net conditional *cdcr*, end-of-period conditional *cdcs* (also known as the conditional *dcs*), and net beginning-of-period expected *edcs*. Demand curves for release are conditioned by the Markov states under the informed release policy assumption. This accounts for conditional inflows, tributary flows and, possibly, state dependent demand. As noted in the above, the nodal CDDP is constructing a set of *cdcrs* for each hydrology state in a particular time period. For a stochastic optimization problem, effectively we are constructing the *cdcrs* for the sub-problems corresponding to multiple states in each time period in the same way as the deterministic case. Dye et al., (2012) illustrate the construction of beginning of period expected demand curves for storage, *edcs*. In their implementation a marginal value in the *edcs*<sub>*h,t*</sub> is formed as the sum of the respective

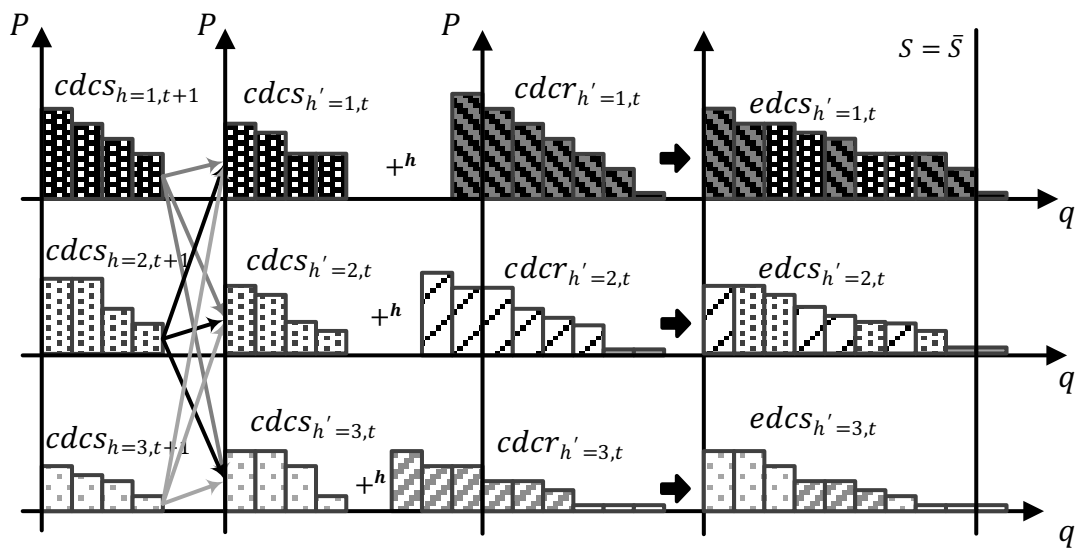
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<sup>118</sup> That is, for each period the water use decisions would have to be the same for each hydrology state but the amount of water flowing between nodes would differ depending on the hydrology state (and corresponding tributary flows). This would mean any non-consumptive use on an arc could not use water flowing on the arc that they had not planned to use (no opportunistic use). The reservoir manager also could follow hybrid/intermediate policy assumptions for reservoir scheduling. However, they are beyond the scope of this thesis.

<sup>119</sup> A first order/lag-one Markov Chain implies that the dependence of the next period inflows on the current and all previous periods' inflows is completely described by its dependence on the current period inflow.

marginal values of horizontally added  $cdcr_t$  and  $cdcs_{t+1}$ 's in all states in the next period  $t + 1$ , each offset by the realised inflows, and taking the probability weighted average of that state occurring  $h$ , given the starting state  $h'$ . Their stochastic CDDP algorithm models total controllable inflows occurring at the reservoir in a particular time period over the planning horizon. As we have already noted earlier, the nodal CDDP procedure constructs the net  $cdcrs$  accounting for all controllable/uncontrollable tributary flows across the network including the inflows at the reservoir node. The algorithm can easily account for this, however given the structure of the  $cdcr$ 's formed in the intra-period problem it is not possible to account for the realised inflows simply with offsets. Instead, the  $dcr$  needs to be recomputed for each Markov state at each stage.  $cdcr_{h,t}$  denotes the dependence on both period and hydrology state.

Figure 5-10 sketches the conditional demand curves for release and the end of period demand curves belonging to three inflow hydrology states: dry, medium and wet (denoted by  $h = 1,2,3$  respectively). Then we show the beginning-of-period marginal value function for stored water  $edcs_{2,t}$  (assumed medium inflow state,  $h = 2$  at the current period,  $t$ ). Note that the algorithm also applies when the inflow distributions are stochastically independent (see Starkey et al., 2012 and Starkey 2014 for further details). In that case, irrespective of the state (e.g., dry, medium or wet) of a particular stage (e.g. time period), the future looks identical and a single  $edcs$  is carried from period to period.



**Figure 5-10.** Constructing the (net) beginning of period demand curve for storage for time period,  $t$  assuming starting at the medium inflow hydrology state,  $h = 2$ .

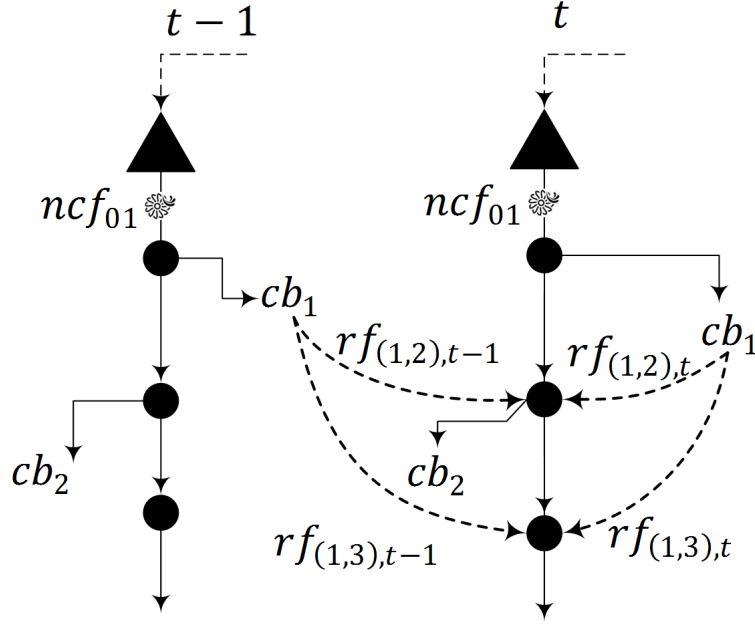


In summary, this section integrates the intra-period nodal CDDP procedure and the stochastic CDDP model to transform into a ‘seamless’ single reservoir nodal stochastic optimization model. But the two models operate independently of one another. Section 5.6 discusses the return flow benefit issue in a nodal water market.

## 5.7 Modelling Return Flow Benefits

Consumptive and distributary flows are assumed to be lost to the water network. In practice some of that water may make its way back to the water network in the same or a later period. This section investigates such return flows in a nodal reservoir network. This section briefly investigates some return flow modelling issues. However, further implementation of return flows in a CDDP will not be pursued in this thesis.

An ideal hydrologic-economic model would represent all major flows. Stream flows, surface evaporation, ground recharge and leakage, and return flows are the main components of water balance. We earlier omitted incorporating evaporation losses and assumed no return flows. To model evaporation, leakage and return flows in time period  $t$ , coefficients of  $x_{ji}$  in equation 2-2 of the previous nodal tree optimization can have fractional values:  $f_i + \alpha_{ji} \sum_{(j,i) \in A} x_{ji} = c_i + d_i + \alpha_{ij} \sum_{(i,j) \in A} x_{ij}$  where  $0 < \alpha_{ij}, \alpha_{ji} \leq 1$ . However, return flows are difficult to model because of their temporal and spatial nature. The complexity depends on various physical and environmental conditions that may be difficult to model effectively.



**Figure 5-11.** Temporal and spatial distribution of return flows ( $t - 1, t$ ) of consumptive off-take

Let us assume a simple return flow scenario to illustrate the issues. Figure 5-11 sketches return flows  $rf_{(1,2),t-1}$ ,  $rf_{(1,2),t}$ ,  $rf_{(1,3),t-1}$ ,  $rf_{(1,3),t}$ ,  $rf_{(2,3),t-1}$  and  $rf_{(2,3),t}$  arriving at downstream nodes 2 and 3 due to the consumptive use at node 1 in the current and the previous periods. The first issue here is that the nodal CDDP procedure has to account for the return flow benefits of an upstream node consumptive use at some downstream nodes without knowing the actual consumptive off-take quantity at the upstream node both spatially and temporally. This is because the nodal CDDP is yet to process the upstream consumptive node.

- The return flows could positively contribute to the downstream tributaries and sometimes the stream flows in the current and/or in the future time periods. This leads us to re-adjust both intra and inter period models. It may be possible for the nodal CDDP to iteratively estimate and adjust the spatial return flow contributions to the downstream tributaries in the current period depending on the upstream consumptive and distributary off-takes.
- However, the stochastic CDDP cannot directly re-adjust the tributaries and the stream flow as in the intra-period model. Further, the estimate-and-adjustment process could draw significant computational resources and there could be convergence issues.

This shows that modelling return flows creates difficulties for CDDP. Such issues would not be present in an LP model, however we might expect the linkages formed within the constraint matrix to reduce computational efficiency.

Finally, the decision to model these auxiliary flows has to be based on the significance of those flows in the final solution. As noted in the beginning of this section, we will not consider return flows in the next chapters of this thesis.

## **5.8 Chapter Conclusions**

In this chapter, CDDP is used to decompose the optimization into a sequence of single period trade-offs between the benefits of immediate release and the expected benefits of storage for later release, where the benefit from immediate release is modelled by a demand curve for release (*dcr*) in each period. This presupposes a tree-connected single reservoir nodal system (where water can be reallocated anywhere), symmetry of information, and price taking users, among other aspects.

This chapter illustrates a nodal catchment model that can be applied to optimise usage, or to clear an intra-temporal market for any catchment with one reservoir and a tree configuration. To solve the model, we proposed a simple and efficient two-level application of CDDP. First, a multi-nodal deterministic CDDP, presented here, constructs aggregate demand curves for release in each period.

Then a stochastic CDDP constructs aggregate demand curves for storage in the single reservoir for each period. Where multiple reservoirs occur in the same catchment, a multi-reservoir version of the intra-temporal algorithm would also be needed. Increasing the number of long-term storage reservoirs will increase the complexity of the algorithm, because the “curse of dimensionality” will eventually apply to any DP-based technique for multiple reservoirs. We investigate such models in Chapter 8.

Another direction for future research is to extend the model to allow non-tree networks. By holding parts of the network fixed and enumerating all such configurations, our model could account for non-tree networks when the number of configurations is small (see Chapter 6). The next chapter applies the nodal CDDP to meet cooling and temperature control in-stream flow demands in parallel arcs assuming the upstream flow may be split either arbitrarily or any other suitable splitting method. This is another way to generalise nodal CDDP for non-tree networks.

# 6.

## Representing Flow Mixing Demands in a Mixed-use Catchment

### 6.1 Introduction

Chapter 5 discussed how CDDP could be used to optimise a multi-nodal water system, or to clear a multi-nodal water market in the presence of competing consumptive demands, such as irrigation for farming, and non-consumptive demands, such as hydro-power generation. This chapter illustrates how the nodal CDDP procedure designed to apply only to tree networks can be adapted to handle certain non-tree structures in the network.

High temperature return flows from thermal power plant cooling can affect the downstream ecosystem. As a result an additional flow past the thermal station is needed to control the temperature. The mixing flow required can be modelled as a fixed ratio of heated to unheated water. The chapter explains how to extend the previous nodal CDDP procedure to incorporate flow mixing demands in a mixed-use single reservoir catchment. The extended model allows the addition of parallel arcs to the previous model discussed in Chapter 5. The parallel arcs extension could be generalised for other situations.

An optimization model seeking primarily to maximise hydropower production typically imposes constraints to meet other requirements such as minimum instream flows for environmental requirements. These constraints lower the hydropower benefits. Increasing competition for flows (e.g., consumptive, urban demands), as witnessed in many places, may compromise the original operating targets. In addition, the environmental constraints can also impose restrictions on the stream flows. These constraints could enforce either a simple set of flow bounds at different locations in the network or more complex flow requirements. For example a thermal power plant located in close proximity to the river may make use of the stream flows for cooling purposes. The Huntly thermal Power Station

(HPS) in the Waikato region of New Zealand draws water for cooling purposes and subsequently discharges warmer water back into the Waikato River. Unless managed, the in-stream temperature level may rise so much that it affects the river ecosystem. To address this, the reservoir releases and other flow controls are required to regulate river flows under (1) hydropower production, (2) irrigation and urban water demands, (3) cooling flow demands, and (4) corresponding temperature control (mixing or diluting) in-stream flow demands. Devising an operating policy that accommodates all the above requirements can be a challenging task.

The rest of this chapter is organized as follows: Section 6.2 briefly describes related previous work. Section 6.3 how to extend the multi-nodal CDDP procedure to handle parallel paths, with additional flow mixing-type constraints. Section 6.4 then briefly describes how the multi-nodal deterministic CDDP can be embedded into the stochastic CDDP approach described in Section 5.5 in the previous chapter for a longer time-horizon to set up a stochastic nodal market. Then, a numerical illustration is presented of the model applied to a hypothetical system styled on the Waikato river catchment. Finally, Section 6.5 presents a chapter summary.

## **6.2 Supplying Water Mixing Demands**

Hydro-reservoir storage targets often conflict with the consumptive and other user water-supply demands. In-stream flows downstream depend on the reservoir release. When the downstream flow is used by a thermal power generator for cooling, the system planner may require additional release to dilute thermal power plant heated effluent flow. This mixing flow demand has to be varied throughout the year with respect to the tributary flows, the stream temperature, discharge water quality (based on actual power production), and stream ecological life (Lence et al., 1992). High temperature discharge occurs in addition to the ecological impacts of river regulation in a hydro-power operation (e.g., Olivares, 2008; Wu et al., 2011).

Economics of water resource literature contains very few hydrologic-economic models that analyse thermal power mixing flow requirements. For example, Lence et al. (1992) used goal programming to model mixing flow requirements of thermal power in a multi-reservoir catchment for multi-periods. Their model seeks to “minimize deviations from specified targets for reservoir storage and for power generation, subject to water quality constraints”. The authors assumed a single cooling process within

the plant that increases the input water temperature. The water-quality constraints determine the allowable thermal power generation for a given reservoir level (for different hydrology scenarios and for different months of a year). Authors further cite the work of Shafer et al. (1981) and Yang (1991), who examined a similar problem assuming constant thermal power mixing demands. However, the literature is not rich with dynamic hydrology-economic optimization models with mixing water use as an objective under stochastic environment. Although the multi-nodal CDDP model discussed earlier could easily provide for the non-consumptive nature of in-stream flow values (e.g., arc flow bound penalties) or consumptive nature environmental flow values (e.g., distributary demands discussed in Chapter 5), the focus of this chapter is to explicitly model non-consumptive thermal power cooling and diluting demands using parallel arcs. Irregular releases as a result of peaking hydro operation and high temperature cooling return flows from a thermal power plant affect the downstream ecosystem. As a result an additional flow has to be released to control the stream temperature which can also affect river management elsewhere. A flow splitting device is used to meet a specific purpose, determining the percentage flow volume to be diverted into an arc.

### **6.3 Intra-period Multi-nodal CDDP Model to Process Parallel Arcs**

This section presents an intra-period multi-nodal CDDP model for the extended model that constructs a demand curve for release  $dcr$  from a long term reservoir, assuming known tributary flows, as described in Chapter 5. The demand curve for release provides the marginal value of the last unit released from long-term storage, for any release level. The constructed demand curve can be used as part of a stochastic optimisation over a longer time horizon, or a market-clearing model, as described below.

The CDDP approach constructs the demand curve for release directly from bid curves, or marginal cost and benefit functions. It effectively works directly with the optimality conditions of an underlying welfare maximisation model. We recall the assumptions established in Section 5.3.1.1 regarding the consumptive, non-consumptive use and the marginal cost/benefit functions.

As shown in the earlier chapter the algorithm processes from the leaf nodes in towards the reservoir at the root, transforming competing demands and supplies into net demand functions. These demands are combined with the net-demand curves for water from any arcs leading away from the reservoir to create a nodal net demand curve for water. A demand with larger marginal return is served before one with

lower marginal return. We abuse the previous notation slightly by assuming all marginal cost/benefit functions have already been transformed to net-demand form. Chapter 5 determines the net-demand curve for water by sorting the marginal net-demands from highest to lowest, which amounts to adding the function inverses:

$$nb_i = cb + {}^h ce + {}^h cf + {}^h \sum_{(j,i) \in A} \widetilde{nb}_j \quad [6-1]$$

Recall that the  $\widetilde{nb}_j$  terms correspond to net demands for water from child nodes. Net demand curves on arcs are processed from child to parent, by truncating the net-demand curve for water,  $nb_i$ , to the arc bounds, to form  $\overline{nb}_i$  as explained above. Where an arc involves a non-consumptive demand, the algorithm adjusts the marginal benefit from each unit of flow directly. This adjustment is accounted for by ‘vertically’ adding (or subtracting) the non-consumptive flow net-demand curve,  $ncf_{ij}$ , to (from) the child node’s net demand curve for water,  $nb_j$ , where the water flow is away from (towards) the reservoir.

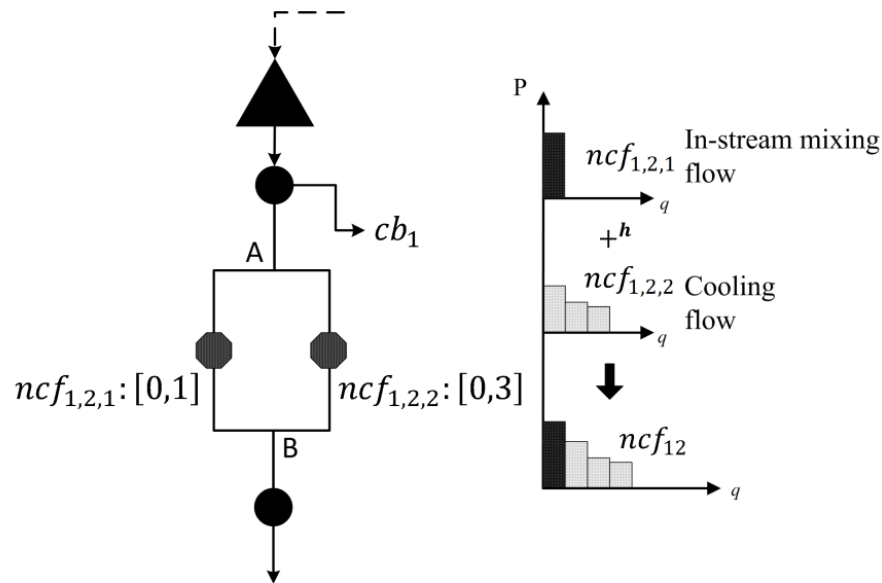
$$\widetilde{nb}_j(x_{ij}) = \overline{nb}_j(x_{ij}) + {}^v ncf_{ij}(x_{ij}) \quad [6-2]$$

The benefits from water flowing between two nodes could be competing or complementary. Competing benefits are served by different flows whereas complementary benefits arise from the same flow. This chapter focuses on the former case with the competing benefits located on parallel arcs. The flow split between the arcs could be controlled by different mechanisms. A sluice gate allows the operator to split flows between two parallel arcs. A boom might split flows in a constant ratio, or a weir might be designed to produce a split that varies with the flow level. In the case of a thermal power plant, a movable gate could divert flows for cooling and in-stream mixing/temperature control. Environmental requirements would constrain the allowed split. The type of splitting decides how the arc is modelled, which essentially comes down to whether we perform horizontal or vertical addition.

### 6.3.1 Case 1: Controlled Flow Splitting

Under controlled flow splitting, we can select the arc that each unit of water should flow on, with no restriction on the balance of flows. Sometimes the parallel arcs can host competing users (e.g., an ecological flow arc versus a hydropower use arc). Let  $ncf_{1,2,1}$  and  $ncf_{1,2,2}$  denote the arc demands for flows on two parallel paths. Since we can choose which path for each extra increment of flow, the

CDDP can construct the composite arc flow demand curve by horizontally adding  $ncf_{1,2,1}$  and  $ncf_{1,2,2}$  as shown in the figure below. In other words, CDDP can handle multiple competing (non-consumptive) uses on arcs in just the same way as it handles multiple competing (consumptive) uses at a node<sup>120</sup>. So that this approach replaces the parallel arcs with an equivalent single arc and then proceeds with the original nodal CDDP procedure.



**Figure 6-1.** Processing parallel arcs to provide flows for two competing non-consumptive users (e.g., cooling flows and in-stream mixing flows).

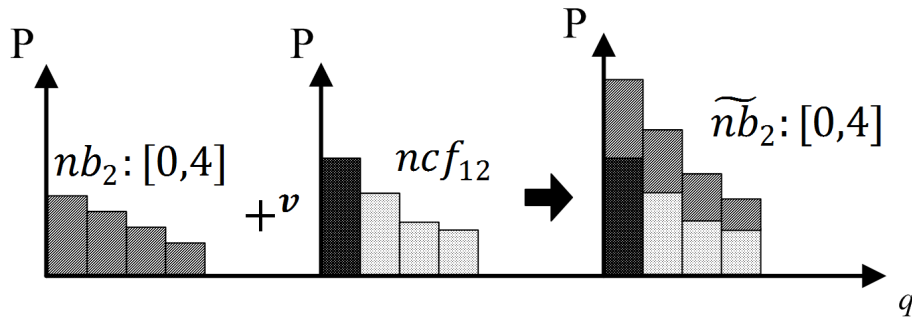
This case assigns a higher marginal water value for the in-stream mixing flow because it provides an ecological service to the system (e.g., special type of distributary flow). The second and subsequent flows would only be required if the cooling flow demand was high enough. In that case the marginal water values for these could be carefully selected to ensure they were inserted in the correct order.

The cooling flow, on the other hand, provides a direct economic benefit from generating using the thermal station. Composite arc flow demand curve  $ncf_{12}$  represents the combined non-consumptive demand between nodes 1 and 2. The algorithm vertically adds this composite flow demand curve to the

<sup>120</sup> So this method could be applied to several competing parallel arc flow users. For example, we could represent the cooling flow demands, mixing flow demands and instream/environmental flow demands using dedicated parallel arcs. Then we can adequately model different user demands at the local level.



truncated nodal demand curve of node 2 to construct the nodal demand curve for flows for node 1 (as illustrated in Figure 6-2 below).



**Figure 6-2.** The demand curve passes to node 1 after processing parallel arc flow demand curves.

In the interest of simplicity, we omit considering any water quality related flow benefit (penalty) functions normally observed in the thermal cooling flow models<sup>121</sup>. This arc flow quality constraint could be a function of stream temperature, thermal flow intake, discharge and temperature standard.

### 6.3.2 Case 2: Constant Splitting of Flows

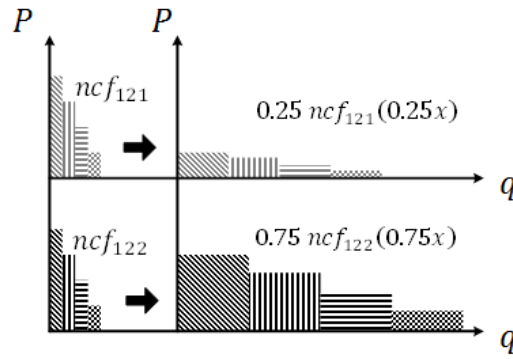
Another situation is where flow is split in a constant ratio, say, 0.25:0.75. The effect is the same as when the ratio defines the probability that each tiny increment would flow one way or the other. The resulting combined net-demand curve is found by vertically adding adjusted net-demand curves for the parallel arcs flows (according to the equation 6-3), as shown in Figure 6-3. Note that integer increments in the aggregate *dcr* do not remain integer, once split. So steps in the aggregate curve are typically formed using parts of adjacent steps in the underlying curves. Here  $\alpha$  and  $\alpha'$  denote the split ratios.

$$ncf_{ij}(x_{ij}) = \alpha ncf_{ij,k}(\alpha x_{ij,k}) + \alpha' \cdot ncf_{ij,k+1}(\alpha' x_{ij,k+1}) \quad [6-3]^{122}$$

Suppose the split gate capacity is 5 units and the ratio of flows is 1:3 in-stream mixing flows to plant cooling. The two graphs (top and bottom) in the Figure 6-3 respectively illustrate the two components of the right-hand side of the above equation graphically. The required scaling is explained below.

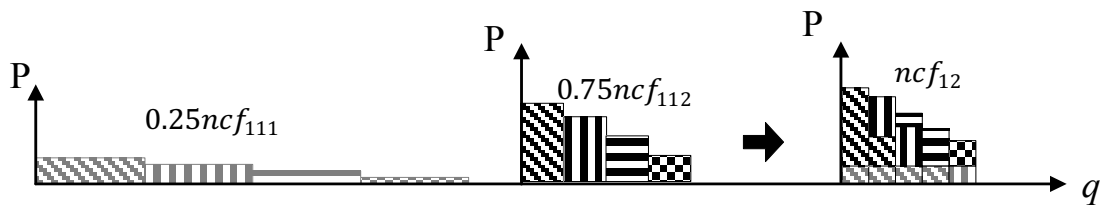
<sup>121</sup> This case involves an implicit assumption that water splitting must occur in full units of water. For example the first unit of water can either be used for mixing flows or for cooling flows (which would violate the ecological constraint). In particular, the first unit of water never produces economic benefit. Also, if the arc capacity was 5, the last unit would only ever be mixing flows when no additional cooling was required.

<sup>122</sup> Relevant benefit function:  $NCF_{ij}(x_{ij}) = NCF_{ij,k}(\alpha x_{ij,k}) + NCF_{ij,k+1}(\alpha' x_{ij,k+1})$



**Figure 6-3.** Horizontal scaling of demand curves for constant splitting flows  $ncf_{12,1}$  and  $ncf_{12,2}$  denote mixing (grey towers) and cooling (black towers) arc flow demand curves.

The split gate splits upstream flow  $x_{ij}$  into two parts  $0.25x_{ij}$  (in-stream mixing) and  $0.75x_{ij}$  (plant cooling). In order to get “1” full unit of in-stream mixing flow requires 4 units of upstream flow, providing  $0.25 \times 4$  units of in-stream mixing flow. Each unit of upstream flow only produces 0.25 units of in-stream mixing flow, gaining only 0.25 of the marginal benefit of a single unit of flow. This is shown in the LHS graph in Figure 6-4 showing marginal in-stream mixing benefit by upstream flow. Similarly, the plant cooling flow requires a non-integer step of  $\frac{4}{3}$  units of upstream flow for each unit of cooling flow. Each unit of upstream flow only receives 0.75 of the marginal cooling benefit (see the middle graph in Figure 6-4). The composite arc flow demand curve is then formed by vertically adding the ‘scaled’ arc flow demand curves. Doing this exactly would create non-integral steps so the marginal values are averaged over each whole unit of flow as shown in Figure 6-4. For example, the first unit of the RHS will be formed with 25% of the first in-stream mixing flow unit and 75% of the first cooling flow unit.



**Figure 6-4.** Vertical scaling of demand curves for constant splitting of flows and the composite arc flow demand curve.

In the above example, the algorithm systematically stacks the marginal value pieces (grey and black towers) and disregards the pieces that cannot be met, because only 5 flow units are assumed to be feasible for  $ncf_{11}$ .

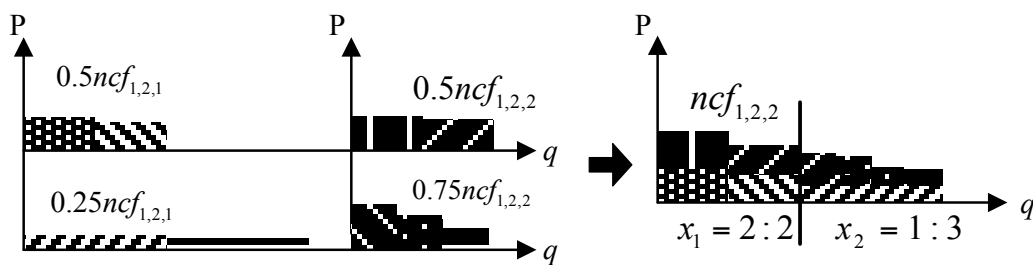
This method allows the first unit to produce some cooling flows and hence economic benefit. Also, this second method does not require artificial prices from the mixing – the marginal benefit for mixing could be zero and the method would still work.

The model might be improved by combining the two methods so there is a minimum ratio for a particular non-consumptive use (e.g., cooling flows only up to 75% of the upstream flows but more mixing flows are allowed giving independent benefits from the mixing flow arc).

The next section describes an advanced version of the constant splitting flow method. Here, the splitting ratio changes based on the upstream flow level. We show a two-level variable splitting flow method but this could be extended for higher number of splits.

### 6.3.3 Case 3: Variable Splitting of Flows

More generally, the splitting ratio may be a pre-determined function of total (aggregate parallel) arc flow,  $x$ . Thus, marginal water value can be described as a monotone decreasing function of the flows. As in the second case, we can apply vertical addition to construct the composite arc flow demand curve, but the “probability weights” now vary as a function of total flow. So,  $\alpha_{ijk} = \alpha_{ijk}(x)$ . Let  $\alpha_{111}:(1 - \alpha_{111})$  denote the split ratio applying up to aggregate incoming flow  $x$ ;  $\alpha_{112}:(1 - \alpha_{112})$  up to  $x_1 + x_2$ .



**Figure 6-5.** Modified vertical addition of arc flow demand curves under a pre-defined flow splitting control function.

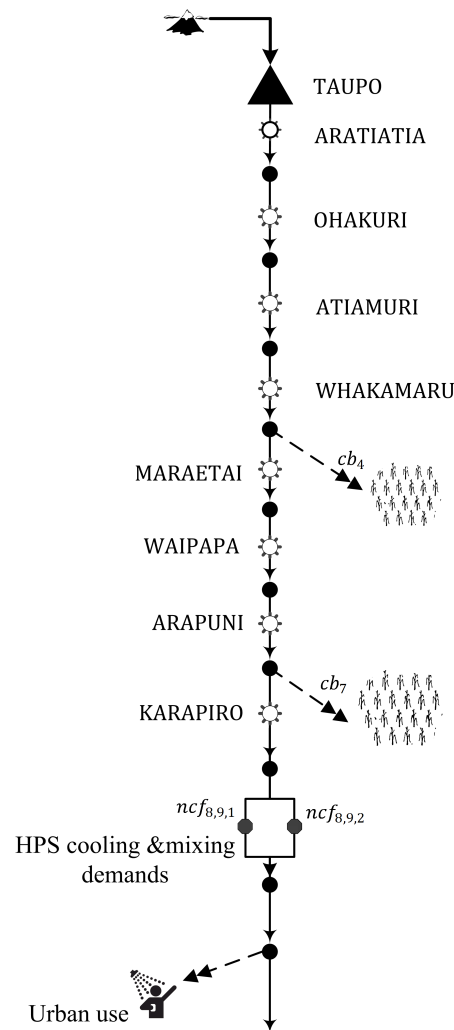
Figure 6-5 illustrates this modified vertical addition of ‘scaled’ demand curves. Here we assume two splitting ratios: 2:2 for the first 4 incoming flow units and 1:3 for the second 4. Thus, we can write the weighted demand curve as follows:

$$ncf_{ij}(x) = \alpha_k(x).ncf_{ij,k}[x.\alpha_k(x) + \beta(x)] + \alpha_{k+1}(x).ncf_{ij,k+1}[x.\alpha_{k+1}(x) + \beta(x)] \quad [6-4]$$

Note that we cannot keep the block width as an integer, even in this simple case, so numerical approximation would be required for integer representation. Therefore a correction term  $\beta(x)$  is added to the breakpoints. Parts of the underlying *dcrs* are discarded once more, because they cannot be met. We present a numerical illustration of the model applied to a hypothetical system styled on the Waikato river catchment in the next section.

#### 6.4 Numerical Example and Results

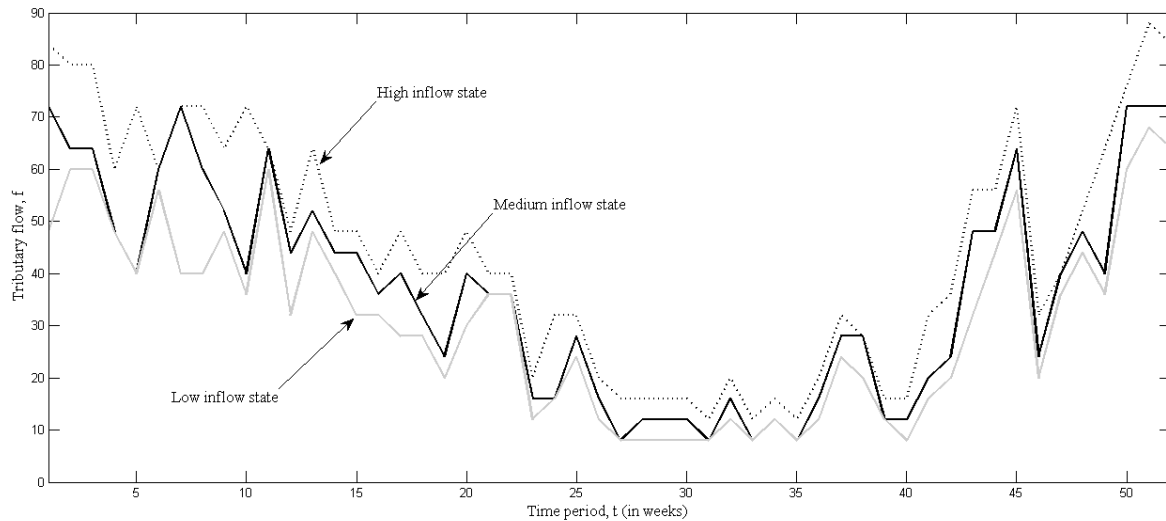
This section presents a numerical example to demonstrate how the algorithm clears the market and then allocates water to the cooling and in-stream mixing uses assuming that the system operator applies the controlled/competitive and constant flow splitting methods. We have developed a combined intra/inter-period CDDP model of this type in MATLAB, and applied it to an example loosely based on the topology of the Waikato River catchment in New Zealand, as shown in Figure 6-6. We assumed that market participants are submitting bids for current and future time periods. The user bid functions are discrete, non-increasing and piecewise constant.



**Figure 6-6.** Waikato styled catchment topology with a set of selected non-consumptive users, a mixing flow user and a consumptive (urban) user.

Lake Taupo plays the long-term storage role in the Waikato river catchment scheme. Downstream from Taupo, the Waikato river supports eight hydroelectric plants, totalling approximately 1000 MW, which, in this simplified framework, are assumed to have no explicit storage capacity. Instead, the ability of each to focus its output into peak periods is assumed to have been accounted for in determining its demand curve for release. For illustrative purposes, we include downstream urban, and some hypothetical irrigation users in the model, assuming monotone decreasing consumptive demand curves for water for each user. The model ignores other local power schemes, including geothermal and the upstream Tongariro hydro power scheme. It represents the 1435 MW Huntly Thermal Power Station (HPS) as a “special” non-consumptive user, because of the ecological impact of the plant, in terms of heated and polluted water (Chapman, 1996). The HPS contains six separate generating units including

combine cycle gas turbine (CCGT) and open cycle gas turbine (OCGT). According to the Huntly PS environmental report of 2013, the plant draws up to 40 cubic meters of flows per second ( $\text{m}^3/\text{s}$ ) from the Waikato River primarily for cooling and 98% of intake is returned. Discharge flow is approximately at a temperature of 8.6 C warmer than when abstracted. However, the maximum downstream river temperature shall not exceed 25 C, forcing the HPS to cease operating units 1-4 sometimes during hot days in the past. An intermediate cooling tower was added to the system to cool the return flows (e.g., Genesis Energy, 2013; New Zealand Electricity Department Huntly power station, 1972). The daily highest and lowest river flow rates are usually observed during winter and summer periods. For example, a peak flow rate of  $915 \text{ m}^3/\text{s}$  was recorded on 25<sup>th</sup> July and the lowest flow rate  $172 \text{ m}^3/\text{s}$  on 12<sup>th</sup> April (Genesis Energy, 2013).



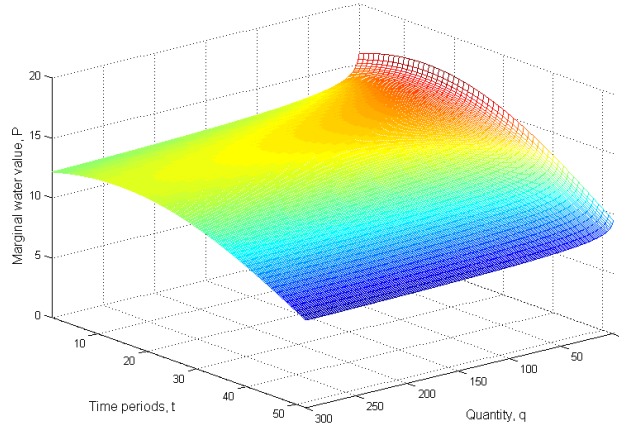
**Figure 6-7.** An example of weekly inflow patterns during the planning horizon according to annual hydrology cycle where hydrology states  $h = \text{low}, \text{med}, \text{wet}$  (source: Appendix A5, Table A5.1)

We assume an annual hydrology cycle with high inflows in weeks 1-5, 40-52, and relatively low in other weeks (increasing from September and decreasing from February/March). The hydrology cycle has three possible hydrology states (high, med and dry) in each week (refer to figure above), described by a lag-1 Markov chain. Relevant transition probabilities, for the example are assumed as:  $\rho_{hh'} = \{(0.5, 0.25, 0.25), (0.25, 0.5, 0.25), (0.25, 0.25, 0.5)\}$ .

We assumed a demand pattern that is relatively constant, state dependent and consumptive (urban use).

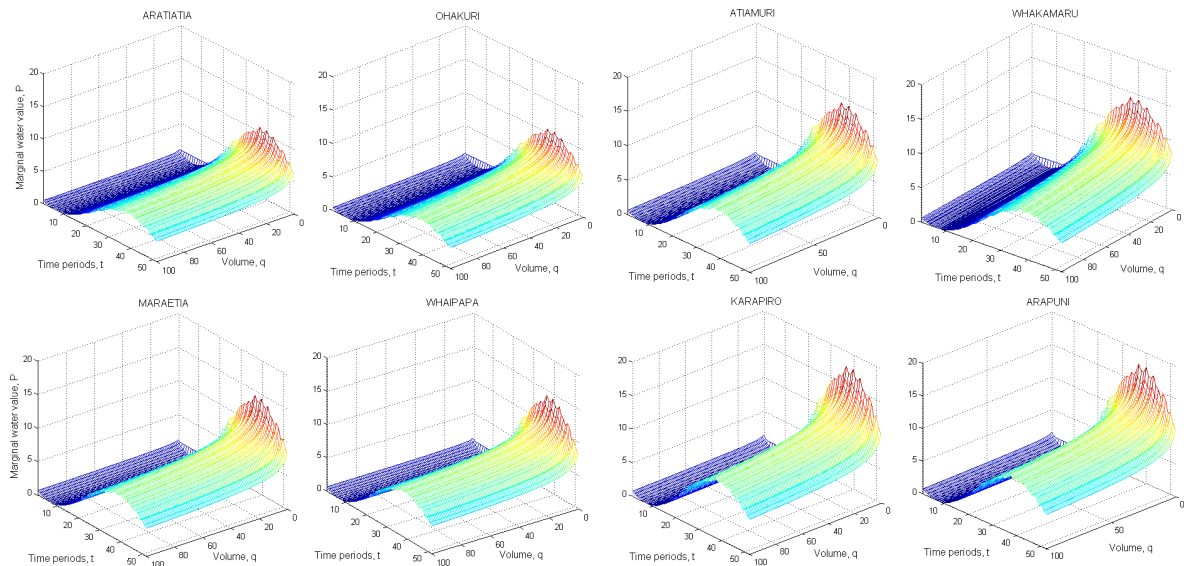
The consumptive use demand curve depends on the cost of production (e.g., crop and other water

requirement as described by Ward and Michelsen, 2002; Gibbons, 1986 & 2013) and the hydrology scale  $\theta(h)$ . Non-consumptive hydropower demands vary during the year in response to the electricity market spot/nodal prices and hydrology scale  $\theta(h)$  (see Figure 6-8).



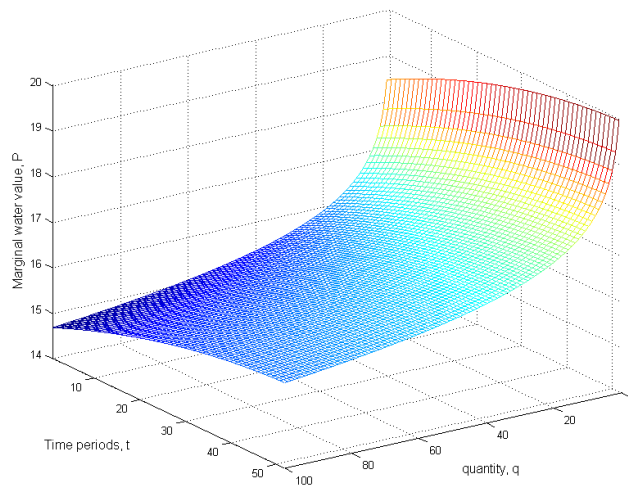
**Figure 6-8.** Urban user demands at the last node (weeks  $t = 1$  to 52, hydrology state  $h = low$ )

Figure 6-8 sketches the marginal values of urban water use for a 52 week planning horizon. The prices record a slight increase in the summer. The arc flow demand surfaces (52 weeks) for medium inflow state belong to all hydro users are shown in Figure 6-9.



**Figure 6-9.** The hydropower demand curves for medium inflow hydrology state and for a 52 week planning period

We illustrate three different cases starting from the base case (Case 1) that ignores modelling HPS in-stream mixing flow requirement<sup>123</sup>. The second case (Case 2) assumes that HPS is submitting both cooling flow demand curves and in-stream mixing flow demand curves for all time periods in the planning horizon based on the controlled/competitive flow splitting method described in the above<sup>124</sup>. Hence, Figure 6-10 depicts final HPS demands (cooling and in-stream mixing) for 52 weeks for medium inflow hydrology state. This case is based on the daily net water abstraction given in the Huntly PS environmental report (Figure 13 of page 10, Genesis Energy, 2013). The third (Case 3) assumes that the system operator is using the constant flow splitting method to provide both cooling and in-stream mixing flow requirements<sup>125</sup>.



**Figure 6-10.** HPS cooling and in-stream mixing flow demand curves (weeks  $t = 1$  to 52, state  $h = \text{medium}$ ) for the controlled/competitive flow splitting method

Based upon the information given, Section 6.4.1 explains the spot market trading and the results of the experiment.

#### 6.4.1 Spot Water Market

For the interest of simplicity, we assume only spot market trading which has some trading restrictions with respect to future water. The market clearing here gives us the current period release given the hydrology state and storage, and the reservoir consumption. Here we assume that market participants

<sup>123</sup> The ecological cost of the water temperature rise is ignored.

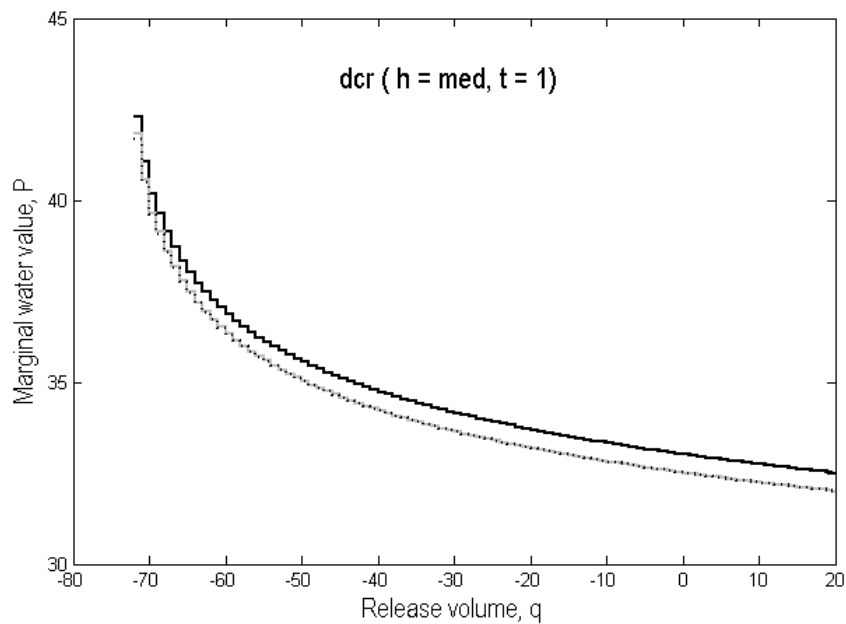
<sup>124</sup> As we have assumed hypothetical date, the mixing flow benefits may not represent real benefit. The bid prices were designed to show relative behavior of the HPS.

<sup>125</sup> This method simply splits the quantities ignoring the marginal water values.



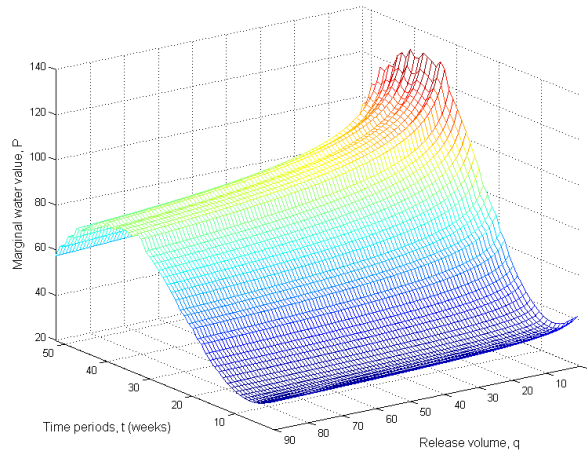
express firm bids for their future water needs (i.e., for modelled future hydrology states). Indeed we could incorporate other complexities into the market design such as future trading under uncertainty, tradable contracts and other hedging instruments.

Figure 6-11 illustrates the conditional net *dcrs* (having negative release volumes) obtained for the first period of the planning horizon for the medium hydrology state, under all the case. The dotted black line indicates the *dcrs* for the base case (Case 1). The *dcr* shifts up when we include mixing water demands (depicted by the black solid line plot). The grey coloured solid line plot denotes the demand curve for release of the third case (this is almost directly under the dotted black line). The curves in the latter two cases almost coincide with each other.



**Figure 6-11.** Single period net *dcr* obtained for the three cases for medium inflow hydrology state in time period  $t = 1$  (source: Appendix A6, Table A6.1).

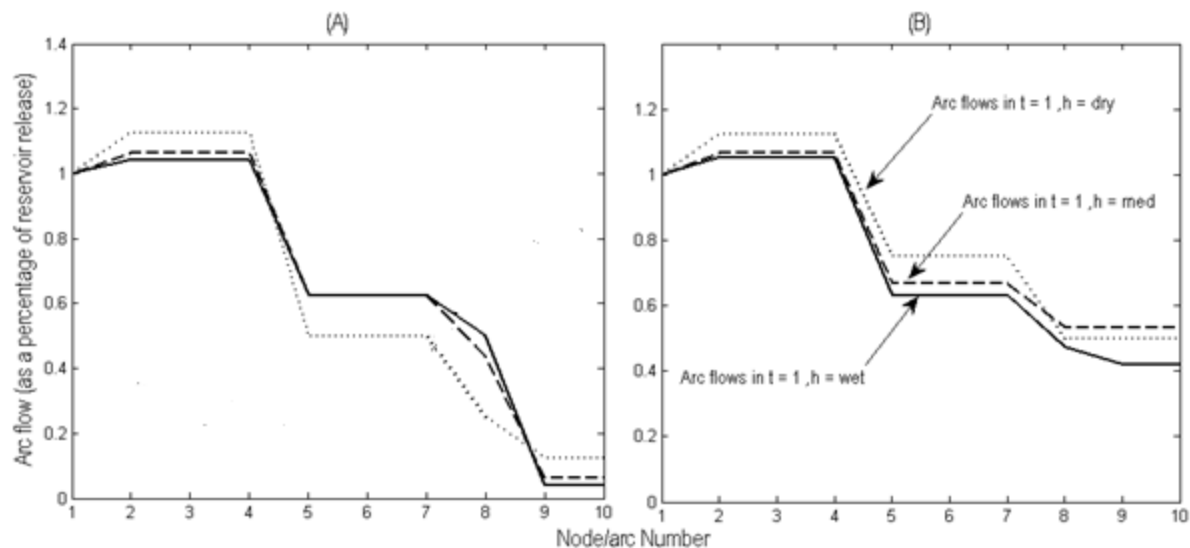
The algorithm forms a demand surface for release for the entire planning horizon for each hydrology state. For example, the figure below illustrates the demand surface for a 52 week planning horizon for the dry hydrology state. The bids (demands) are scaled accordingly to represent state dependent prices to construct the  $dcr_{h,t=1} \forall h = \{dry, med, wet\}$  (see Figure 6-12).



**Figure 6-12.** Demand surface for release for a 52 week planning horizon at low inflow hydrology state (scenario 2).

Figure 6-13 (A) shows the percentage arc flow volumes for the second case. The black solid line is the arc flow for wet hydrology state. The dashed black line and the dotted black line represent the medium and dry inflow states respectively. The tributary flow contribution at nodes 2 and 3 increases overall flow volumes on arcs 2 and 3.

The model allocates a higher percentage flow to the HPS during the dry state, particularly when it is prepared to pay more, in Figure 6-13 (B). Here accounting for the cooling flow demand curve yielded a 2% increase in net benefit for Case 2, and 10% when HPS increases her bid value two-fold. We would use the *dcr* to compute the above values. This should not be taken to indicate anything about the actual situation pertaining in the Waikato catchment, because the data we employed were purely illustrative. But it suggests that this type of integrated optimisation, or market, could yield economic benefits, in situations of this type.



**Figure 6-13.** Single period arc flows as a percentage of release recorded for Case 1 and Case 2 for all hydrology states (week,  $t=1$ , state,  $h = low, med, wet$ ).

We illustrated a nodal catchment model that can implicitly clear an inter-temporal multi-nodal market in presence of external flow uses for a single reservoir catchment with a tree configuration. We processed cooling and temperature control flow demands as non-consumptive uses on arcs. The multi-nodal deterministic CDDP constructed aggregate demand curves for reservoir release in each period. The stochastic CDDP algorithm then constructed the aggregate demand curves for reservoir storage. The results demonstrate how the nodal CDDP can be extended to handle some non-tree aspects of a water network such as required flow splitting in parallel arcs.

## 6.5 Chapter Conclusions

This chapter has described a means to model cooling and in-stream temperature control flow demands represented on parallel arcs that can be processed as for normal arc flow demands. We model these flow requirements assuming that an upstream flow unit can be diverted to any arc of our choice, but other assumptions can be made.

While applied here to an example involving hydro power production and mixing of thermal power station cooling water, the technique could be applied in a variety of situations in which flow mixing is important for other reasons, such as dilution of polluted water flows. Hydropower production too, can disturb river regulation causing negative environmental impacts because of their abrupt water releases during switching between peak and off-peak (Edwards et al., 1999). Our approach could be used to

impose a penalty charge for hydropower generators, rather than taxing other users in a wholesale water market. The results demonstrate how net benefits can be increased by accounting for the demand curve for cooling water, across all hydrology conditions and time periods.

# 7.

## **A Modelling Framework for Analysing Two-reservoir Multi-nodal Mixed-use Catchments**

### **7.1. Introduction**

Chapter 6 presented an integrated basin-wide planning method based on the intra-period nodal CDDP procedure to meet different types of ecological service flow requirements in a single reservoir catchment.

The objective of this chapter is to develop an intra-period two-reservoir CDDP procedure referring to the multi-reservoir deterministic LP model to construct intra-period marginal value surfaces for two inter-connected reservoirs in a mixed-use catchment to be used in a higher stochastic CDDP model that forms the demand surfaces for stored water. The remainder of this chapter is structured as follows.

- Section 7.2 presents the background and the research motives for the two-reservoir modelling. We begin by outlining computational issues that must be addressed in extending the intra-period CDDP to two reservoirs.
- Section 7.4 formulates an optimization model for jointly operating two reservoirs together with an additional set of assumptions and definitions, deemed important to set up the two-reservoir models. This helps us to understand the nature of demand surfaces underlying each sub-problem.
- Section 7.4 is about the demand surface for transfer for the inter-reservoir chain procedure. It involves several simple steps including pruning of sub-trees, process inter-reservoir nodal chains to form a marginal water value strip, and then create surfaces by concatenating these

strips using critical prices and price differences. There we present a numerical example regarding the construction of demand curves for transfer. We then use this example to note a procedure that construct the demand surface for transfer for the inter-reservoir chain.

- Finally, Section 7.5 concludes this chapter with a summary.

## **7.2. Optimal Joint Releasing of Two Reservoirs in a Wholesale Water Market**

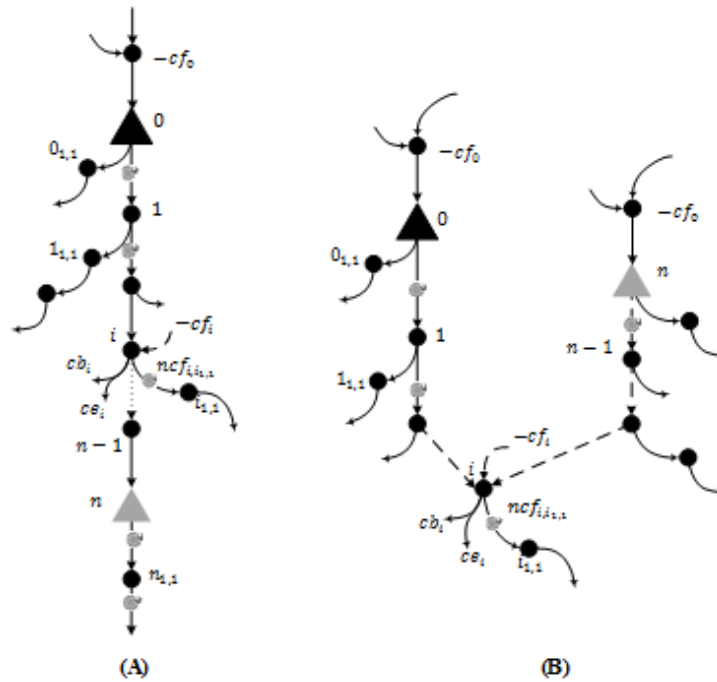
The objective of this section is to model optimal joint releasing of two reservoirs in a wholesale water market. Section 7.2.1 presents two reservoir optimization primal and dual models. With two inter-linked reservoirs we are now faced with the following new CDDP modelling issues.

The first modelling issue is that we are forced to construct marginal value *surfaces* for each reservoir based on the *joint* release or storage vector. Irrespective of the nature of the connection between the two reservoirs (e.g., series or parallel), in economic terms an optimal joint water release should allocate flows to the highest valued uses in the catchment while respecting capacities and requirements.

For example, assume two reservoirs connected in a series have to meet a set of upstream and downstream demands, as in Exhibit A of Figure 7-1. These two reservoirs are linked by a unique path, which we will call the “inter-reservoir chain”. Subject to various flow constraints, release from the upstream reservoir can be used to meet consumptive demands occurring anywhere along this interlink, or on branches linked to it. The upstream releases could pass right down through the inter-reservoir chain, delivering non-consumptive benefits, to then be stored in downstream storage to meet future consumption demands. The release from the top reservoir could reach the lower reservoir via the inter-reservoir chain providing the lower reservoir with the opportunity to release more water to meet more downstream demand. The marginal value of this met demand will depend on the downstream release. It will also affect the marginal value of that release and the marginal value of the top reservoir release which replaces it in the lower reservoir.

Exhibit B of Figure 7-1 shows two parallel connected reservoirs. There is an inter-reservoir chain linking the two reservoirs, with the potential to flow to a sink (e.g., the sea) from points along the inter-link. Obviously, there may be demands attached to the inter-reservoir chain that can be met by either

reservoir. The marginal value at the ‘black’ reservoir will depend, in part, on whether the release from the ‘grey’ reservoir is large enough to meet that demand or not. Subject to various flow constraints, an extra unit of water released from the grey reservoir could be used to meet demand outside this interlink<sup>126</sup>.



**Figure 7-1.** Series and parallel connected two reservoirs and nodes radiating in a tree network.

The second modelling issue is that the direction of processing is not uniquely determined, because for arcs in the inter-reservoir chain ‘toward the reservoir’ is different for each reservoir. In a two-reservoir tree structure, both reservoirs can act as the root as needed. The reservoir acting at the root is labelled the target and the other storage is labelled as passive.

With these explanations, it is clear that optimising joint operation of the two reservoirs may not be as straightforward as it was for the single reservoir case. The following discussion in Section 7.3 provides us with an opportunity to explain the joint operating two-reservoir problem. The main objective of this section is to describe a general mathematical framework for the two-reservoir optimization problem.

<sup>126</sup> However, the extra unit will often be used to meet consumptive demand along this interlink, or on branches linked to it, that would otherwise have been met by release from the ‘black’ reservoir. Thus, the net effect could be to increase storage in the ‘black’ reservoir. Again, the value of water in the ‘grey’ reservoir is not independent of the value of water stored in the ‘black’ reservoir, or vice versa.

### 7.3. Two Reservoir Optimization Model

#### 7.3.1. Assumptions and Nomenclature

For now, assume two series connected reservoirs under deterministic settings. In addition to the assumptions set forth in chapter 3, assume that the reservoirs are located sufficiently close to each other, so that we may ignore any flow transportation delays between the two reservoirs.

##### *Indices*

- $i, j$ : Denote any nodes in the two-reservoir tree, and  $(i, j)$  denotes an arc belonging to the set of arcs  $A$  in the inter-reservoir network.  $1, 2, 3, 4 \dots (n - 2), (n - 1)$  are the nodes in the inter-reservoir chain. The upstream and downstream reservoirs are labelled as 0 and  $n$  respectively.
- $k_i$ : Denotes the base node of the sub-tree(s) connected to the inter-reservoir chain node  $i$ . Thus,  $(i, k_i) \in A_i$  is the sub-tree linking arc. Here  $A_i$  is the set of arcs in the sub-tree  $T_i$ .
- $t = 1, 2, \dots T + 1$ : The operational time horizon. Period  $t = 1$  is the initial/current period (e.g., week).  $T + 1$  denotes the future periods of the planning horizon..

##### *Functions and Parameters*

- $NTB_{1:n-1,t}$ : Inter-reservoir chain benefits function, in time period  $t$ .
- $NB_{0,t}$ : Upstream nodal benefit function, in time period  $t$ .
- $NB_{n,t}$ : Downstream nodal benefit function, in time period  $t$ .
- $V_{T+1}(s_0^{T+1}, s_n^{T+1})$ : End-of-horizon marginal water values for the two reservoirs.
- $X_{(i,j)}$ : Arc flow bound/Release bounds for the two reservoirs.

##### *Decision Variables*

- $c_i^t, d_i^t$ : Nodal flow off-takes in time period  $t$ .  $c_i^t$  denotes consumptive demands and  $d_i^t$  represents tributary demands at node  $i$ .

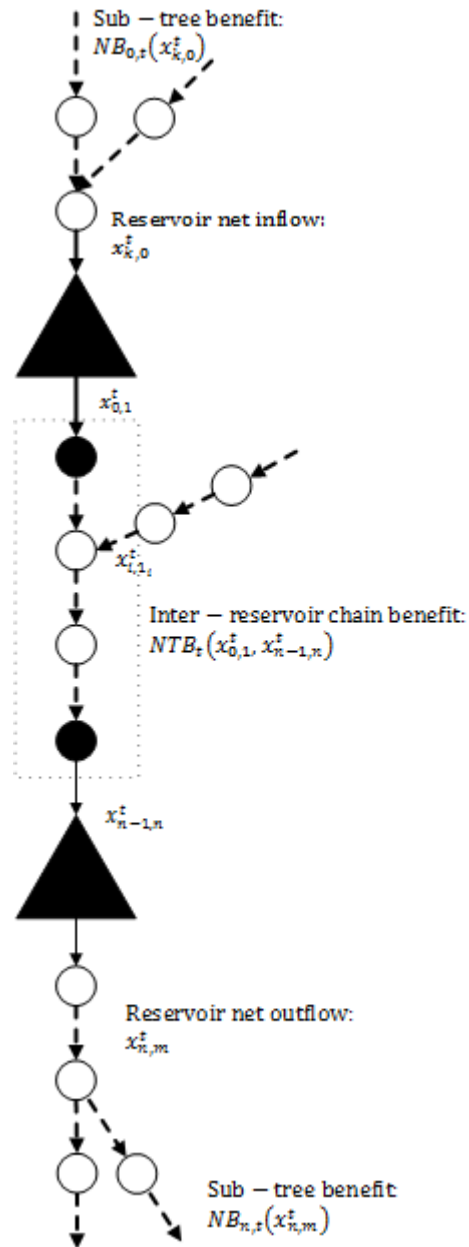


$x_{ij}^t$ : Flow through arc  $(i, j)$  in time period  $t$  (flow injection/release from a parent node to a child node).

$s_i^t$ : Beginning-of-period volume at storage  $i$  in time period  $t$ .

$NTB_{1:n-1,t}$ ,  $NB_{0,t}$ ,  $NB_{n,t}$ , and  $V_T(s_0^T, s_n^T)$  are piecewise linear convex benefit functions in their respective domains. The optimal intra-period net nodal chain benefit function  $NTB_{1:n-1,t}$  is to be obtained by solving an inter-reservoir chain optimization sub-problem. The net intra-period benefit functions of the upstream reservoir node,  $NB_{0,t}$  and the downstream reservoir node,  $NB_{n,t}$  are the optimal objective function values of respective sub-problems. Both  $NB_{0,t}$  and  $NB_{n,t}$  may include independent subtrees leading from each reservoir node where such sub-tree nodes are denoted by indices  $k$  and  $m$ . Finally, the end of horizon sub-problem provides the optimal the end-of-horizon marginal water value function,  $V_T(s_0^T, s_n^T)$  for the two reservoirs.

Figure 7-2 depicts a generalized series-linked two-reservoir network architecture. A single link, referred to as the inter-reservoir chain as in the previous discussion, connects the two reservoirs. Suppose that sub-trees lead from both upstream and downstream reservoirs. Outflows from the upstream sub-tree contribute to the upstream reservoir inflows. The sub-tree radiating from the downstream reservoir node accepts the downstream releases. For the interest of simplicity, we assume a tree like network structure although the single reservoir nodal CDDP procedure can handle parallel arcs.



**Figure 7-2.** Details of an inter-reservoir nodal chain network

The inter-reservoir chain optimization problem, which produces  $NTB_{1:n-1,t}(x_{0,1}^t, x_{n-1,n}^t)$ , could be modelled as a sub-problem within the two-reservoir optimization model. This sub-problem provides the maximum possible inter-reservoir chain benefit given release from the upstream reservoir,  $x_{0,1}^t$ , and outflow into the downstream reservoir,  $x_{n-1,n}^t$ . Here we assume that the optimal solution values for the inter-reservoir chain optimization sub-problem are available for any release and outflow.

### 7.3.2. Two-reservoir Optimization Model (Primal Problem)

This section formulates the two-reservoir deterministic LP optimization master problem. Then we decompose the above problem into three sub-problems as discussed below.

#### 7.3.2.1. Deterministic Two Reservoir Optimization Problem

$$\max_{s^t, x^t} \left( \sum_{t=1}^T NTB_{1:n-1,t}(x_{0,1}^t, x_{n-1,n}^t) + \sum_{t=1}^T NB_{0,t}(x_{k,0}^t) + \sum_{t=1}^T NB_{n,t}(x_{n,m}^t) + V_{T+1}(s_0^{T+1}, s_n^{T+1}) \right) \quad [7-1]$$

Subject to,

Storage continuity equations:

$$x_{0,1}^t - x_{k,0}^t - s_0^t + s_0^{t+1} = 0, \forall t \quad \lambda_{0,t} \quad [7-2]$$

$$x_{n,m}^t - x_{n-1,n}^t - s_n^t + s_n^{t+1} = 0, \forall t \quad \lambda_{n,t} \quad [7-3]$$

Initial storage conditions:

$$s_0^0 = S_0^0 \quad \gamma_{0,t}^0 \quad [7-4]$$

$$s_n^0 = S_n^0 \quad \gamma_{n,t}^0 \quad [7-5]$$

Storage bounds:

$$\underline{S}_0 \leq s_0^t \leq \bar{S}_0 \quad \gamma_{0,t} \quad [7-6]$$

$$\underline{S}_n \leq s_n^t \leq \bar{S}_n \quad \gamma_{n,t} \quad [7-7]$$

Bounds for the relevant water channels:

$$\underline{X}_{(i,j)} \leq x_{i,j}^t \leq \bar{X}_{(i,j)} \quad \forall (i,j) \in \{(0,1), (k,0), (n-1,n), (n,m)\}, t \quad \mu_{(i,j),t} \quad [7-8]$$

The next section includes interpretations and explanations of the prices and allocations generated by the above model.

#### Explanation (primal model):

The objective function of the master problem seeks to maximise the intra-period benefits of two reservoirs ( $i = 0, n$ ), the inter-reservoir chain ( $1:n-1$ ) and future storage benefits over the planning horizon.

Equations [7-4], [7-5], [7-6], [7-7] and [7-8] denote the primal constraints for initial storage levels, a pair of primal constraints for the storage and the relevant water channels bounds. Each primal bound pair  $[\underline{S}_0, \bar{S}_0]$ ,  $[\underline{S}_n, \bar{S}_n]$ , and  $[\underline{X}_{(i,j)}, \bar{X}_{(i,j)}]$  will have two individual dual variables that can be denoted by

assigning superscripts – and + to the (non-positive) dual price on the lower bound, and the (non-negative) dual price on the upper bound, respectively. We can construct an unrestricted “composite” dual variable by taking the sum of these two variables. For example, from constraint equation [6-6],  $s_0^t \geq \underline{s}_0$  and  $s_0^t \leq \bar{s}_0$  have the corresponding (respective) dual variables  $\gamma_{0,t}^+$  and  $\gamma_{0,t}^-$  where  $\gamma_{0,t} = \gamma_{0,t}^+ + \gamma_{0,t}^-$ .

Here, we do not set any constraints for the end-of-horizon storages but we could follow the formulations in Chapter 4.

The dual variables  $\lambda_{0,t}$  (constraint [7-2]) and  $\lambda_{n,t}$  (constraint [7-3]) are unrestricted in sign.  $\lambda_{0,t}$  indicates the marginal water values at the upstream reservoir node and  $\lambda_{n,t}$  is the marginal water values at the downstream reservoir node.

In the interests of simplicity, we ignore denoting spill<sup>127</sup>. We can further decompose the above primal problem into a set of single period sub-problems. Each sub-model maximises the current period and the future period benefits as in the DP/CDDP model.

Suppose that  $V_{t+1}(s_0^{t+1}, s_n^{t+1})$  denotes the accumulated optimal benefits for time periods  $t + 1, t + 2, \dots, T + 1$  the optimal release policy for period  $t$  is given by:

$$V_t^*(s_0^t, s_n^t) = \max_{x_{0,1}^t, x_{n,m}^t} \left[ \left( NTB_{1:n-1,t}(x_{0,1}^t, x_{n-1,n}^t) + NB_{0,t}(x_{0,1}^t) + NB_{n,t}(x_{n,m}^t) \right) + V_{t+1}(s_0^{t+1}, s_n^{t+1}) \right]$$

This gives the maximum system benefits / performance and optimal release  $(x_{0,1}^{t*}, x_{n-1,n}^{t*})$  for any set of storage volumes  $(s_0^t, s_n^t)$  in that period subject to appropriate constraints from above.

The next section develops a computational routine for deriving the optimal release policy for the two-reservoir problem based on the Lagrangian dual formulations given here.

### 7.3.2.2. Lagrangian Problem

Based on the above assumptions, the primal model ([7-1], [7-2], [7-3], [7-4], [7-5], [7-6], [7-7], and [7-8]) is convex. We write the Lagrangian problem using Lagrange multipliers  $\lambda_{0,t}$ ,  $\lambda_{n,t}$ ,  $\mu_{(0,1),t}^+$ ,  $\mu_{(0,1),t}$ ,

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<sup>127</sup> This modelling choice is made to reduce the complexity of the two-reservoir CDDP procedure. The model form does not preclude spill, it just does not label it explicitly.

$\mu_{(n,n-1),t}$ ,  $\gamma_{0,t}$ , and  $\gamma_{n,t}$ , and then consider the first order conditions to investigate the dual price relationships.

$$\begin{aligned} \mathcal{L}(\lambda, x, s) = & \sum_{t=1}^T \left( NTB_{1:n-1,t}(x_{0,1}^t, x_{n-1,n}^t) + NB_{0,t}(x_{k,0}^t) + NB_{n,t}(x_{n,m}^t) - \lambda_{0,t}(x_{0,1}^t - x_{k,0}^t - s_0^t + \right. \\ & s_0^{t+1}) - \lambda_{n,t}(x_{n,m}^t - x_{n-1,n}^t - s_n^t + s_n^{t+1}) - \mu_{(k,0),t}^+(x_{k,0}^t - \bar{X}_{k,0}) - \mu_{(k,0),t}^-(\underline{X}_{k,0} - x_{k,0}^t) - \\ & \mu_{(n,m),t}^+(x_{n,m}^t - \bar{X}_{n,m}) - \mu_{(n,m),t}^-(\underline{X}_{n,m} - x_{n,m}^t) - \mu_{(0,1),t}^+(x_{0,1}^t - \bar{X}_{0,1}) - \mu_{(0,1),t}^-(\bar{X}_{0,1} - x_{0,1}^t) - \\ & \mu_{(n,n-1),t}^+(x_{n,n-1}^t - \bar{X}_{n,n-1}) - \mu_{(n,n-1),t}^-(\bar{X}_{n,n-1} - x_{n,n-1}^t) - \gamma_{0,t}^+(s_0^t - \bar{s}_0) - \gamma_{0,t}^-(\underline{s}_0 - s_0^t) - \\ & \left. \gamma_{n,t}^+(s_n^t - \bar{s}_n) - \gamma_{n,t}^-(\underline{s}_n - s_n^t) \right) + V_{T+1}(s_0^{T+1}, s_n^{T+1}) \end{aligned} \quad [7-9]$$

Suppose that the terms  $NTB_{1:n-1,t}(x_{0,1}^t, x_{n-1,n}^t)$ ,  $NB_{0,t}(x_{k,0}^t)$ ,  $NB_{n,t}(x_{n,m}^t)$  and  $V_{T+1}(s_0^{T+1}, s_n^{T+1})$  in  $\mathcal{L}(\lambda, x, s)$ , are (partially) differentiable with respect to  $x_{0,1}^t$ ,  $x_{n,n-1}^t$ ,  $x_{k,0}^t$ ,  $x_{n,m}^t$ ,  $s_0^t$ ,  $s_n^t$  and  $\lambda_{0,t}$ ,  $\lambda_{n,t}$ ,  $\mu_{(0,1),t}^+$ ,  $\mu_{(0,1),t}^-$ ,  $\mu_{(n,n-1),t}^+$ ,  $\gamma_{0,t}$ ,  $\gamma_{n,t}$ , the necessary conditions for the objective function to be a maximum are given by the following first order conditions:

$$\frac{d\mathcal{L}}{dx_{k,0}^t} = \frac{dNB_{0,t}}{dx_{k,0}^t} + \lambda_{0,t} - \mu_{(k,0),t}^+ + \mu_{(k,0),t}^- = 0 \quad [7-10]$$

$$\frac{d\mathcal{L}}{dx_{n,m}^t} = \frac{dNB_{n,t}}{dx_{n,m}^t} - \lambda_{n,t} - \mu_{(n,m),t}^+ + \mu_{(n,m),t}^- = 0 \quad [7-11]$$

$$\frac{\partial \mathcal{L}}{\partial x_{i,j}^t} = \nabla NTB_{1:n-1,t}(x_{0,1}^t, x_{n-1,n}^t) + \begin{bmatrix} -\lambda_{0,t} \\ \lambda_{n,t} \end{bmatrix} - \begin{bmatrix} \mu_{(0,1),t}^+ \\ \mu_{(n,n-1),t}^+ \end{bmatrix} + \begin{bmatrix} \mu_{(0,1),t}^- \\ \mu_{(n,n-1),t}^- \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad [7-12]$$

$$\frac{d\mathcal{L}}{ds_0^t} = \lambda_{0,t} - \lambda_{0,t-1} - \gamma_{0,t}^+ + \gamma_{0,t}^- = 0 \quad [7-13]$$

$$\frac{d\mathcal{L}}{ds_n^t} = \lambda_{n,t} - \lambda_{n,t-1} - \gamma_{n,t}^+ + \gamma_{n,t}^- = 0 \quad [7-14]$$

$$\frac{\partial \mathcal{L}}{\partial s_i^{T+1}} = \nabla V_{T+1}(s_0^{T+1}, s_n^{T+1}) + \begin{bmatrix} -\lambda_{0,T+1} \\ -\lambda_{n,T+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad [7-15]$$

The next section interprets and explains the dual prices generated by the Lagrangian formulation.

### Explanation (dual model)

Marginal benefit foregone in the subtree above the upstream reservoir is equal to the marginal water value of that reservoir if the bounds are not binding (see equation [7-10]). The marginal benefit from

release from the downstream reservoir is equal to the marginal water value of that reservoir if the bounds are not binding (see equation [7-11]).

Equation [7-12] shows the interaction of the current water values of the two reservoirs,  $(\lambda_{0,t}$  and  $\lambda_{n,t})$  with the inter-reservoir chain and the Equation [7-13] and Equation [7-14] shows the interaction of the reservoir water values over (that is water values between the current and the previous period's water values of the sub-tree at 0, and the sub-tree at  $n$ , respectively). For example, according to equation [7-13], if the arc flow bounds are not binding, the current and the next period marginal water values for the upstream reservoir are equal ( $\lambda_{0,t} = \lambda_{0,t+1}$ ).

Equation [7-12] shows the first order conditions of the inter-reservoir chain benefit function with respect to  $x_{0,1}^t$  and  $x_{n-1,n}^t$ . For example, when the relevant primal variables are away from their bounds, the partial derivative of  $NTB_{1:n-1,t}$  with respect to  $x_{0,1}^t$  will be equal to the marginal water value  $\lambda_{0,t}$  of the upstream reservoir. Similarly, the negative of the partial derivative of  $NTB_{1:n-1,t}$  with respect to  $x_{n-1,n}^t$  is equal to the downstream storage  $\lambda_{n,t}$ .

Equation [7-15] represents the end-of-horizon marginal water values for the two storages. This values end-of-horizon storage volume that will be carried beyond the current planning horizon. The end-of-horizon effects can be modelled as in Chapter 3 by specifying the bounds on end-of-horizon storage bid tranches and the end-of-horizon storage conditions. However, this discussion does not require detail modelling of the end-of-horizon effects of the two-reservoir problem.

In order to focus our study on the implications of the release and storage variables, we rewrite the Lagrangian function in [7-9] to form the Lagrangian dual sub-problem by grouping storage/nodal continuity variables and including the bound constraints explicitly as follows:

$$\begin{aligned} \max_{x,s} \sum_{t=1}^T & (NTB_{1:n-1,t}(x_{0,1}^t, x_{n-1,n}^t) - \lambda_{0,t}x_{0,1}^t + \lambda_{n,t}x_{n-1,n}^t + NB_{0,t}(x_{k,0}^t) + \lambda_{0,t}x_{k,0}^t + NB_{n,t}(x_{n,m}^t) - \\ & \lambda_{n,t}x_{n,m}^t + \lambda_{0,t}s_0^t - \lambda_{0,t}s_0^{t+1} + \lambda_{n,t}s_n^t - \lambda_{n,t}s_n^{t+1}) + V_{T+1}(s_0^{T+1}, s_n^{T+1}) \end{aligned}$$

Subject to,

$$s_0^0 = S_0^0; s_n^0 = S_n^0; \underline{s}_0 \leq s_0^t \leq \bar{s}_0; \underline{s}_n \leq s_n^t \leq \bar{s}_n; \underline{X}_{(i,j)} \leq x_{i,j}^t \leq \bar{X}_{(i,j)} \quad [7-16]$$

The Lagrangian sub-problem [7-16] naturally decomposes into the following set of sub-problems, which separate the other constraints such as arc flow bounds and storage bounds in time periods  $t \leq T$ . The reformulation of sub-problem [7-16] creates the following sub-problems [7-17] to [7-22]. Thus, we could individually optimise the following sub-problems.

The upstream reservoir ( $i = 0$ ) sub-tree sub-problem:

$$\begin{aligned} & \max_x (NB_{0,t}(x_{k,0}^t) + \lambda_{0,T}x_{k,0}^t) \\ & \text{s. t. } \underline{X}_{k,0} \leq x_{k,0}^t \leq \bar{X}_{k,0} \quad \forall t \leq T \end{aligned} \quad [7-17]$$

The downstream reservoir ( $i = n$ ) sub-tree sub-problem:

$$\begin{aligned} & \max_x (NB_{n,T}(x_{n,m}^t) - \lambda_{n,T}x_{n,m}^t) \\ & \text{s. t. } \underline{X}_{n,m} \leq x_{n,m}^t \leq \bar{X}_{n,m} \quad \forall t \leq T \end{aligned} \quad [7-18]$$

Inter-reservoir chain ( $1:n-1$ ) sub-problem:

$$\begin{aligned} & \max_x (NTB_{1:n-1,T}(x_{0,1}^t, x_{n-1,n}^t) - \lambda_{0,t}x_{0,1}^t + \lambda_{n,T}x_{n-1,n}^t) \\ & \text{s. t. } \underline{X}_{0,1} \leq x_{0,1}^t \leq \bar{X}_{0,1}, \underline{X}_{n-1,n} \leq x_{n-1,n}^t \leq \bar{X}_{n-1,n} \quad \forall t \leq T \end{aligned} \quad [7-19]$$

Reservoir storage sub-problem:

For the inter-period upstream storage benefits:

$$\begin{aligned} & \max_s (\lambda_{0,t} - \lambda_{0,t-1})s_0^t \\ & \text{s. t. } \underline{S}_0 \leq s_0^t \leq \bar{S}_0 \quad \forall t < T + 1 \end{aligned} \quad [7-20]$$

For the inter-period downstream storage benefits:

$$\begin{aligned} & \max_s (\lambda_{n,t} - \lambda_{n,t-1})s_n^t \\ & \text{s. t. } \underline{S}_n \leq s_n^t \leq \bar{S}_n \quad \forall t < T + 1 \end{aligned} \quad [7-21]$$

For the end-of-horizon effects:

$$\begin{aligned}
& \max_s V_{T+1}(s_0^{T+1}, s_n^{T+1}) - \lambda_{0,T} s_0^{T+1} - \lambda_{n,T} s_n^{T+1} \\
& \text{s. t. } \underline{s}_0 \leq s_0^{T+1} \leq \bar{s}_0, \underline{s}_n \leq s_n^{T+1} \leq \bar{s}_n
\end{aligned} \tag{7-22}$$

An optimal solution to [7-17]-[7-22] under certain conditions optimises [7-1]-[7-8]. The optimality conditions for the full model [7-1]-[7-8] are the combination of all feasibility conditions for the sub-problems above plus feasibility for the two storage balance constraints.

The following section further describes the nature of the optimal solutions of each sub-problem. This will be later useful to explain the construction of various marginal water value curves and surfaces.

The sub-problems [7-17], [7-18], and [7-19] seek to find the optimal releases for any dual pair  $(\lambda_{0,t}, \lambda_{n,t})$ . Note that  $x(\lambda)$  in the above sub-problems provides the optimal primal solution value based on the  $\lambda$ . In general the  $x(\lambda)$  functions associated with the above sub-problems can be used to create marginal water value surfaces/curves that contain the optimal releases/storages sets. In this section we study the conditions under which we can assure the existence feasible solutions for the above sub-problems. For simplicity, we will consider the above solutions in periods  $t = T$  and  $t = T + 1$ .

### 7.3.3. Sub-problem Analysis

This section analyses the sub-problems given in the previous section. For simplicity, the following assumes unique optimal primal solutions to each sub-problem. It is straightforward to extend to cases involving multiple optimal primal solutions.

Suppose that  $x_{k,0}^*(\lambda_{0,T})$  and  $x_{n,m}^*(\lambda_{n,T})$  give the optimal solutions of the sub-problems [7-17] and [7-18] respectively based on the values of the coefficients  $\lambda_{0,T}$  and  $\lambda_{n,T}$  in the respective objective functions. The optimal solution of sub-problem [7-17] has the following structure. The demand curve  $x_{k,0}^*(\lambda_0)$  is decreasing with  $x_{k,0}^*(\lambda_0) = \bar{X}$  when  $\lambda_0 > -\frac{dNB_0(\bar{X})}{dx_{k,0}}$  and  $x_{k,0}^*(\lambda_0) = \underline{X}$  when  $\lambda_0 < -\frac{dNB_0(\underline{X})}{dx_{k,0}}$ . Assuming  $NB_0$  is differentiable, when  $\lambda_0$  is between the bounds  $-\frac{dNB_0(\underline{X})}{dx_{k,0}}$  and  $-\frac{dNB_0(\bar{X})}{dx_{k,0}}$ ,

we have  $\underline{X} \leq x_{k,0}^*(\lambda_0) \leq \bar{X}$  such that  $\frac{dNB_0(x_{k,0}^*(\lambda_0))}{dx_{k,0}} = \lambda_0$  (refer to the first order conditions [7-10]).

Similarly, for sub-problem [7-18], when  $\frac{dNB_n(\underline{X})}{dx_{n,m}} \leq \lambda_n \leq \frac{dNB_n(\bar{X})}{dx_{n,m}}$  we have  $\underline{X} \leq x_{n,m}^*(\lambda_n) \leq \bar{X}$  such that



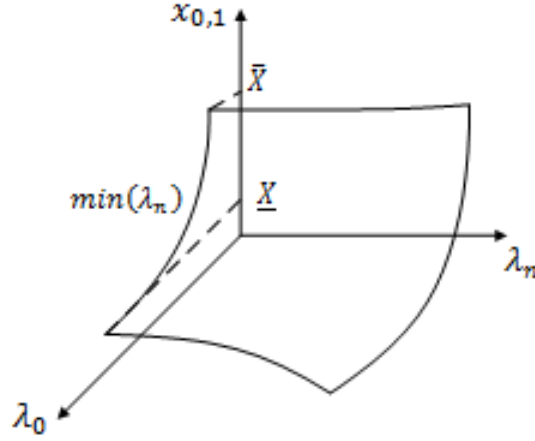
$\frac{dNB_n(x_{n,m}^*(\lambda_n))}{dx_{n,m}} = \lambda_n$ . Using the equation [7-11], we check the boundary conditions of  $x_{n,m}^*(\lambda_n)$  with  $x_{n,m}^*(\lambda_n) = \bar{X}$  when  $\lambda_n < \frac{dNB_n(\bar{X})}{dx_{n,m}}$  and  $x_{n,m}^*(\lambda_n) = \underline{X}$  when  $\lambda_n > \frac{dNB_n(\underline{X})}{dx_{n,m}}$ . The former set of price-quantity pairs denoted by  $x_{k,0}^*(\lambda_0)$  is the demand curve for inflows for the upstream reservoir and the latter set of price-quantity pairs denoted by  $x_{n,m}^*(\lambda_n)$  is the demand curve for release for the downstream reservoir. Obviously, this single dimension demand curve construction can be easily done with the nodal CDDP procedure. Details are shown in later sections.

Sub-problem [7-19] seeks to maximise the inter-reservoir chain benefits subject to inter-reservoir chain arc flow constraints. Suppose  $x_{0,1}^*(\lambda_{0,T}, \lambda_{n,T})$  and  $x_{n-1,n}^*(\lambda_{0,T}, \lambda_{n,T})$  give the optimal solutions of sub-problem [7-19] dependent on the value of scalar coefficients.  $\lambda_{0,T}$  and  $\lambda_{n,T}$  in the objective function.

Marginal inter-reservoir benefit function satisfies  $\frac{\partial NTB_{1:n-1}(x_{0,1}^* x_{n-1,n}^*)}{\partial x_{0,1}} - \lambda_0 = 0$  and  $\frac{\partial NTB_{1:n-1}(x_{0,1}^* x_{n-1,n}^*)}{\partial x_{n-1,n}} + \lambda_n = 0$  when the respective bounds are slack at the optimal solution of this sub-problem. This observation, coupled with the [7-16] yield the following inequalities and their implications.

For the upstream release flow conditions:  $\lambda_0 < \frac{\partial NTB_{1:n-1,t}(\bar{X}_{0,1}, x_{n-1,n}^*)}{\partial x_{0,1}} \Rightarrow x_{0,1}^*(\lambda_0, \lambda_n) = \bar{X}_{0,1}$  and  $\lambda_0 > \frac{\partial NTB_{1:n-1,t}(\bar{X}_{0,1}, x_{n-1,n}^*)}{\partial x_{0,1}} \Rightarrow x_{0,1}^*(\lambda_0, \lambda_n) = \underline{X}_{0,1}$ . The downstream release flow conditions are:  $\lambda_n > -\frac{\partial NTB_{1:n-1}(x_{0,1}^*, \bar{X}_{n-1,n})}{\partial x_{n-1,n}} \Rightarrow x_{n-1,n}^*(\lambda_0, \lambda_n) = \bar{X}_{n-1,n}$ , and  $\lambda_n < -\frac{\partial NTB_{1:n-1}(x_{0,1}^*, \bar{X}_{n-1,n})}{\partial x_{n-1,n}} \Rightarrow x_{n-1,n}^*(\lambda_0, \lambda_n) = \underline{X}_{n-1,n}$ . Sub-problem [7-19] creates a region  $(x_{0,1}^*, x_{n-1,n}^*, \lambda_0, \lambda_n)$  in the price-quantity space, which could be considered to be a demand curve for transfer. Then  $x_{0,1}^*(\lambda_0, \lambda_n)$  and  $x_{n-1,n}^*(\lambda_0, \lambda_n)$  are the demand surfaces for release into and receipt from the inter-reservoir chain, respectively. The two surfaces are monotonic as  $\lambda_0$  or  $\lambda_n$  increases. The above motivates us to describe the corners and interior points of the feasible region. The following conditions,  $x_{0,1}^* = \bar{X}_{0,1}$  when  $\lambda_0 < \frac{\partial NTB_{1:n-1}(\bar{X}_{0,1}, x_{n-1,n}^*)}{\partial x_{0,1}}$  and  $x_{0,1}^* = \underline{X}_{0,1}$  when  $\lambda_0 > \frac{\partial NTB_{1:n-1}(\underline{X}_{0,1}, x_{n-1,n}^*)}{\partial x_{0,1}}$  indicate that the  $\min(\lambda_n)$  edge of

the surface<sup>128</sup> is decreasing along  $\lambda_0$ -axis (refer Figure 6-3). The first order conditions  $\frac{\partial NTB_{1:n-1}(x_{0,1}^* x_{n-1,n}^*)}{\partial x_{0,1}} = \lambda_0$  and  $\frac{\partial NTB_{1:n-1}(x_{0,1}^* x_{n-1,n}^*)}{\partial x_{n-1,n}} = -\lambda_n$  implicitly describe the correlation between  $\lambda_0$  and  $\lambda_n$  (i.e., the bottom edge or the interior of the demand surface). This will eventually connect to a large flat area which indicates that  $x_{0,1}^*$  cannot respond to additional increase in  $\lambda_0$  because  $\lambda_0$  could get arbitrary high when  $x_{0,1}$  is at its lower bound.



**Figure 7-3.** Shape of the marginal water value surface for transfer seen at the upstream reservoir.

Furthermore, the optimal solutions  $(x_{0,1}^*(\lambda_0^*, \lambda_n^*), x_{n-1,n}^*(\lambda_0^*, \lambda_n^*))$  for the above sub-problem implicitly define the demand surface. This is independent of the storage demand curves/surfaces referred in the next set of sub-problems. Since ultimately  $NTB_{1:n-1,t}$  arises from optimising an inter-reservoir chain sub-problem, we defer discussion of the construction of this four dimensional marginal value surface to Section 7.4.

Sub-problems [7-20] and [7-21] seek to maximize the storage benefits for the upstream and the downstream reservoirs respectively. Like in the previous sub-problems, the  $\lambda$ 's function as scalar coefficients of  $s$ 's in the two objective functions. The storage bounds will only be slack at the optimal solution when the marginal water values of respective reservoirs are equal between two periods:  $\lambda_{0,t} = \lambda_{0,t-1}$  and  $\lambda_{n,t} = -\lambda_{n,t-1}$ . The following conditions show when the storage limits of the upper storage

<sup>128</sup> Similarly, the  $\min(\lambda_n)$  edge of the demand surface  $x_{n-1,n}^*(\lambda_n, \lambda_0)$  is described by the following conditions,  $x_{n-1,n}^* = \bar{X}$  when  $\lambda_n > \left(-\frac{dNB_{1:n-1}(\bar{X})}{dx_{1:n-1}}\right)$  and  $x_{n-1,n}^* = \underline{X}$  when  $\lambda_n < \left(-\frac{dNB_{1:n-1}(\underline{X})}{dx_{1:n-1}}\right)$ .

are binding:  $s_0^{*t}(\lambda_0, \lambda_n) = \bar{S}_0$  when  $\lambda_{0,t} > \lambda_{0,t-1}$  and  $s_0^{*t}(\lambda_0, \lambda_n) = \underline{S}_0$  when  $\lambda_{0,t} < \lambda_{0,t-1}$ . Similarly, we can write the following for the downstream storage:  $s_n^{*t}(\lambda_0, \lambda_n) = \bar{S}_n$  when  $\lambda_{n,t} > \lambda_{n,t-1}$  and  $s_n^{*t}(\lambda_0, \lambda_n) = \underline{S}_n$  when  $\lambda_{n,t} < \lambda_{n,t-1}$ .

For any  $(\lambda_0, \lambda_n)$ , using [7-2] and [7-3], we put  $\tilde{s}_0^t(\lambda_0, \lambda_n) = s_0^{t+1*}(\lambda_0, \lambda_n) + x_{0,1}^{t*}(\lambda_0, \lambda_n) - x_{k,0}^{t*}(\lambda_0)$  and  $\tilde{s}_n^t(\lambda_0, \lambda_n) = s_n^{t+1*}(\lambda_0, \lambda_n) + x_{n,m}^{t*}(\lambda_0, \lambda_n) - x_{n-1,n}^{t*}(\lambda_0, \lambda_n)$  where  $\tilde{s}_0^t(\lambda_0, \lambda_n)$  and  $\tilde{s}_n^t(\lambda_0, \lambda_n)$  are between their bounds they are consistent with the optimal solutions of [7-20] and [7-21] with  $\lambda_{0,t-1} = \lambda_{0,t}$  and  $\lambda_{n,t-1} = \lambda_{n,t}$ . We will first consider the upstream storage. For any such  $(\lambda_0, \lambda_n)$  we put  $s_0^{*t}(\lambda_0, \lambda_n) = \underline{S}_0$  where  $\tilde{s}_0^t(\lambda_0, \lambda_n) < \underline{S}_0$ . Then  $s_0^{*t}(\lambda_0, \lambda_n) = \underline{S}_0$  is still consistent with  $\lambda_{0,t-1} > \lambda_{0,t}$ . Similarly, for the upper bounds we put  $s_0^{*t}(\lambda_0, \lambda_n) = \bar{S}_0$  where  $\tilde{s}_0^t(\lambda_0, \lambda_n) > \bar{S}_0$  and  $s_0^{*t}(\lambda_0, \lambda_n) = \bar{S}_0$  is consistent with  $\lambda_{0,t-1} < \lambda_{0,t}$ . For the lower bounds of the downstream storage, we set  $s_n^{*t}(\lambda_0, \lambda_n) = \underline{S}_n$  where  $\tilde{s}_n^t(\lambda_0, \lambda_n) < \underline{S}_n$  for any  $(\lambda_0, \lambda_n)$ . Then  $s_n^{*t}(\lambda_0, \lambda_n) = \underline{S}_n$  is consistent with  $\lambda_{n,t-1} > \lambda_{n,t}$ . Similarly, for the upper bounds  $s_n^{*t}(\lambda_0, \lambda_n) = \bar{S}_n$  where  $\tilde{s}_n^t(\lambda_0, \lambda_n) > \bar{S}_n$  which is consistent with  $\lambda_{n,t-1} < \lambda_{n,t}$ . So there will be many  $s_0^{*t}$  and  $s_n^{*t}$  that satisfy [7-2] and [7-3] respectively, and are consistent with the prices. According to sub-problems [7-20] and [7-21], the marginal water value between periods are linked and furthermore equal if storage bounds are slack. But we cannot guarantee that these storage volumes are in the feasible set of solution of the two sub-problems. Hence, we are motivated to consider sub-problem [7-22] which maximises the end-of-horizon benefit function subject to the end-of-horizon storage constraints of the two storages. Using this, we will find the optimal price-volume solution set for the end-of-horizon conditions. For the upstream end-of-horizon conditions, when  $-\lambda_{0,T}$  is between  $\frac{\partial V_{T+1}(\underline{S}_0)}{\partial s_0^{T+1}}$  and  $\frac{\partial V_{T+1}(\bar{S}_0)}{\partial s_0^{T+1}}$ , we have  $\underline{S}_0 < s_0^{T+1*}(\lambda_{0,T}, \lambda_{n,T}) < \bar{S}_0$  such that  $\frac{\partial V_{T+1}(s_0^{T+1*}, s_n^{T+1*})}{\partial s_0^{T+1}} = \lambda_{0,T}$ . The following conditions hold when the storage limits are binding: for the upper storage bounds,  $\lambda_{0,T} > \frac{\partial V_{T+1}(s_0^{T+1*}, s_n^{T+1*})}{\partial s_0^{T+1}}$  when  $s_0^{T+1*}(\lambda_{0,T}, \lambda_{n,T}) = \bar{S}_0$  and for the lower storage bounds,  $\lambda_{0,T} < \frac{\partial V_{T+1}(s_0^{T+1*}, s_n^{T+1*})}{\partial s_0^{T+1}}$  when  $s_0^{T+1*}(\lambda_{0,T}, \lambda_{n,T}) = \underline{S}_0$ . For the downstream end-of-horizon conditions, when  $-\lambda_{n,T}$  is between  $\frac{\partial V_{T+1}(\underline{S}_n)}{\partial s_n^{T+1}}$  and  $\frac{\partial V_{T+1}(\bar{S}_n)}{\partial s_n^{T+1}}$ , we have  $\underline{S}_n \leq s_n^{T+1*}(\lambda_{0,T}, \lambda_{n,T}) \leq \bar{S}_n$  such

that  $\frac{\partial V_{T+1}(s_0^{T+1*}, s_n^{T+1*})}{\partial s_n^{T+1}} = \lambda_{n,T}$ . The demand surface  $s_n^{T+1*}(\lambda_{0,T}, \lambda_{n,T})$  is decreasing along the  $s$  axis with

$$s_n^{T+1*}(\lambda_{0,T}, \lambda_{n,T}) = \bar{s}_n \quad \text{when} \quad \lambda_{n,T} > \frac{\partial V_{T+1}(s_0^{T+1*}, s_n^{T+1*})}{\partial s_n^{T+1}} \quad \text{and} \quad s_n^{T+1*}(\lambda_{0,T}, \lambda_{n,T}) = \underline{s}_n \quad \text{when}$$

$$\frac{\partial V_{T+1}(s_0^{T+1*}, s_n^{T+1*})}{\partial s_n^{T+1}} > \lambda_{n,T}. \quad \text{Then the end-of-horizon sub-problem determines a set of possible optimal}$$

storage price-volume solutions  $(s_0^{T+1*}, s_n^{T+1*}, \lambda_{0,T}^*, \lambda_{n,T}^*)$  which can be passed on to the previous period sub-problem like in the reverse recursive CDDP. So we start from the end-of-horizon storage sub-problem [7-22] which gives us the truncated marginal water value surface for the end-of-horizon storage demand surfaces  $s_0^{T+1}(\lambda_{0,T}, \lambda_{n,T})$  and  $s_n^{T+1}(\lambda_{0,T}, \lambda_{n,T})$ . Unlike the current period demand surface for storage, constructing  $s_0^{T+1}(\lambda_{0,T}, \lambda_{n,T})$  and  $s_n^{T+1}(\lambda_{0,T}, \lambda_{n,T})$  can be straightforward as the model does not have to maintain  $V_{T+1}$  from period to period.

When the upper and lower storage bounds are slack (i.e.,  $\lambda_{0,T} = \lambda_{0,T-1}$ ,  $\lambda_{n,T} = \lambda_{n,T-1}$ ), the optimal  $s_0^{T*}$  and  $s_n^{T*}$  for the storage sub-problems [7-20] and [7-21] can be found directly using the storage balance constraints:  $s_0^T(\lambda_{0,T}^*, \lambda_{n,T}^*) = x_{0,1}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*) - x_{k,0}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*) + s_0^{T+1*}(\lambda_{0,T}^*, \lambda_{n,T}^*)$  in [7-2] and  $s_n^T(\lambda_{0,T}^*, \lambda_{n,T}^*) = x_{n,m}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*) - x_{n-1,n}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*) + s_n^{T+1*}(\lambda_{0,T}^*, \lambda_{n,T}^*)$  in [6-3]. So that  $s_0^T(\lambda_{0,T}, \lambda_{n,T})$  and  $s_n^T(\lambda_{0,T}, \lambda_{n,T})$  are 2-D overall monotone decreasing demand surfaces. The beginning-of-period storage optimal solution can be denoted as  $(s_0^{T*}, s_n^{T*}, \lambda_{0,T}^*, \lambda_{n,T}^*)$  for the storage sub-problems [7-20] and [7-21] in the price/quantity set  $\Theta$  defined as:  $(s_0^T(\lambda_{0,T}^*, \lambda_{n,T}^*), s_n^T(\lambda_{0,T}^*, \lambda_{n,T}^*), \lambda_{0,T}^*, \lambda_{n,T}^*, x_{0,1}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*), x_{k,0}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*), x_{n-1,n}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*), x_{n,m}^{T*}(\lambda_{0,T}^*, \lambda_{n,T}^*), s_0^{T+1*}, s_n^{T+1*})$ . But the above discussion implies recording of quantity demands for every possible price pair which is not a mandatory requirement. Thus  $s_0^T(\lambda_{0,T}, \lambda_{n,T})$  and  $s_n^T(\lambda_{0,T}, \lambda_{n,T})$  amount to horizontal addition of several demand surfaces as described in the inter-period CDDP but in the two dimensional space. So that we could store prices as a function quantity given that we know the inter-reservoir sub-problem solutions. The nodal CDDP procedure can be easily applied to form the truncated demand surfaces for transfer for the inter-reservoir chain as described in Section 7.4.

### 7.3.4. Interim Summary

In summary, the above two-reservoir main optimization problem comes down to a set of sub-models that allow construction of a number of demand curves and surfaces. Constructing the 1- dimensional curves (e.g.,  $x(\lambda)$ ) efficiently are as for the single reservoir nodal CDDP procedure using the demand curve adding methods. But sub-problems [7-19], [7-20], [7-21] and [7-22] require finding optimal price and release/storage pairs as multi-dimensional surfaces. In the case of sub-problem [7-19], we could think of filling the price-quantity grid directly or tracking particular marginal water value contours to form the intra-period 4-d marginal water value surfaces. The marginal water value along a particular contour on the demand surface will be constant. However, sub-problems [7-20], [7-21] and [7-22] require merging multi-dimensional demand surfaces in abstract to form the demand surfaces for storages which determine the optimal  $\lambda$ s where release and reservoir consumption demands allow the storage balance constraints to hold. One could consider implementing the above surface adding using different methods such as point wise addition of marginal water value points in the price-quantity space, or filling the marginal prices and quantities of one reservoir by holding the quantities of the other.

Section 7.3 describes the structure and development of 4D price-quantity demand surfaces, for example, the demand surface for transfer characterising primal-dual solutions,  $(x_{0,1}^*, x_{n-1,n}^*, \lambda_0^*, \lambda_n^*)$ , of sub-problem [7-19]. In addition, various projections and diagrams will be sketched to visualise and illustrate these marginal water value surfaces. The primal-dual nature allows us to treat the four three-variable projections separately, for example,  $x_{0,1}^*(\lambda_0, \lambda_n)$ ,  $x_{n-1,n}^*(\lambda_0, \lambda_n)$ ,  $\lambda_0^*(x_{0,1}, x_{n-1,n})$  and  $\lambda_n^*(x_{0,1}, x_{n-1,n})$ . For referencing convenience, we also define two diagrams: the ‘quantity diagram’ and the ‘price diagram’. The quantity diagram is a projection of price contours onto the quantity plane. The quantity diagram partitions the quantity plane into connected areas corresponding to fixed prices, that is, sets of the following form:

$$\{(x_{0,1}, x_{n-1,n}) | \lambda_0^*(x_{0,1}, x_{n-1,n}) = \hat{\lambda}_0, \lambda_n^*(x_{0,1}, x_{n-1,n}) = \hat{\lambda}_n\}.$$

Similarly, the price diagram is a projection of quantity contours onto the price plane, partitioning it into connected areas of fixed quantities.

The above analysis helps us develop the following computational routine to construct the intra-period demand curve for transfer for the jointly operating two reservoirs.

#### **7.4. Intra-period Demand Surface for Transfer**

This section describes how to form the demand surfaces for transfer for the inter-reservoir chain. The process begins by pruning all sub-trees leading from the inter-reservoir nodes leaving a single nodal chain. Then we can solve the inter-reservoir nodal chain sub-problem to construct the demand surface for transfer. Section 7.4.1 sets the stage for the nodal CDDP tree reduction procedure description by presenting the notation and assumptions required in addition to those used in the single reservoir nodal CDDP procedure. Then we illustrate how to form the demand surface for transfer in section 7.4.2.

##### **7.4.1. Inter-reservoir Chain Optimization Model**

The inter-reservoir chain optimization sub-problem and sub-tree optimization sub-problems are presented in the following section. We describe a CDDP based sub-tree pruning procedure that solves the sub-tree sub-problems. Finally, this section develops marginal water value surfaces using the CDDP approach, to find the optimal solutions to the inter-reservoir chain sub-problem.

###### **7.4.1.1. Inter-reservoir Chain Sub-problem**

The inter-reservoir chain includes flows into/out of both reservoir nodes and the minimal set of connecting arcs. The remaining collection is formed by removing all arcs in the reservoir tree from the original (and any resulting singleton nodes). This forms a set of sub-trees independently radiating from each node in the inter-reservoir chain

The following sub-problem seeks to maximise the inter-reservoir chain benefits by maximising benefits strictly of the set of nodes and arcs that form the chain between the two reservoirs including the reservoir nodes and accounting for benefits from the subtrees connected to the nodes on that chain,  $1, 2, \dots, n-1$ , but not those connected to the reservoirs. For modelling convenience, we will assume that each node on the inter-reservoir chain is only connected to a single sub-tree.

$$NTB_{1:n-1}(x_{0,1}, x_{n-1,n}) = \max_{x,q} \sum_{i=1}^{n-1} \left( NCB_i(q_i) + NSB_i(x_{i,k_i}) \right) + \sum_{i=0}^{n-1} NCF_{i,i+1}(x_{i,i+1})$$

Subject to:

$$q_i + x_{i,k_i} + x_{i,i+1} - x_{i-1,i} = 0 \quad \forall i = 1, \dots, n-1$$

$$\underline{X}_{i,i+1} \leq x_{i,i+1} \leq \bar{X}_{i,i+1}, \quad \underline{X}_{i,k_i} \leq x_{i,k_i} \leq \bar{X}_{i,k_i}, \quad \underline{Q}_i \leq q_i \leq \bar{Q}_i, \quad [7-23]$$

where  $k_i$  is the base node for the subtree connected to node  $i$  on the subtree chain and the  $NSB_i$  is the benefits from that subtree, defined below. Here  $NCB(q)$  denotes piece-wise linear convex nodal consumptive benefit function<sup>129</sup> which is the net sum of the local consumptive benefit functions  $CB(c)$ ,  $CE(d)$ , and  $CF(f)$  (see Section 3.2 in Chapter 3) at any node in the chain. Other terms carry their usual meaning. A negative lower bound for  $q_i$  is required to allow nodal tributary flows. Importantly, sub-problem [7-23] defines  $NTB_{1:n-1}$  and then sub-problem [7-19] seeks to maximise the inter-reservoir chain benefits  $NTB_{1:n-1}$ . As noted in the previous section the dual version of [7-19] can be used to construct the marginal water value surface defined by  $x_{0,1}(\lambda_0, \lambda_n)$  and  $x_{n-1,n}(\lambda_0, \lambda_n)$ . Section 7.4.3 explains how to form the above demand surface for inter-reservoir transfer using the CDDP procedure.

We can explicitly show the sub-tree benefit maximisation problem for a set of sub-trees leading from any node on the inter-reservoir chain. To understand this, we will consider any sub-tree  $T_i$  with  $N_i$  nodes, on which consumptive, distributary, tributary and non-consumptive demands are accommodated. Thus subtree sub-problems is:

$$NSB_i(x_{i,k_i}) = \max_{x,q} \sum_{j \in N_i} NCB_j(q_j) + \sum_{(j,k) \in A_i} NCF_{j,k}(x_{j,k})$$

Subject to,

$$q_j + \sum_{(j,k) \in A_i} x_{j,k} - \sum_{(k,j) \in A_i} x_{k,j} = 0$$

$$\bar{X}_{j,k} \leq x_{j,k} \leq \underline{X}_{j,k}, \quad \underline{Q}_j \leq q_j \leq \bar{Q}_j \quad [7-24]$$

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<sup>129</sup> Similar to consumptive/distributary marginal benefit functions,  $\frac{dNCB(q)}{dq} = nsb(q)$  denotes the sub-tree marginal benefit function for sub-tree off-takes.

Without loss of generality,  $(i, k_i)$  denotes a single arc connecting node  $i$  on the ‘chain’ with its subtree. In both cases we have not shown the time index because these sub-problems are independent over time. The sub-problems [7-23] and [7-24] decompose the two-reservoir tree network into an ‘inter-reservoir chain’ and a ‘remaining collection’ of sub-trees leading from the inter-reservoir chain nodes.

The sub-tree sub-problems [7-24] cover the remaining collection of sub-trees. One can consider optimising these as pruning these subtrees to form the inter-reservoir chain. The solutions of the set of sub-tree sub-problems prune all the sub-trees radiating from the inter-reservoir chain nodes without distorting the economic benefits of sub-tree flow off-takes. The structure of [7-24] is similar to the nodal optimization model discussed in Chapter 5. Further, the dual of the primal sub-tree sub-problem defines the sub-tree net consumptive marginal water value function. Hence, we could use the single reservoir nodal CDDP procedure to form a nodal net consumptive demand curve for each node in the inter-reservoir chain.

#### **7.4.2. CDDP Sub-tree Pruning Procedure**

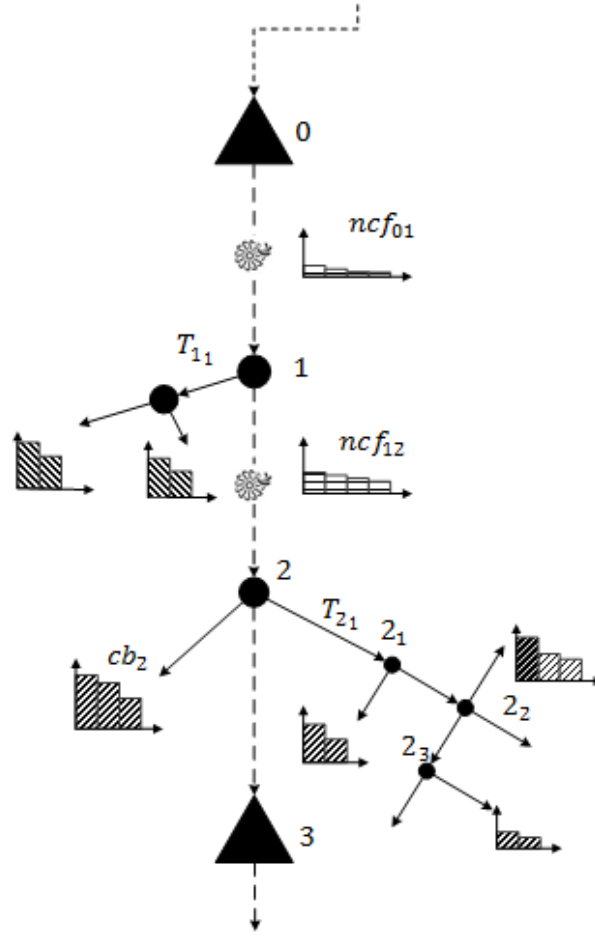
To illustrate the sub-tree reduction procedure, we use the above two hypothetical long-term storages within a connected water network modeled as a tree. The following descriptions accompany a set of examples based on the two series linked reservoirs. However, we include some hypothetical design features to the network to demonstrate some aspects of the method. The first illustration shows the full network and the node labels. Then it prunes the sub-trees in the inter-reservoir chain. Note that we will use this numerical example in other places in this chapter.

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#### **Illustration 7.4. A**

Consider two series connected reservoirs and a few sub-trees radiating from the inter-reservoir chain nodes, as in Figure 7-4, on which the demands for consumptive and non-consumptive use in main and sub-trees must be accommodated. We will label the network as an upstream nodal CDDP where  $i = 0$  indicates the upstream/root storage.  $i$  is increasing as we traverse along the chain towards the leaf storage.

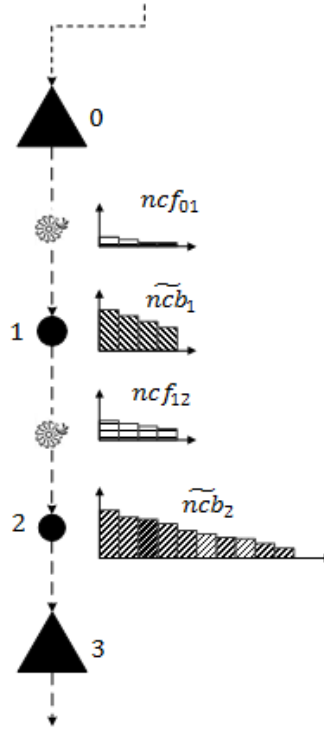




**Figure 7-4.** Schematic diagram of a series linked two-reservoir tree network

The above figure includes no sub-trees radiating from the two reservoirs.  $T_{2,0}$  denotes the main stream, and  $T_{2,1}$  represents an irrigation diversion sub-tree.  $T_{0,1}$  represents another consumptive flow off-taking sub-tree at node 1.

As we have already noted in the above section, it is possible to include a compensating marginal benefit function  $nsb_1$  in place of all sub-tree demand curves of  $T_{1,1}$  to ensure that the resulting benefits of the sub-tree are unchanged. Similarly, we construct the local consumptive demand curve  $ncb_1$  in place of local consumptive, distributary and tributary marginal costs/benefits and the marginal sub-tree benefit function  $nsb_1$  by pruning sub-tree  $T_{2,1}$ . Hence,  $\widetilde{ncb}_2$  is the net nodal marginal benefit of diverting flows for local uses and also the different uses in the sub-tree  $T_{2,1}$ . Note that this is not just the direct benefits from such consumptive use, but includes associated non-consumptive uses in each sub-tree. The sub-trees are processed towards the inter-reservoir chain.



**Figure 7-5.** Example of a pruned inter-reservoir nodal tree network

Figure 7-5 sketches the reduced tree after separately applying the nodal CDDP procedure for each sub-tree radiating from the inter-reservoir chain.  $\widetilde{ncb}_i$  accounts for both local consumptive demand curve ( $ncb_i$ ) and the demand curve of the sub-tree nodes ( $nsb_i$ ) leading from node  $i$ . For example,  $\widetilde{ncb}_1$  and  $\widetilde{ncb}_2$  represent the net consumptive demand curves of the inter-reservoir chain nodes 1 and 2 respectively.

In summary, this illustration describes how to prune any sub-tree leading from inter-reservoir chain nodes. The nodal CDDP simply prunes any sub-tree by forming relevant sub-tree marginal benefit functions and horizontally adding those sub-tree marginal functions to the respective local consumptive marginal benefit functions in the chain nodes. Finally, we end up with a set of net nodal marginal benefit functions for each node in the inter-reservoir chain.

#### End of illustration 7.4 A

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##### 7.4.3. Demand Surface for Transfer

This section is about the construction of  $x_{0,1}(\lambda_0, \lambda_n)$  and  $x_{n,n-1}(\lambda_0, \lambda_n)$  surfaces, the ‘transfer demand’ as described above, using the CDDP approach. Note that although we use the term ‘demand surface for

transfer', certain price pairs may invoke a release from one reservoir without a corresponding receipt at the other, and vice versa. The inter-reservoir chain transfer benefits function is defined as the maximum benefit from a set of inter-linked nodes between two connected reservoirs, based on the injection and extraction into and out of the chain from and to the reservoirs (i.e., using the sub-problems [7-19] and [7-23]).

The variable  $x_{0,1}$  represents the injection to the chain from the upstream reservoir  $i = 0$ .  $x_{n,n-1}$  is the release flow from the chain to the downstream reservoir  $i = n$ . Note that although  $x_{0,1}(\lambda_0, \lambda_n)$  and  $x_{n-1,n}(\lambda_0, \lambda_n)$  are written as two quantities given as a function of prices, they really define a four dimensional surface which maps out optimal quadruples  $(x_{0,1}^*, x_{n-1,n}^*, \lambda_0^*, \lambda_n^*)$ . This set of all optimal primal/dual solutions to the inter-reservoir chain sub-problem can be used to construct the demand surfaces for storage in the reservoir sub-problem. This leads us to look for suitable methods to form the demand surfaces  $x_{0,1}(\lambda_0, \lambda_n)$  and  $x_{n-1,n}(\lambda_0, \lambda_n)$  in the price-quantity space, efficiently.

The method<sup>130</sup> we describe here is to utilise the single reservoir nodal CDDP procedure to build up the surface in “strips”. Each strip corresponds to either a critical downstream marginal water value that induces a downstream versus inter-reservoir demand swap or a critical marginal price difference  $(\lambda_0 - \lambda_n) = \varepsilon$  between the two reservoirs. However, in the development that follows, we will follow the latter option.  $\varepsilon$  is an arbitrary discrete price which can take any value based on one or more non-consumptive demands. A downstream critical price induces a change in the pattern of optimal releases according to the upstream marginal price.

If the downstream end-of-chain marginal water value is fixed,  $\lambda_n = \check{\lambda}_n$ , a demand curve for release from the upper reservoir can be constructed using the single reservoir nodal CDDP procedure. This gives us a demand curve  $x_{0,1}(\lambda_0, \check{\lambda}_n)$  by systematically incrementing  $\lambda_0$  to find the corresponding  $x_0$ . Label this demand curve  $\Gamma^{\lambda_n}$  which will be referred to as the demand curve for transfer (*dct*)

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<sup>130</sup> We could think of different approaches either based upon adjusting quantities to determine prices or adjusting prices to determine quantities, to fill the price-quantity space. A straightforward means of doing this would be to iterate through a grid of prices for both reservoirs recording the relevant optimal flows. Any method of solving the inter-reservoir chain with fixed reservoir prices would be sufficient. This effectively constructs the demand surface one “grid-square” at a time as described later.

corresponding to any  $\varepsilon$ , being the set of all optimal release-water value pairs  $(x_{0,1}(\lambda_0, \check{\lambda}_n), \lambda_0)$ . Let  $\Gamma^{\check{\lambda}_n}$  denote a strip if the water value is critical for the increment,  $\varepsilon$ . The release water values,  $\lambda_{0,\check{\lambda}_n}^*$ , will effectively relate to the upstream reservoir water values,  $\lambda_0$ , via sub-problem [6-19]. This effectively defines a strip of  $x_{0,1}(\lambda_0^*, \check{\lambda}_n)^{131}$ .

- A strip  $(\Gamma^{\check{\lambda}_n})$  refers to a set of sorted marginal water values that forms a *dct* corresponding to a critical marginal water value  $\tilde{\varepsilon}$ .
- A band is made up of sections bordered by two strips (i.e., critical marginal price differences).
- A diagonal line connects marginal water value points in two adjacent strips across a band.

This poses the question of how  $x_{0,1}(\lambda_0^*, \check{\lambda}_n)$  changes as  $\check{\lambda}_n$  changes. We would like to be able to construct a small number of these demand strips and effectively knit them together into the full demand surface. In order to explore the issues involved, we run through a worked example, Illustration 7.4 B. This will lead to a clear description of the structure of the demand surface in the section following the numerical example. We then outline a process for constructing the demand surface in Section 7.4.3.

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#### **Illustration 7.4. B**

The objective of this illustration is to examine the structure of the demand surface  $x_{0,1}(\lambda_0, \lambda_n)$  for a specific example, illustrating the various features and motivating possible ways to construct the surface.

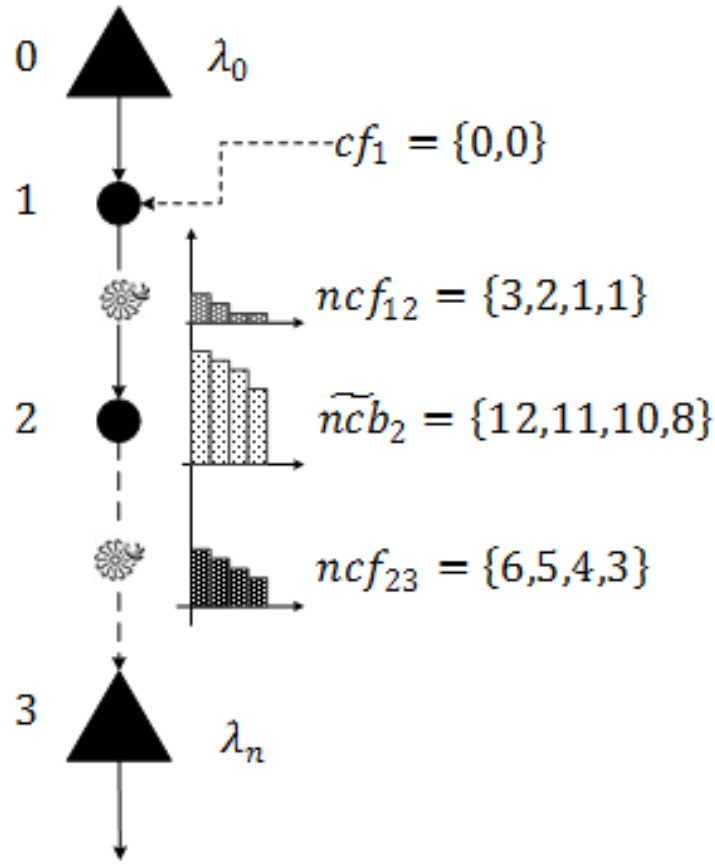
#### ***Data***

Figure 7-6 shows a two-reservoir nodal network. The storages are labelled as ‘0’ (upstream) and ‘3’ (downstream) respectively. There are two nodes on the inter-reservoir chain. Hypothetical non-consumptive (e.g.,  $ncf_{01} = \{3,2,1,1\}$ ,  $ncf_{12} = \{6,5,4,3\}$ ) and inter-reservoir consumptive (e.g.,  $nch_1 = \{12,11,10,8\}$ ) use are benefiting from the two storages. An inflow of two units are attached to node 1.

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<sup>131</sup> In addition, for each  $(x_{0,1}, \check{\lambda}_0^*)$  the procedure will have implicitly determined the optimal downstream reservoir values,  $x_{n-1,n}^*$ . For example, these could be reconstructed using forward simulation. It is also possible to keep track of these on the inter-reservoir chain, effectively constructing  $x_{n-1,n}^*(x_{0,1}, \lambda_0)$  as a part of the CDDP procedure.

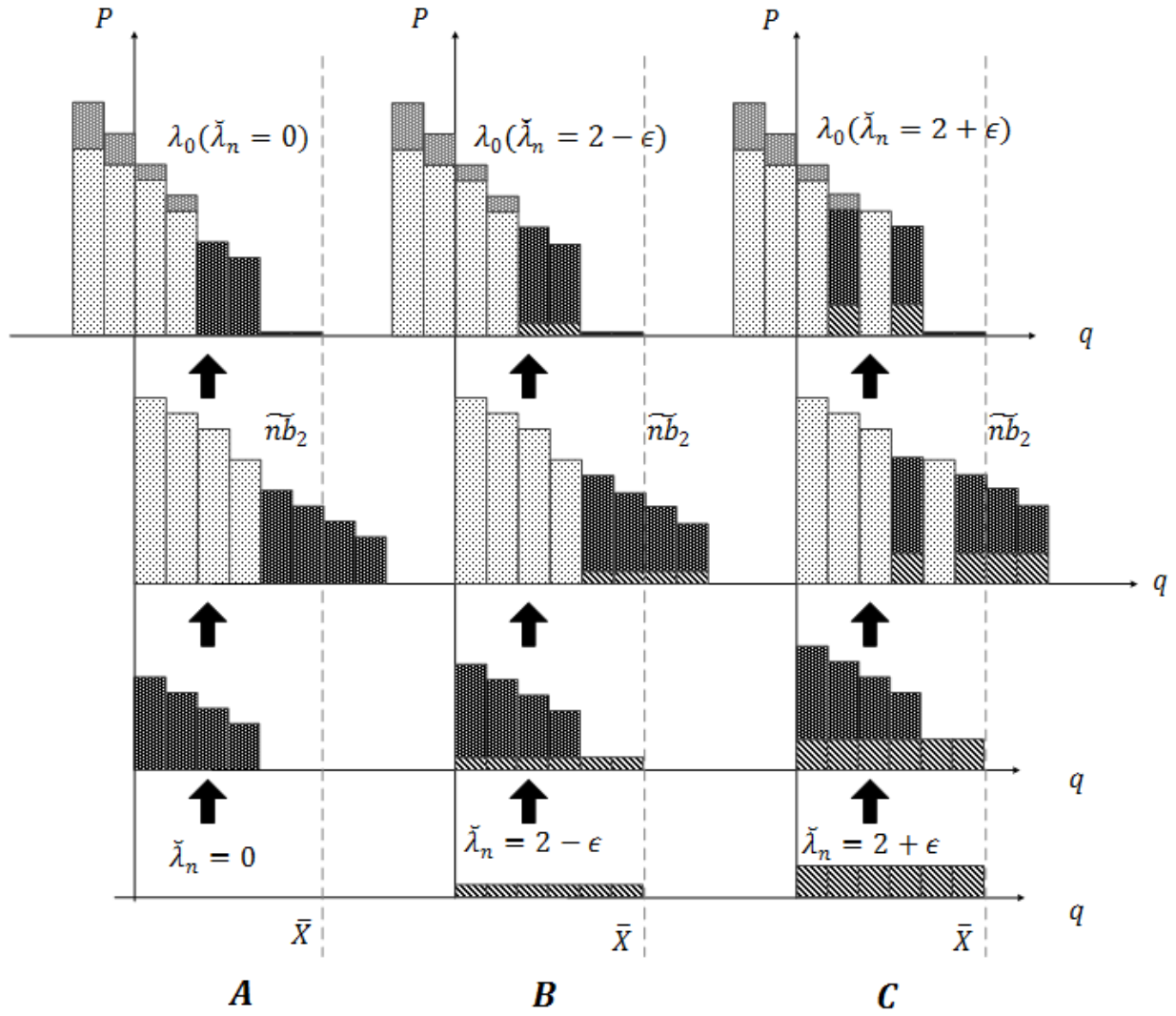
Let the flow capacities for all arcs in the inter-reservoir chain be  $[\underline{X}, \overline{X}] = [0, 6]$ . Any extra water will be a loss to the system.



**Figure 7-6.** A simple series connected two-reservoir tree network s

### **Processing**

This section is about processing the inter-reservoir tree by setting different downstream water values. Firstly, we set the downstream water value  $\lambda_n = 0$ . This will invoke a set of marginal price tranches at the upstream reservoir. Figure 7-7 sketches the demand curves seen below node 2, below node 1 and at the upstream reservoir when the nodal CDDP is processing the inter-reservoir chain from the downstream to upstream. Figure 7-7-A constructs the set of marginal water value tranches at the upstream reservoir when  $\lambda_n = 0$  are  $\{11, 9, 6, 5, 0, 0\}$ . Similarly, Figure 7-7 B shows the upstream marginal water values when the  $\lambda_n$  is marginally below '2'.



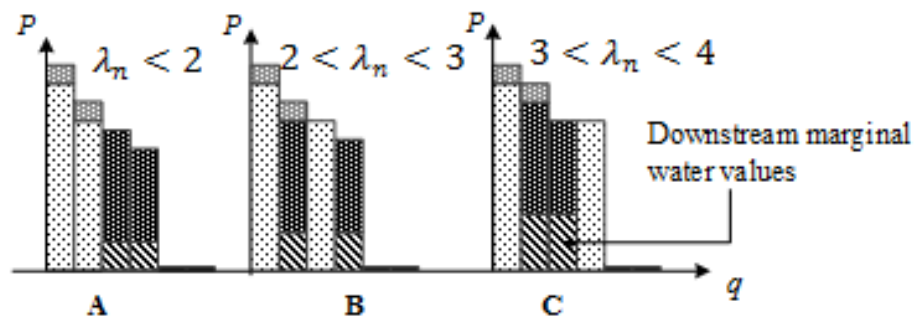
**Figure 7-7.** Downstream-upstream processing to construct the demand curves for transfer for  $\lambda_n = 0$ ,  $\lambda_n = 2+$ .

The top of Figure 7-7-A shows demand curve  $\Gamma^{\lambda_n=0}$ , which is piecewise constant with steps at the upstream critical water values 11, 9, 6, and 5. Note the two tranches to the left of the vertical axis are truncated. Between these critical values the release is constant.  $\Gamma^{\lambda_n}$  describes the edge of the demand surface  $x_{0,1}(\lambda_0, \lambda_n)$  which also forms an edge strip on the demand surface. When  $\lambda_n = \Delta$ , the release schedule/ordering is unchanged but some upstream critical prices have been raised by  $\epsilon$ . For example, Figure 7-7-B sketches the upstream demand curve corresponding to  $\Delta = 2 - \epsilon$ . Also, Figure 7-7-C corresponds to  $\Delta = 2 + \epsilon$ . The upstream critical water values become 11, 9,  $6 + \epsilon$ , and  $5 + \epsilon$ . This holds for any  $0 < \epsilon < 2$ . The ‘critical water values’  $6 + \epsilon$ , and  $5 + \epsilon$  correspond to critical water value differences (see below). As  $\lambda_n$  is increased from 0 to 2, a band on the demand surface is traced out.

So  $\lambda_n = 2$  becomes the next downstream critical value and Diagrams A and B show the two strips bordering the first two bands in the marginal water value space. (This band is illustrated below in Figure 7-9) The band is divided into regions where the release decisions do not change. Some borders between regions correspond to fixed critical upstream marginal water values e.g.,  $\lambda_0 = 11$ . Others correspond to critical upstream marginal water value differences, e.g.,  $\lambda_0 - \lambda_n = 6$ .

When the downstream marginal water value reaches the next critical value (i.e.,  $\lambda_n = 2$ ), the last consumptive bid at node 2 and the first combined reservoir/non-consumptive downstream bid take the same value. This means that there is indifference between which of the two demands will be met by the incoming flow. Slightly increasing the  $\lambda_n$  by  $\epsilon$  (that is  $\Delta = 2 + \epsilon$ ), the first combined reservoir/non-consumptive bid is valued at  $\Delta + 6$  and will take the position previously held by the last consumptive bid valued at “8”. This will induce a change in the bid stack order at node 2. So not only are the upstream marginal water values increasing, but there is a change in order of the final bid tranche (see Figure 7-7-B). Notice that since the swap occurs at node 2, the non-consumptive demand between nodes 1 and 2 is no longer associated with the consumptive bid valued at “8”.

We continue marginally increasing the downstream critical marginal water value  $\lambda_n$  and examine how the upstream marginal water values change. For example, the diagrams A, B, and C in Figure 7-8 sketch the marginal water values seen at the upstream reservoir for downstream marginal water values ( $\lambda_n < 2, 2 < \lambda_n < 3, 3 < \lambda_n < 4$ ).



**Figure 7-8.** Marginal water values seen at the upstream reservoir for the  $\lambda_n = 0, 2$  and  $3$ .

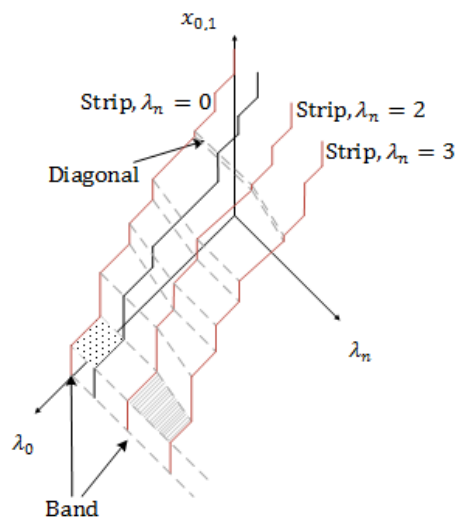
Bids with fixed heights in the final *dct*, (denoted by lighter towers) are meeting inter-reservoir consumptive demands. Others (which include the striped tower sections) shift upwards as the

downstream marginal water values increase, and they will ultimately be passed through to the downstream reservoir. As already noted in the above, the bid stack ordering will be unchanged for a range of  $\lambda_n$ s demarcated by two adjacent critical downstream  $\lambda_n$ s. It is possible to use CDDP to generate these bands by recording the non-critical ranges and tracking them at each node in the chain. This gives us  $x_{0,1}$ ,  $x_{n-1,n}$  and  $\lambda_0$  changes corresponding to the critical  $\lambda_n$ s.

#### 7.4.3.1. Discussion

In what follows, we sketch the above *dcts* and their projections on to the quantity space and the price space, to clearly visualise the nature of the 4-D surface and then to describe it.

The *dcts* corresponding to the downstream water values  $\lambda_n = 0, 1, 2, 3$  are plotted in Figure 7-9. The values  $\lambda_n = 0$ ,  $\lambda_n = 2$  and  $\lambda_n = 3$  indicate the critical downstream marginal water values and thus corresponding *dcts* construct the strips or the edges of the bands. Over each band there is no change to the order in which demands are met over the inter-reservoir chain as the upstream marginal value is increased. We could say the general pattern of inter-reservoir flows is unchanged. The upstream critical prices which are unchanged as  $\lambda_n$  is increased over this band form square steps in this band. The upstream critical prices which are raised as  $\lambda_n$  is increased add diagonal steps. Along these diagonals the marginal water value difference is constant (refer the marginal water values ‘6’ and ‘5’ corresponding Strip 0 and Strip 2).



**Figure 7-9.** Demand curves for transfer, projections of demand curves for transfer onto quantity (net receipts) and marginal water value diagrams.



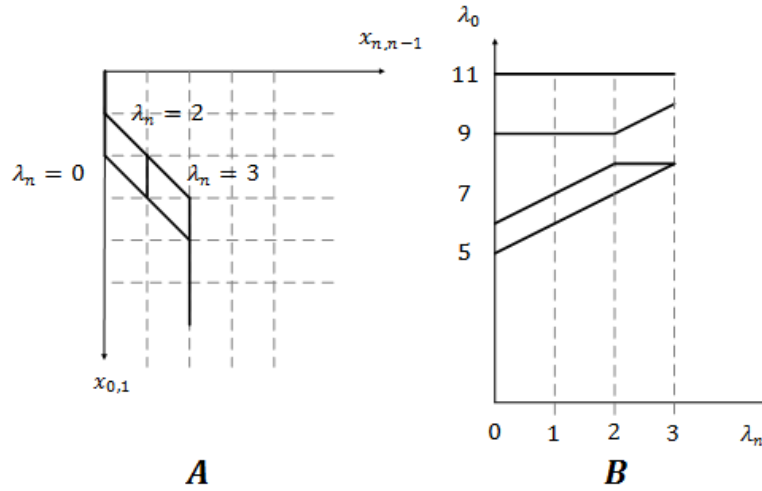
These square and diagonal steps (dotted and hashed areas) can be seen in the price diagram shown in Figure 7-10 as edges of constant release areas. Diagram-A shows the quantity diagram of demand surface  $\lambda_0^*(x_{0,1}, x_{n-1,n})$  along guidelines of contours of  $\check{\lambda}_0$  in the release-receipt space. As expected there are curve segments (i.e.,  $\check{\lambda}_0$  guidelines) parallel to the upstream release  $x_{0,1}$  and angular curve segments as depicted in the diagram. From Sub-problem [7-19], given a fixed  $\lambda_n$ , one can induce a change to  $\lambda_0$  by invoking a change to  $x_{n-1,n}^t$  through changing  $x_{0,1}^t$ . This is illustrated in Figure 7-7 and Figure 7-8. Here in Figure 7-9, the angular curve segments denote the release-receipt quantity difference<sup>132</sup>. All solutions in the top parallelogram correspond to  $\lambda_n = 2$  and all solutions in the lower parallelogram correspond to  $\lambda_n = 3$ . In both cases, though, there is indifference between which of the two demands will be met by the incoming flow.

Similarly, there will be curve segments parallel to the  $x_{n-1,n}$  axis in case of the  $\lambda_n$  contours as shown in the quantity diagram projected from  $\check{\lambda}_n$  contour line of demand surface for transfer for the downstream reservoir (Figure 7-10-A). A curve shows an offset when the downstream water value reaches a critical value. Finally, in Figure 7-10-B we plot the curves in the marginal water value space ( $\lambda_0 - \lambda_n$ ).

The demand curves corresponding to the remainder of the  $\lambda_n$ s indicate all possible order-swaps between inter-reservoir consumptive demands and the downstream demands as further described in the following. We record  $x_{0,1}, x_{n-1,n}$  and  $\lambda_0$  of all occurrences that would eventually form the marginal water value transfer function.

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<sup>132</sup> Angular curve segments correspond to fixed quantity differences  $(x_{0,1} - x_{n-1,n})$ . That is, changes to the release  $x_{0,1}$  are exactly matched by changes to the receipt  $x_{n-1,n}$  corresponding to water transferred from one reservoir to the other.



**Figure 7-10.** Projections of the demand curves for transfer corresponding to  $\lambda = 0, 2$ , and 3 onto  $x_{0,1}$ - $x_{n,n-1}$  quantity diagram and  $\lambda_0$ - $\lambda_n$ .

#### End of Illustration.

As we have already noted in the above, a strip corresponds to the downstream critical marginal water values ( $\tilde{\lambda}_n$ ). In the development of the procedure that follows, it will be convenient to distinguish between a band and a strip.

The critical water values occur as a result of the interaction between inter-reservoir consumptive demands and the downstream demands. These marginal price differences only arise within a band, forming diagonals (see grey solid lines in Figure 7-9) between constant release-receipt pairs.

The release order changes when the downstream marginal water value is marginally above “2”. So the new downstream critical marginal water value creates the second band<sup>133</sup>. This means that the end-of-chain pass through non-consumptive demand step swaps position with the inter-reservoir consumptive step somewhere along the new *dct*.

<sup>133</sup> At each step in the CDDP it is simply a matter of going through these tranches to see the least they can be increased before invoking a change in order. That will give the increment to the next downstream critical water value. In addition, there is a single upstream-downstream critical marginal water value difference for a network with no non-consumptive demand or consumptive off-takes between the two reservoirs; this is zero. This implies no incentive for the upstream reservoir to release water if  $\lambda_n < \lambda_0$ . On the other hand,  $\lambda_n > \lambda_0$  makes a full upstream release.

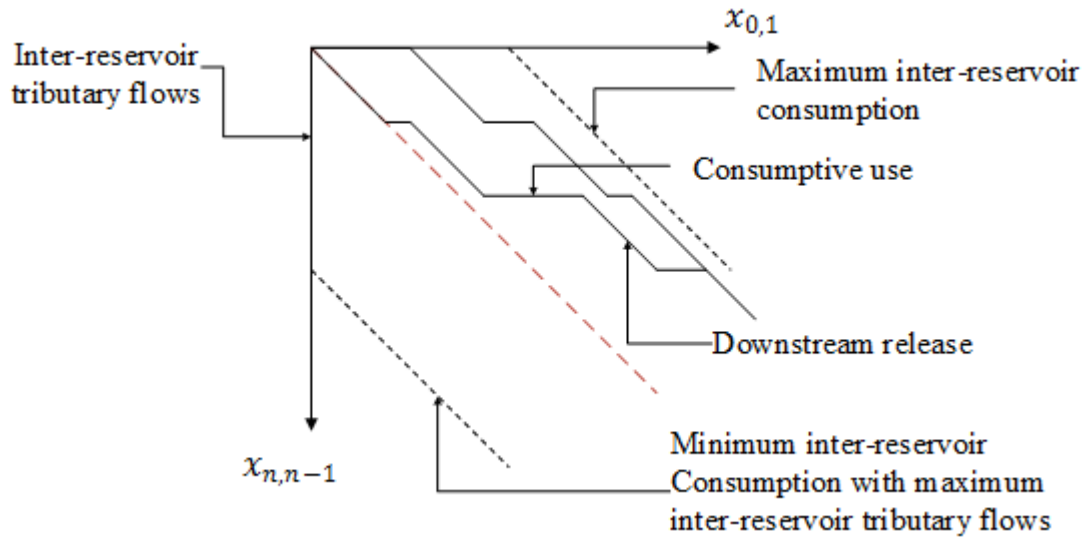
The *dct* curve segments parallel to  $\lambda_0$ -axis represent a range of increasing optimal upstream release units  $x_{0,1}$  along the flat for the same  $\lambda_0$ . This means that we are trading off a marginal return of  $\lambda_0$  at the upstream reservoir with the same marginal return from releasing into the inter-reservoir chain. The horizontal flats are where there are a range of optimal  $\lambda_0$ s for the same upstream release unit  $x_{0,1}$ . The bottom of the *dct* with upstream marginal water value of zero (near  $\bar{X}$  bound) denotes maximum inter-reservoir consumption and then each upward step corresponds to reduced consumption.

$\lambda_0$  is continually changing its value along a diagonal line as shown in the  $\lambda_0$ - $\lambda_n$  projection in Figure 7-10-B. Both bands and strips together will complete the marginal water value surface. Bands together with diagonals will make a quadrilateral cell grid of the marginal water value surface. The upstream release is unchanged within a cell but the upstream-downstream marginal water value difference can change (e.g., the *dct* corresponding to  $\lambda_n = 1$  in Figure 7-9-A).

Furthermore, the diagonal line  $\lambda_0 = \lambda_n$  indicates sending water from the upstream storage to downstream storage with no interaction between the downstream bids and inter-reservoir consumptive bids. The rest of the diagonals are corresponding to different levels of interaction between the two.

In summary, a strip corresponds to a downstream marginal critical value. A critical value implies one or more swaps are taking place between downstream and inter-reservoir consumptive demands. A band represents a fixed pattern of flows. It may correspond to the upstream-downstream critical price differences. The critical prices themselves introduce interactions between the critical price differences and other critical prices. Diagonal lines section the bands according to critical price differences, reshaping the cells into a set of small irregular patterns.

Figure 7-10-B in the above illustrative example briefly shows the relationship between the inter-reservoir consumptive demands and the downstream demands. Based on that, Figure 7-11 sketches a generic projection of the demand surface for transfer in the upstream release-downstream receipt space. This helps us to observe the interactions between  $x_{0,1}$  and  $x_{n-1,n}$  along the *dcts*. Sometimes, it may be convenient for us to imagine a series configuration when describing these curves.



**Figure 7-11.** Example of a set of demand curve strips as seen in the upstream release – downstream receipt space for the upstream reservoir.

There will be a unique *dct* projected on to the red coloured diagonal line that implies the upstream releases are equal to the downstream receipts when there is no inter-reservoir consumptive use between the series connected two reservoirs (e.g., some typical cascade hydropower systems). The rest of the *dct* contours (corresponding to inter-reservoir consumptive use in the quantity diagram) will spread across a diagonal band indicate by the two black coloured dotted lines in the above diagram. The lower boundary *dct* is the maximum possible downstream receipts. Minimum inter-reservoir consumption occurs when marginal water values of the two reservoirs are very high. As noted in the above, the maximum possible inter-reservoir consumptive use will be attained when the lowest upstream/downstream marginal water values for any possible upstream release level. We could imagine respective *dcts* in Figure 7-10-A is sitting “above” a diagonal and in a dimension coming out of the  $(x_{0,1} - x_{n-1,n})$  plane. So the *dcts* are defined along a locus point within the band and sideways steps imply inter-reservoir consumptive swaps.

In general, the above is applicable for the parallel two-reservoir configuration but the diagonal band between the two dotted lines will expand possibly covering the whole space in order to accommodate large consumptive use. This is because the downstream confluence normally discharges to a sink (e.g., sea) which has infinite consumptive use. There are a variety of ways in which we could use the above insights to define a process to generate the required demand surface.

#### 7.4.3.2. Interim Summary

This computational routine considers the implications associated with the critical marginal water value differences and the downstream critical marginal water values. We provide a nodal CDDP procedure to trace out the transfer surface in which each successive unit of upstream reservoir release is used to meet the next most attractive inter-reservoir consumptive use. If non-consumptive values combined with the marginal water value at the downstream reservoir are high enough, water is transferred through to the downstream reservoir<sup>134</sup>. This means that as the downstream marginal water value increases, some upstream marginal water values will shift upwards without changing their order in the bid stack. In that, a profile of demand curves corresponding to critical marginal price differences will be neatly stacked together to form a band. This implies that a large number of *dcts* are required to accurately approximate the demand surface. But we may still form the surface sufficiently by storing fewer *dcts* in an irregularly discretised grid space, if we could carefully select the *dcts* corresponding to strips<sup>135</sup>. So the *dct* shape progressively changes along the  $\lambda_n$ - axis. The following description of the CDDP process considers strips at downstream critical marginal water values<sup>136</sup>.

#### 7.4.4. CDDP Procedure for the Inter-reservoir Demand Surface for Transfer

This section presents the CDDP procedure for the inter-reservoir demand surface for transfer. To simplify the presentation here we assume non-negative water values and a downstream-upstream network configuration. Let all water value stacks be given as lists of marginal water values for each water unit. At the start, this procedure calls the sub-tree pruning procedure to reduce the inter-reservoir sub-trees and then constructs the net demand surface for transfer.

##### 7.4.4.1. Description

###### Nomenclature:

$k$ : Indexes the critical downstream marginal water value incrementing iteration

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<sup>134</sup> Similarly, the reverse processing of the inter-reservoir chain will generate the demand surface for transfer at the downstream reservoir  $x_{0,1}(x_{n-1,n}, \lambda_n)$ .

<sup>135</sup> Regular discretization of the downstream marginal water values, would require us to store all *dcts*. So we will only select the strips corresponding to critical water values and interpolate the diagonals.

<sup>136</sup> One could construct a model that considers a fine grid of critical marginal water value difference bands.

- $\widetilde{ncb}_i$ : Net consumptive demand curve for node  $i$
- $ncf_{ij}$ : Non-consumptive demand curve for arc  $(ij)$
- $b \in B_i$ : Index to represent the bid position or sequence in a bid tranche
- $A, A'$ : Index set for keeping track of the lower reservoir water value increments.
- $\lambda_i$ : Indicates the marginal water values at node  $i$ .
- $(\lambda_{i_{kb}})_{b \in B}$ : Denotes the sequence of bids at the upstream reservoir node corresponding to  $k^{\text{th}}$  iteration whose  $b^{\text{th}}$  element is given by  $\lambda_{i_{kb}}$
- $\varepsilon$ : Critical marginal water value increment for  $k^{\text{th}}$  iteration
- $\Gamma^{\lambda_{nk}}$ : Demand curve for transfer corresponding to  $\varepsilon$  (marginal water value slice)

Procedure

Start.

Pre – computation procedure:

Reduce all sub-trees in the inter-reservoir nodal chain using sub-tree pruning procedure to construct a set of net consumptive demand curves  $[\widetilde{ncb}_1, \widetilde{ncb}_2, \widetilde{ncb}_3, \dots, \widetilde{ncb}_i, \dots]$ , one for each node of the inter-reservoir chain.

Main procedure:

Initialization

Set  $k = 0 \rightarrow (\lambda_{n_{kb}})_{b \in B} = [0]$ .

Set  $\Delta = 0.01$  // A small discrete increment

Do

Set  $\varepsilon = \infty$  // Set a finite initial critical value

Set  $\widetilde{ncb}_0 = \widetilde{ncb}_n = [0]$  // Starting values of upstream & downstream nodes

$\lambda_{n_{kb}} \leftarrow \Delta$

$nb_n \leftarrow (\lambda_{n_{kb}})_{b \in B}$

$A = \{b | 1 \leq b \leq B\}$

For  $i = n$  to 1 Step  $-1$  do

```

 $\overline{nb}_i \leftarrow nb_i + {}^h \widetilde{ncb}_i$ 
set  $(j, i) = (par(i), i)$  or  $j = par(i)$ 
 $\hat{A} = h\{A, nb_i, \widetilde{ncb}_i\}$  //Index set after horizontal addition of  $nb$  and  $\widetilde{ncb}$ 
If  $A - A' \neq \emptyset$  then //check symmetrical difference  $b \in A, b-1 \notin A$ 
     $\varepsilon_i \leftarrow \min(\varepsilon_i, \overline{nb}_{i,b} - \overline{nb}_{i,b-1})$  //Select smallest critical value
Else
     $\varepsilon_i \leftarrow \varepsilon$  // Non existence of critical values at node  $i$ 
End
 $nb_{par(i)} \leftarrow \widetilde{nb}_i = \overline{nb}_i + {}^v ncf_{ji}$  // Net nodal demand curve for flows
Next  $i$  // Repeat process until reach node 0.

```

$(\lambda_{0_{kb}})_{b \in B} \leftarrow nb_0$  //Updates upstream marginal water value

$\Gamma^{\lambda_{n_k}} \leftarrow \{(\lambda_{0_{kb}})_{b \in B}, \lambda_{n_k}, B\}$  //Store marginal water value slice

$\lambda_{n_{k+1}} \leftarrow \lambda_{n_k} + \varepsilon_i$  //Increment downstream value to search next critical value

$k = k + 1$  //Search next slice

While  $\varepsilon_i \leq 0$

Output:  $\{\Gamma^{\lambda_{n_0}}, \Gamma^{\lambda_{n_1}}, \dots \Gamma^{\lambda_{n_k}}, \dots \Gamma^{\lambda_{n_K}}\}$

End

#### 7.4.4.2. Explanation of Main Procedure

The nodal CDDP is applied to construct the demand curve for transfer  $\Gamma^{\tilde{\lambda}_{n_0}}$  (initial strip/edge) corresponding to starting end-of-chain critical marginal water value  $\lambda_{n_{k=0}}$ . Here  $k$  indexes the critical downstream marginal values. For a general case, as we have already noted in the above, a negative end-of-chain critical marginal water value may be required to begin the process. But to avoid unnecessary complexities, we assume  $\lambda_{n_{k=0}} = 0$  as the starting value. Furthermore, for the interest of simplicity, we imagine a small set of discrete critical downstream price increments.

Conceptually, the transfer surface construction can be started by creating the *dct* corresponding to maximum inter-reservoir consumption<sup>137</sup> ( $\lambda_n = 0$  in the above illustration). The procedure will continually increase through the critical  $\lambda_n$  values until it generates the *dct* that incentivises the least inter-reservoir consumptive off-takes. Practically, though, the procedure could use a very large negative value if negative water values are allowed.

In the following, we set the initial *dct* for zero downstream marginal water value. As noted in the above the initial *dct* involves some finite maximum consumptive water off-takes in the inter-reservoir chain between the two storages.

The next critical  $\lambda_{n_k}$  will arise from the downstream bids<sup>138</sup>. The CDDP procedure keeps track of which tranches include the downstream water value directly. The new downstream marginal water value will change these tranches but not the inter-reservoir consumptive bids. The next downstream critical marginal water value (e.g.,  $\lambda_{n_{k=1}}$ ) is found when the pattern of releases and flows change. The downstream critical value implies a change in the primal solution (i.e., different demands are met)<sup>139</sup>.

At each step in the CDDP it is simply a matter of going through these tranches to see the least they can be increased before invoking a change in order. That will give the increment either to the next downstream critical water value or a smaller increase at another node marginal water values. For example, it can be seen from Figure 7-10 -B that an increase in the downstream marginal water value from 2 to 3 can allow the fifth and sixth tranches to switch order at node 2.

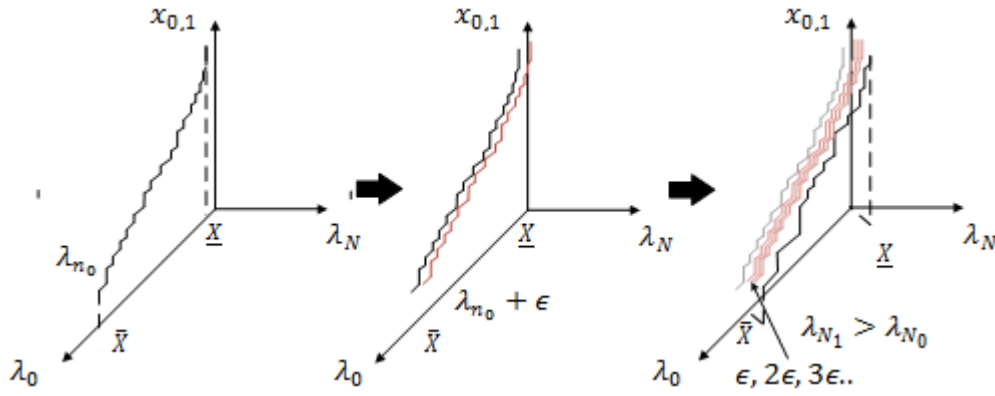
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<sup>137</sup> In that, there could be negative water values at the downstream reservoir.

<sup>138</sup> The upstream prices are depending on the downstream prices regardless of whether the downstream price is critical or not.

<sup>139</sup> The critical prices are strictly involved with the net demands in the inter-reservoir chain. One could attempt to construct the demand surface for transfer by incrementing the lower reservoir prices iteratively and then record the upper prices. However, there is a risk of omitting critical values and sometimes thin strips. The water value surface constructed is only an approximation.





**Figure 7-12.** Example *dcts* plotted for the downstream critical marginal water values and for the upstream-downstream critical value differences in the upstream-downstream marginal water value space

This step repeats the process for all  $\lambda_{n_k}$ s keeping a track of  $x_{0,1}$  and  $x_{n-1,n}$  changes. The CDDP progressively creates a set of *dcts* (e.g.,  $\Gamma^{\lambda_{n_0}}, \Gamma^{\lambda_{n_1}}, \Gamma^{\lambda_{n_2}}, \dots, \Gamma^{\lambda_{n_K}}$ ), determining the critical water values as it goes. Then the critical upstream water values and diagonals connect the strips implicitly creating the bands and filling the space between *dcts*. The critical marginal value differences in the bands will not invoke a change in order in those tranches but they may increase some upstream marginal water values (see the set of red solid curves lying between the two strips in Figure 7-12).

Both upstream and downstream marginal water values increase gradually as we move along  $\lambda_n$  axis. This means that as the downstream water value increases, the marginal value of holding onto more units of water in the upstream reservoir must increase to make this worthwhile, and that passing them right through to the downstream reservoir, becomes preferred over consuming them along the way.

The procedure stops after processing the final critical marginal water value  $(\check{\lambda}_{n_K})^{140}$ . This water value is easy to identify since when processing the CDDP for this water value, no tranche reordering can be invoked at any inter-reservoir chain node by an increase in the downstream water value. In conclusion, this section applied a “lower level” intra-period CDDPs, firstly to reduce the sub-trees leading from the inter-reservoir nodes and then to form demand surfaces for transfer for the inter-reservoir nodal chain. A “higher level” inter-period CDDP can be developed to find the trade-offs among the end-of-period demand surface for storage, inter-reservoir demand surface for transfer and the net demand curves of

<sup>140</sup> This leads us to ask how to identify the final  $\lambda_{n_K}$  which is the stopping rule. For any  $\lambda_{n_k}$  beyond  $\lambda_{n_K}$  will only produce flat areas with no visible strips and hence no quantity swaps.

sub-trees leading from the reservoirs. The usual CDDP horizontally and vertically merges demand curves. The notion of creating a demand surface for transfer deserves consideration as it provides a greater insight to the intra-period two-reservoir optimization problem. However, the inter-period problem requires adding multi-dimensional demand surfaces and this lies outside the scope of this thesis.

## **7.5. Chapter Conclusions**

This chapter develops an intra period CDDP framework to assess the marginal value of water stored for two inter-connected reservoirs in a mixed-use catchment. The two-reservoir optimization is decomposed into a set of sub-problems in order to study the nature of demand surfaces underlying each sub-problem. We deploy a “low level” intra-period nodal CDDP procedure to construct the demand surfaces for transfer for the inter-reservoir chain. Then a “higher level” (inter-period) CDDP could be used to find the trade-off between future storage demands and the current period releases to the inter-reservoir chain and other direct sub-trees. The CDDP generates two-dimensional marginal water value surfaces when processing the inter-reservoir chain. So the two-reservoir problem can be computationally difficult to handle unless there is limited consumptive use in the inter-reservoir chain. That is, the computational effort depends on the size of the feasible region which depends on the amount of inter-reservoir chain consumptive use. Stochastic Programming appears more suitable to handle multi-reservoir problems.

# 8.

## Explorations of Intra-period CDDP Extensions

### 8.1. Introduction

This chapter describes some multi-dimensional intra-period CDDP model extensions, to model time-of-use (consumptive and non-consumptive) water allocations, and the demand curve for power for inter-plant operation in a competitive water market setting. Although we anticipate some modelling issues related to time-of-use intra-period model, we believe this work contributes to the existing body of literature in terms of cyclic sub-period LP modelling and still explains the problem context and importance of time-of-use bidding in a water market.

Section 8.2 explains the issue of transportation/release delays in a water market setting and the impacts of the temporary, relatively small intermediate storages. Following the developments in Chapter 3, an LP model in Section 8.2.1 formulates the above problem. This chapter then develops a release delay CDDP algorithm that constructs time-of-use demand curves for all nodes, accounting for flow delays but assuming “run-of-river” operations, with no intermediate storage capacity at the nodes in Section 8.3. The following section explains the time-of-use storage delay CDDP algorithm that forms a multi-dimensional nodal marginal value surface as a result of incremental sub-period flow arrivals under the intermediate storage capacity constraints. The two algorithms are then combined to construct the time-of-use demand surface in Section 8.4. Finally, Section 8.5 provides a summary of the chapter.

### 8.2. Modelling Time-of-use Demand for a Mixed used Catchment

The notion of allocating water at different “times of use” is not discussed in the water resource literature. Apparently, timing of water availability for a particular use in the catchment could be as important as certainty of quantity and locational delivery of the resource for a successful market design.

Practically, the downstream consumptive and non-consumptive demands may require water in different time slots within a day than the flows desired by hydropower firms. For example, a hydro generator typically prefers water to be released around 6 p.m., so she could dispatch more energy to meet the peak electricity demand. On the other hand, consumptive users will receive that water at various times after it is released from the reservoir, due to flow delays. It is unlikely that all will receive flows at the time of day that they most prefer<sup>141</sup>. So the downstream users may need to delay demands, and/or buffer received flows using intermediate storage facilities. Or they may be prepared to pay enough to have water delivered at their preferred times to influence the upstream release schedule.

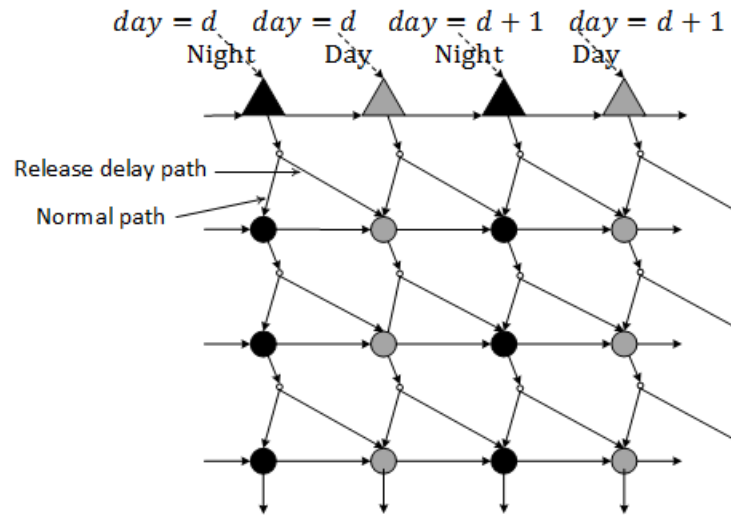
We use a generic LP optimization model to formulate the release delay and storage transfer problem. An LP formulation helps to clarify the situation being addressed and to highlight the issues that arise so as to formulate a better model and then to identify the sub-problems we could use CDDP to model. Due to the flow delays, release not only implies water transfers between nodes, but between sub-periods. A release delay (normally measured at the point of release or at the dam face) denotes the time taken for a release (or spill) to reach a downstream location (node). The release delay could depend on the size of the release (volume), state of the arcs (e.g., canals and conveyors), and the state of the river bed (e.g. wet, dry).

### **8.2.1. LP Modelling of Release Delays and Storage Transfers using Consecutive Sub-periods**

Multi-nodal intra-period model constructs net benefit functions for aggregate release within a period, so that they can be used in an inter-period stochastic model for long-term scheduling of the reservoir. In reality, consumptive and non-consumptive users may require water in more specific time slots than simply “anytime within the month”, or “in a constant stream over the month”. Therefore, some users would prefer to be able to submit demand bids that can guarantee water at the time of highest utility, for them. The time-requirements are likely to follow common patterns within a period.

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<sup>141</sup> Similarly, a domestic water supplier may be required to draw water at a specific time of a day depending on their storage capacity and consumer demands.



**Figure 8-1.** Flow transportation delays in a single reservoir nodal network. The first release occurs in the sub-period night, (on day  $d$ ) and subsequent releases are in the day (on day  $d$ ), night (on day  $d+1$ ) and day (on day  $d+1$ ),

A release at a “night” sub-period will eventually be available for downstream non-consumptive, and perhaps for a consumptive use at some other node. However, the distance between nodes may cause a participant to receive water released at in some later sub-period (e.g., a “day” sub-period). Figure 8-1 sketches a river chain network that shows the transportation delays associate with the two period releases over the planning horizon, modelling the sub-periods as in the last section. A small node between the two decision nodes plays the role of a chance node. The chance node splits the upstream release flows into two or more flows (referred to as sub-period flows). In addition, a river chain network can have several intermediate buffering facilities to transfer flows between the sub-periods (day to night and night to day).

This section formulates a short-term nodal chain optimization with the transportation delays as an LP which seeks to maximise the release and the storage benefits of each node/arc in the network over a planning horizon (for example, see Read (1979)). This model accounts for release delays, intermediate storage, and, the consecutive nature of the time periods. In particular, the time horizon could be one month. This model seeks to maximise the net benefit based on different “times of release”. We assume a single long-term reservoir node ( $i = 0$ ) connected to a nodal chain with intermediate short term buffering capacities.

## Notation

### Indices

$i, j = 0, 1, 2, 3, \dots, n$ : Nodes, where  $(i, j)$  denotes an arc belonging to the set  $A$ , of arcs in the network. Unlike in the previous models, any node  $i$  could allow short-term storage<sup>142</sup>.

$k$ : Indexes the parallel arcs/delay paths connecting nodes  $i$  and  $j$ .  $\Omega(ij)$  denotes the set of parallel arcs in the network. Any parallel arc is labelled as  $(i, j, k)$ . In contrast to the previous models, the parallel arcs create a release delay profile from one node to the other over time. This means that a parallel arc delivers the water to the same location (e.g., node) but in a different sub-period.

$t$ : indexes the sub-periods  $t = 1, 2, 3, \dots, T$ . For example, we assume half-day (12-hour) releases over a month planning horizon ( $T = 60$ ).

### Parameters

$\rho_{ijk}$ : denotes the flow splitting weights at any arc  $k \in \Omega(ij)$  where  $\sum_{k \in \Omega(ij)} \rho_{ijk} \leq 1$ .

$\tau_{ijk}$ : denotes the release delays as a whole number of sub-periods.

$r$ : denotes the total release allocated over all sub-periods.

$\bar{S}_i, \underline{S}_i$ : Upper and lower bounds on  $s_i^t$ , assumed constant for  $t = 1, 2, 3, \dots$ . Let the lower bounds on  $s_i^t$  is  $\underline{S}_i = 0$ .  $\bar{S}_i = 0$  indicates a node with no short-term storage.

### Decision variables

$x_{ij}^t$ : denotes the flow (or release) into an arc  $(i, j)$  in the sub-period  $t$ .

$s_i^t$ : represents the storage transferred from the sub-period  $t$  to  $t + 1$  at node  $i$ .

$q_i^t$ : denotes the net consumptive diversions at the node  $i$ . A negative lower bound for  $q_i$  is required to allow nodal tributary flows.

## Deterministic Multi-nodal Consecutive Sub-period LP Model

In this chapter, as in Chapter 7,  $NCB_{i,t}(q_{i,t})$  and  $NCF_{ij,t}(x_{ij}^t)$  denote piece-wise linear convex nodal consumptive benefit and non-consumptive benefit functions respectively. This is the net sum of the

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<sup>142</sup> Node  $i = 0$  represents the long-term storage as in the previous chapters. But, here the inter-period (weekly) storage is not modelled. Instead, we model the release from node 0 and the intra-period (hourly) storage of other nodes.

local consumptive benefit functions  $CB_i(c_i)$ ,  $CE_i(d_i)$ , and  $CF_i(f_i)$  at any node  $i$ . To avoid modelling difficulties, here we assume that the form  $NCB_{i,t}(q_{i,t})$  implicitly accounts for all consumptive demands are specified by sub-period. This means that there are no demands based only on the total quantity and independent of when the water arrives<sup>143</sup>.  $\frac{dNCB_t}{dx}$  and  $\frac{dV_{T+1}}{ds_i}$  denote the piecewise linear or piecewise step monotone non-increasing current sub-period release marginal benefit and the end-of-horizon marginal benefit functions respectively.

$$\max_{s,x,q} \left( \sum_{t=1}^T \left( \sum_{i=1}^n NCB_{i,t}(q_{i,t}) + \sum_{k \in \Omega(ij)} NCF_{ij,t}(x_{ij}^t) \right) + \sum_{i=1}^n V_{T+1}(s_i^{T+1}) \right) \quad [8-1]$$

Subject to:

Net nodal flow injections:

$$s_i^t = s_i^{t-1} - q_i^t + \rho_{(jik)} x_{ji}^{t-\tau_{ijk}} - x_{ij}^t \quad \forall i, j, t \quad [8-2]$$

Storage bounds:

$$\underline{s}_i^t \leq s_i^t \leq \bar{s}_i^t \quad \forall i, t \quad [8-3]$$

Arc flow capacity bounds:

$$\underline{X}_{ij} \leq x_{ij}^t \leq \bar{X}_{ij} \quad \forall i, j, t \quad [8-4]$$

Reservoir release:

$$\sum_t x_{0j}^t = r, \quad \forall r \quad [8-5]$$

### 8.2.1.1. Explanation

The objective function seeks to maximize the current period release benefits and future period benefits across all nodes over the planning horizon. We assume set target end-of-horizon values  $V_{T+1}(s_i^{T+1})$  for the intermediate storages in the network. Further, these intermediate storage targets may not adequately account for flows in transit.

Constraint [8.2] represents the flow balance at node  $i$  in time period  $t$ . The nodes represent either temporary storages or simple flow off-takes for consumptive and/or distributary uses. Different

<sup>143</sup> Same is true for non-consumptive demands.

proportions of upstream arc flows belonging to different time periods contribute to the incoming flows to any node  $i$  are denoted by:  $\sum_{ij} \sum_{k \in \Omega(ij)} \rho_{ijk} x_{ijk}^{t-\tau_{ijk}}$ . To avoid modelling difficulties, here we ignore flow delays prior to period  $t = 1$  but flows arriving during the modelled time horizon (i.e., when,  $t - \tau_{ijk} \leq 0$ ).

Constraint [8.3] denotes the upper and lower bounds of the storage,  $i$ .

Constraint [8.4] is the arc capacity upper and lower bounds of arc  $(ij)$ .

This LP model could be transformed into a parametric LP problem by setting the release,  $r$  belonging to different values in the constraint [8.5] to obtain a set of sub-period intermediate storage/transfer solutions across the network. But, we will not explore this further.

The number of sub-periods is a potential issue for both LP and CDDP implementations needing to model the sub-period sub-problem repeatedly in each period. Assuming no delays, the multiple short-term storages under the above consecutive sub-period formulation act almost similar to the multiple long-term storages in the previous models. Ignoring the short-term storage, the network becomes no-tree as a result of release delay paths (see Figure 8-1). However, there is still well defined partial ordering of node-sub-period pairs that would allow the construction of demand curves for water constructed from consumptive and non-consumptive demand curves and previously constructed ‘downstream’ demand curves for water.

### **Modelling Cyclic Sub-periods**

As mentioned in the previous section, too many sub-periods might make the previous model impractical. To keep the number of sub-periods small each sub-period modelled does not need to represent an unbroken time interval. Instead they represent a regularly occurring set of periods of time with similar demand characteristics. For example, two sub-periods might represent days and nights within a month. Effectively all of the ‘daytimes’ within the month are combined into a single sub-period and all of the ‘night times’ into another.

Ideally, a user might prefer the system to allow her to specify demand bids for more precise sub-periods (hours for example). For example, a hydroelectric generator may bid high for flows around 6 to 8 pm in line with the daily electricity production schedule. But it is unrealistic to assume that participants will



provide different bids for each hour in some future month, or that a market would clear on that basis. More detail may be required for trading closer to real time, but a distinction between night and day, and perhaps between weekday and weekend, is probably all that can be realistically expected.

Thus, while the method we describe here can be generalized to handle more sub-periods, our discussion here only considers two equal (discontinuous) sub-periods, or more exactly time-of-use bidding intervals, (day and night) within a planning period and for several planning horizons (e.g. monthly or weekly). We assume that users provide (possibly indicative) demand bids for each sub-period. But it is important to understand that the relationship between these sub-periods is not to be considered “consecutive”, but “cyclic”. The day sub-period represents all daytime hours in the week, and the daytime release level determined for the (generic) “day”, is assumed to occur at a constant rate over those daytime hours. Thus some of that release will arrive downstream during the (generic) “day”. But some will arrive during the (generic) “night”, rather than in “the next night”. And that is equally true of releases scheduled for the generic “night”. We will simplify the problem slightly by assuming each market interval to be self-contained, and ignore carryover from the last night of one market interval to the first day of the next. This is a very good approximation if the market interval is a month, but potentially less good if it is a week. It is still consistent, though, with assuming that the system is likely to operate according to a stable cyclic pattern that only shifts slowly from week to week.

The impact of delay means that a pre-determined proportion of the daytime release from one node will arrive as a daytime flow at the next downstream node, with the remainder arriving as a night-time flow. For example, a delay of 4 hours implies a 2/3: 1/3 split. Imposing this ratio restriction has a similar effect to assuming that a randomly chosen unit of “daytime release” has a pre-determined probability of arriving downstream as day-time versus night-time flow. To formulate the cyclic sub-period LP model, we change the arc flow variable period index and the storage variable period index to modulo formats as follows:

$t$ : Indexes the sub-periods  $t = D, N$ .

$k$ : Indexes the parallel delay paths connecting nodes  $i$  and  $j$  in  $t = D, N$ .

### Deterministic Multi-nodal LP Cyclic Sub-period Model

Unlike in the previous model, an end-of-period value function is no longer required. It is effectively assumed to be the same as the first period value function and to be in equilibrium. The day-night cyclic period release marginal benefit function is:

$$\max_{s,x,q} \left( \sum_{i=1}^n NCB_{i,D}(q_{i,D}) + NCB_{i,N}(q_{i,N}) + NCF_{ij,D}(x_{ij}^D) + NCF_{ij,N}(x_{ij}^N) \right) \quad [8-6]$$

Subject to:

Net nodal flow injections:

$$\begin{aligned} s_i^N &= s_i^D - q_i^N + \rho_{jiN}(x_{ji}^D + x_{ji}^N) - \sum_{ij} x_{ij}^N \quad \forall i, j \\ s_i^D &= s_i^N - q_i^D + \rho_{jiD}(x_{ji}^D + x_{ji}^N) - \sum_{ij} x_{ij}^D \quad \forall i, j \end{aligned} \quad [8-7]$$

Storage bounds:

$$\underline{s}_i^t \leq s_i^t \leq \bar{s}_i^t \quad \forall i, t = D, N \quad [8-8]$$

Arc flow capacity bounds:

$$\underline{X}_{ij} \leq x_{ij}^t \leq \bar{X}_{ij} \quad \forall i, j, t = D, N \quad [8-9]$$

Reservoir release:

$$\sum_t x_{0j}^t = r, \quad \forall r, t = D, N \quad [8-10]$$

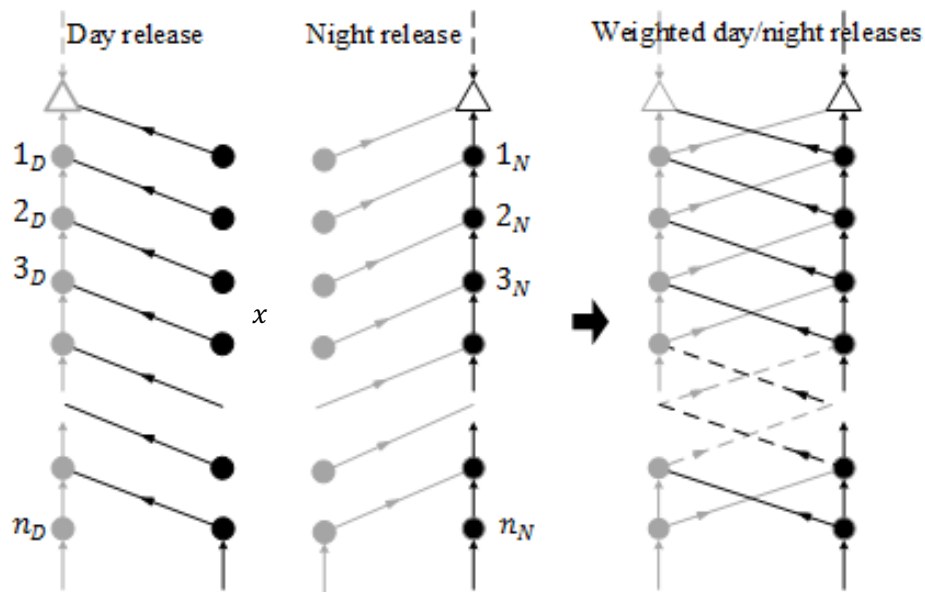
In the following, we attempt to develop a range of CDDP computational routines to solve the above cyclic sub-period optimization problem.

### 8.3. CDDP Modelling of Release Delays

This section is about developing a sequence of CDDP computational routines which step-by-step attempt to develop a full version including short-term storage. The sequence of CDDP developments in this section assumes no short-term storage, fixed storage and dynamic storage for the purpose of avoiding modelling difficulties. However, developing a CDDP beyond the leaf node is not practical due to the dimensionality problem of the sub-period solutions. But, we have continued to develop the sub-period CDDP models despite being unsuccessful as they provide useful insights about time of use of water to modelers and river-chain operators.

The CDDP algorithm which accounts for this effect is essentially the same as a stochastic CDDP algorithm, in which we use the “probability” weighted average of downstream time-of-use nodal demand curves to construct the upstream demand curve for water. (To simplify the discussion, we are assuming, here, that we are dealing with a simple chain downstream from the reservoir, so that each child node is downstream from its parent). So the *dcr* processing can always be considered as occurring ‘up’ an arc, from the child (downstream) end to the parent (upstream) end of the arc. Also, for the purposes of discussion, all non-consumptive use on an arc is assumed to occur at the parent end of the arc, and hence prior to any delay).

Assuming no short-term storages, we repeatedly process the nodes in the expanded time-of-use nodal network to produce time-of-use demand curves for release. Figure 8-2 illustrates the order in which we process “time-of-use bids” (day bids in grey and night bids in black), working towards the reservoir.



**Figure 8-2.** Time-of-use bids to represent release flow delays in a single reservoir nodal network.  $i_D$  and  $i_N$  denote a day-time (e.g., grey colour), and a night-time node (e.g., black colour).

### 8.3.1. A Nodal CDDP Procedure with Release Delays

We assume a nodal tree network with  $n$  nodes similar to the previous cases. Let  $i$  denote any node in the network. We start labelling the nodes from the reservoir node ( $i = 0$ ). The leaf node is labelled as  $i = n$ . Let  $(i, j)$  represents the arc between nodes  $i$  (parent node) and  $j$  (child node). We sub or post

script nodes, arcs and demands to represent their association with any sub-period (e.g.,  $i_D, i_N$  denote day and night nodes,  $nb_D, nb_N$  denote the day and night nodal demand curves).

## Procedure

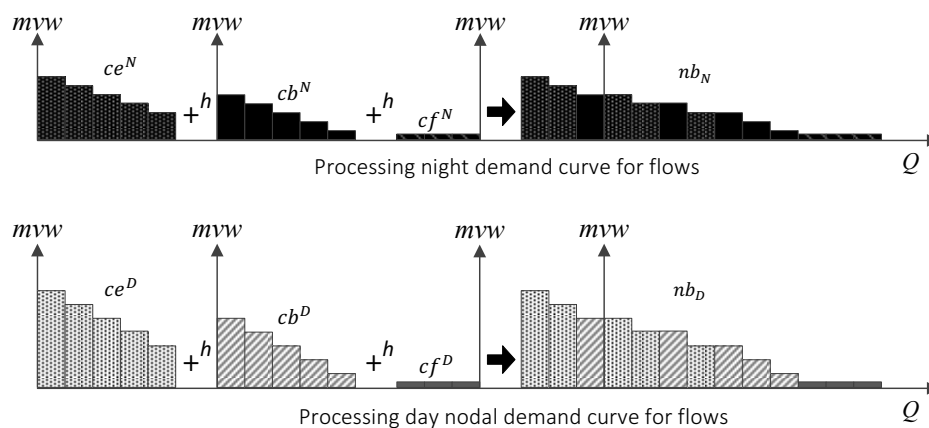
This procedure contains 6 steps starting at Step A.0 and ending at Step A.6. This constructs the demand curve for release from the reservoir that is determined by horizontally adding the day and night demand curves for flow at the reservoir. Then the reservoir can meet the demands of the most valuable uses first regardless of whether they arise from day or night releases. Note that the current development is limited to a single nodal chain.

### Step A.0

The algorithm starts at the leaf node of the chain,  $i = n$ . All nodes other than the reservoir node are split into a day and a night version with the day node arbitrarily processed first.

### Step A.1

First, the CDDP algorithm “horizontally adds” the daytime demand curves for outflow from node  $i$ , consumption at node  $i$ , and tributary flows into node  $i$ , to produce a net demand curve for daytime inflow into node  $i$  from its parent, in the normal CDDP fashion (see Figure 8-3). Night-time curves are processed similarly, but separately.



**Figure 8-3.** Time-of-use (day and night) demand curves for flows. Subscripts D and N denote the day and the night demands.

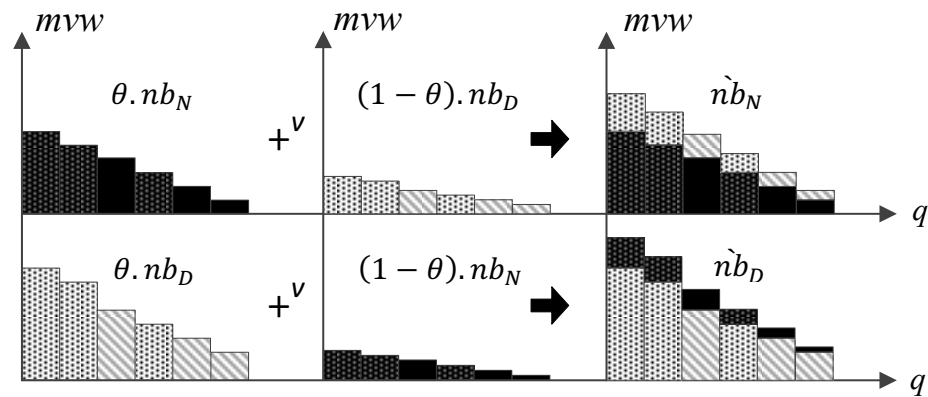
Dark shaded (black background) bid patterns in the upper part of the diagram show the night demands, while the day demands are illustrated by light shaded (white background) patterns in the lower part. The day demands are high priced compared to the night demands.

### Step A.2

Construct demand curves for day (then, separately, for night) flow into the parent end of arc  $(i, j)$ , ignoring any non-consumptive uses at the parent end. Here,  $i$  denotes the child node.

- First, the arc flow capacity truncation  $[L_{i,j} U_{i,j}]$  is applied to the time-of-use demand curves. This restriction occurs at the child node end.
- Second, the delay on the arc is accounted for by taking the probability weighted average of the child end day/night demand curves. Given the day-to-day splitting weight  $0 \leq \theta \leq 1$  (and, symmetrically, the day-to-night splitting weight is  $1 - \theta$ ), we can form time-of-use demand curves for flows out of the parent end of the arc as follows.

$$\hat{n}b_D = \theta \cdot nb_D + (1 - \theta)nb_N, \hat{n}b_N = \theta \cdot nb_N + (1 - \theta)nb_D \quad [8-11]$$



**Figure 8-4.** The time-of-use demand curves at any day and night sub-period node.  $0 \leq \theta \leq 1$  denotes the sub-period flow splitting weights.

The ‘new’ probability weighted day demand curve is constructed by vertically adding probability weighted day and night demand curves. So every day bid now has a night marginal value component because of the release delays. Similarly, we obtain the probability weighted night demand curve.

### **Step A.3**

Non-consumptive use is assumed to occur at the point of release, and hence accounted for after the probability weighted parent-end demand curves have been formed. Non-consumptive uses are accounted for using vertical addition of the parent-end net demand curve and the non-consumptive demand curve. This is done separately for both the day and night nodes.

### **Step A.4**

Unless we have reached the reservoir node, the CDDP algorithm then returns to **Step A.1**, to combine these day/ night demand curves for the parent's parent node. Relabel  $i$  as  $i - 1$  then return to Step A.1. Otherwise, we proceed to Step A.5

### **Step A.5**

Finally, we process the reservoir node ( $i = 0$ ), which is assumed to have full control over release timing. Thus the demand curve for release from the reservoir is determined by horizontally adding the day and night demand curves for flow at the reservoir. That is, the reservoir can release to the most valuable uses first regardless of whether they arise from day or night releases.

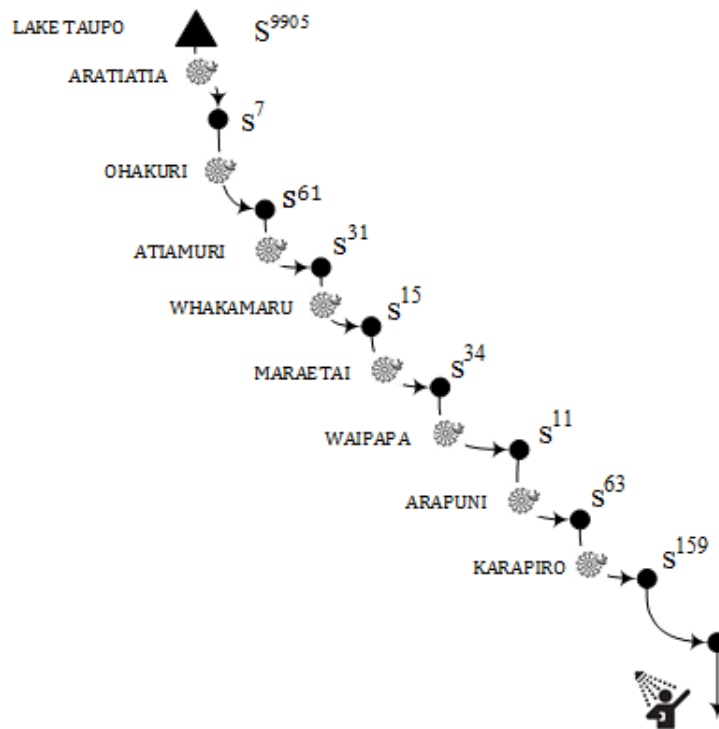
### **End of procedure.**

Section 8.3.2 presents a numerical example to demonstrate the above procedure.

#### **8.3.2. Numerical Example of CDDP Procedure with no Intermediate Storages**

This section constructs the demand curve for release in the presence of arc flow delays using the above time-of-use CDDP procedure. The procedure is applied to an example loosely based on the topology of the Waikato River catchment in New Zealand, in which Lake Taupo plays the long term storage role (see Figure 8-5). Some hypothetical data is used to analyse the network release delay problem.

We perform a set of experiments to show how different assumptions of flow splitting methods impact the final *dcrs*. Scenario 1 assumes that there are no release delays. Scenario 2 assumes that there are flow delays between nodes, and hence that there is a variable contribution of upstream sub-period releases to the incoming flows for different arcs in the network.



**Figure 8-5.** Topological arrangement of the Waikato catchment depicting hydro-electric users and a downstream urban use.  $S^q$  denotes the size of each storage. (source: Read, 1979)

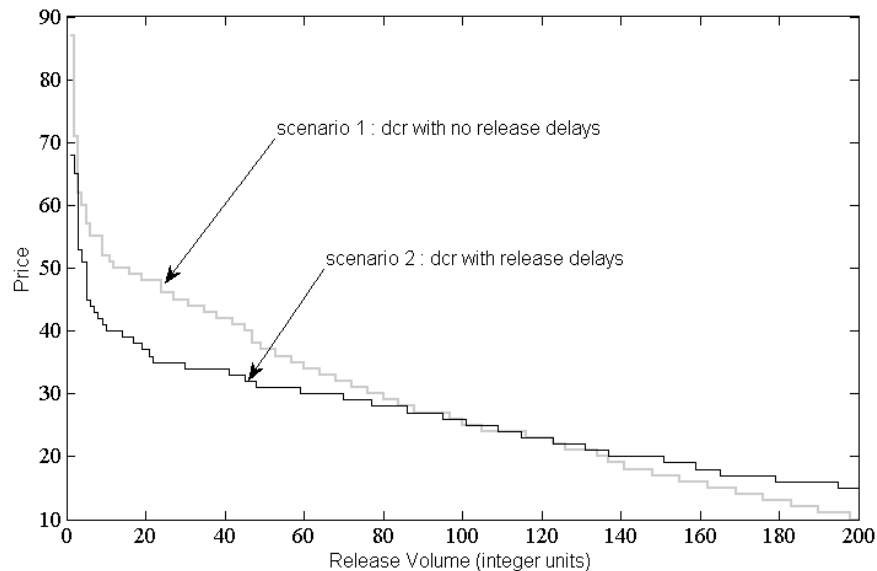
Downstream from Taupo, the Waikato River supports eight hydroelectric plants, totalling approximately 1000 MW, which, in this simplified framework, are assumed to have no explicit storage capacity. We include downstream urban uses in the model, assuming monotone decreasing consumptive demand curves for water. Suppose that consumptive and hydropower users submit separate bids for day and night water use. Other local power schemes, including geothermal and the upstream Tongariro hydro power scheme, are ignored. The release delays (for scenario 2) are shown in Table 8-1. This uses actual flow delays between nodes to determine the sub-period flow splitting weights (using 12-hour sub-periods).

**Table 8-1** Release delays in hours with reference to upstream nodes and relevant sub-period flow splitting weights. An empty cell represents no release flow delays between the two nodes.

Description	TAUP	ARAT	OHAK	ATIA	WHAK	MARA	WIP	ARAP	KARA
Delay (hrs)	0	1.5	7	0.1	0.4	0.1	0.3	2.5	2.0
Splitting weights	-	0.1:0.9	0.5:0.5	-	-	-	-	0.75:0.25	0.75:0.25

Source: Read (1979)

This section illustrates how the *dcr* changes when the procedure applies variable time-of-use flow splitting weights (scenario 2) based on the information given in the above table. Figure 8-6 contains the two *dcrs* constructed with no release delays (scenario 1, grey curve) and variable (scenario 2, black curve) time-of-use flow splitting weights assumptions.



**Figure 8-6.** Example demand curves for release for scenario 1(no release delays) and scenario 2 (release delays) represented by the variable time-of-use weights.

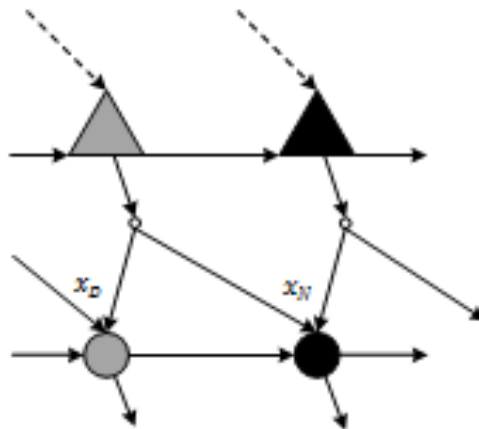
According to Figure 8-6, the time-of-use *dcr* represented using the variable time-of-use flow splitting weights generally records relatively low marginal water values. In the absence of modelling of release delays, prices for low release volumes can be a relatively higher and some of the high release volume prices can be lower. This provides a good case for us to model release delays properly.

The variable splitting method described above models the release delays simply by taking the weighted average of the two sub-period demand functions. This approach cannot explicitly model short-term storages. Section 8.3.3 uses a calibrated flow splitting weight as a function of upstream incremental flows to represent the flows shifted between sub-periods using short-term storage. The method provides some degree of control to the system operator to meet specific time-of-use demands (if she is chosen to) with the upstream release flows in a particular sub-period.



### 8.3.3. A CDDP Procedure with Fixed Intermediate Storages (An Incremental Flow Splitting Method)

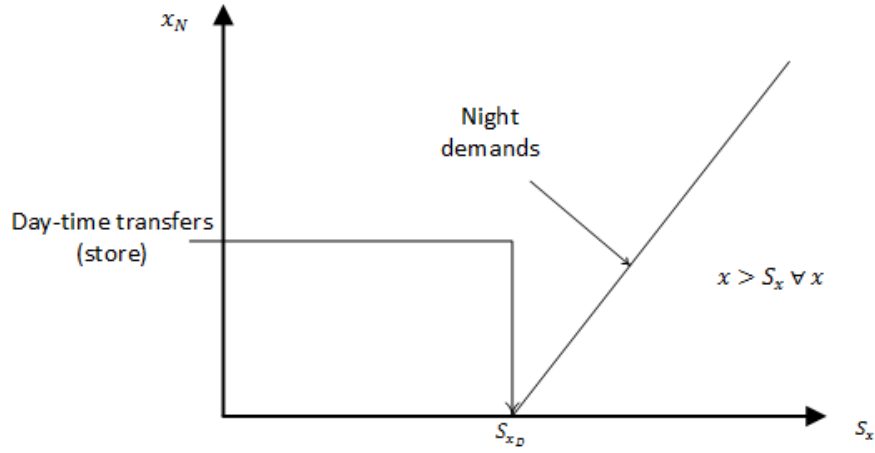
The flow splitting methods discussed so far only model the release delays. However, the flows in transit may also be stored in temporary buffers in the network, and thus deliberately shifted to be available in a more attractive sub-period. Using CDDP this storage cannot be modelled directly without the curse of dimensionality causing a problem. This section describes a flow splitting rule that adjusts the delay times/ splitting proportions (e.g., based on historical data) to account for short-term storage without modelling these buffers directly. The objective of this section is to investigate how to efficiently and flexibly use a flow splitting rule to model the flows in transit. As a result the system operator could sometimes divert upstream release flows in a particular sub-period to meet more valuable time-of-use demands in the other sub-period assuming an intermediate storage.



**Figure 8-7.** Arrival of flows into day and night nodes in different sub-periods

One of the simplest rules is to store some percentage of arrivals (initial flows) in a particular sub-period for the next sub-period use and use the remaining arrivals to meet the demands of that sub-period (see Figure 8-7). Suppose, the night period transfers water to the day period using the intermediate storage up to some pre-specified storage limit. Any remaining water is released in the current ‘night’ period and day period normally would not use the intermediate storage. This leads us to ask how to prioritise the type of sub-period node. A rule for any sub-period arrival flows can be formed to show the flow apportionment as  $\min\{x, S_x\}$  where  $x$  denotes the arrival flows in a particular sub period and  $S_x$  represents the maximum flows to be transferred to the next sub-period. To avoid modelling difficulties, one could assume that the priority is given to flow transfer over the release for all nodes and all sub-

periods. Figure 8-8 depicts a linear rule that stores the night arrivals  $x$  up to  $S_x$  and remaining arrivals will be allocated for the current night-time demands.



**Figure 8-8.** Linear rule to allocate flows in the night sub-period node

This approach uses two different sub-period delay/apportionment functions where each could be calibrated using the historical data. The day-time apportionment function first allocates water for day release (to meet the most valuable day demands) up to some pre-specified level,  $X$ . Water beyond this level is stored (for night use) up to some maximum storage level,  $S_x$ . Any remaining water is again allocated to day release (to meet the least valuable day demands).

Suppose that  $x$  denotes the arrival flows in a particular sub-period. The flow splitting weights  $\tilde{\rho}(x)$  are defined as a function of the arrival flows.

$$\tilde{\rho}(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq X \\ X/x & \text{if } X \leq x \leq X + S_x \\ (x - S_x)/x & \text{if } x > X + S_x \end{cases}$$

This means the water staying the same sub-period is  $x\rho(x)$  and the water stored to the other sub-period is  $x(1 - \tilde{\rho}(x))$ . That means,

$$x\tilde{\rho}(x) = \begin{cases} x & \text{if } 0 \leq x \leq X \\ X & \text{if } X \leq x \leq X + S_x \\ (x - S_x) & \text{if } x > X + S_x \end{cases}$$

And,

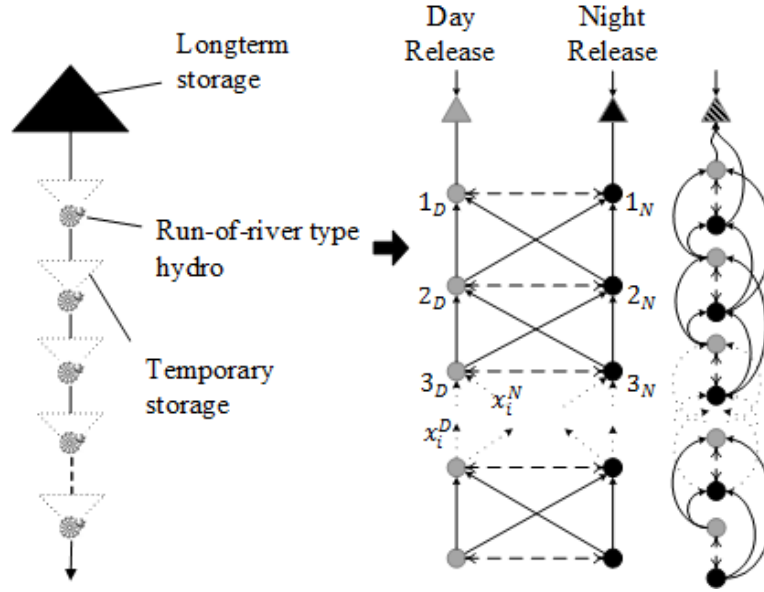
$$x(1 - \tilde{\rho}(x)) = \begin{cases} 0 & \text{if } 0 \leq x \leq X \\ x - X & \text{if } X \leq x \leq X + S_x \\ S_x & \text{if } x > X + S_x \end{cases}$$

Obviously, the system operator could use past data and her experience to calibrate  $\tilde{\rho}$ . Conversely, the calibration process could suggest several splitting stages for either  $\tilde{\rho}$ . Even though it has defined a set of linear splitting rules, the final solution for the two sub-period incremental arrivals flows for a node would be non-linear. The best split depends on the relative sizes of the flow split between the sub-periods. This method may not be suitable in the CDDP because the expected arrival split between sub-periods will depend on the split choices made both above and below a node. However, a simple but hard splitting method to allocate/store the flows arriving in different sub-periods may not provide sufficient flexibility to the system operator to meet the time-of-use demands efficiently. Therefore, in the following section we introduce a relatively dynamic incremental flow splitting weight function.

#### **8.3.4. Time-of-use Demand Curve for Release with Intermediate Storages**

The main objective of this section is to develop CDDP computational routines to construct the time-of-use nodal demand curves considering the intermediate storing of water and release delays. The CDDP ends up forming a 2-dimensional marginal water value surface for the leaf node. Thus, this section does not get past processing the leaf node.

The effect of these intermediate storages in a hydro-catchment is illustrated using a series of nodes (see Figure 8-9). Two sub-periods are modelled: day and night. The RHS tree represents the processing direction (against the stream flow direction, towards the reservoir). Horizontal (checked) arrows between the sub-period nodes (day-night) represent the net storage transfers (see figure below). Each node is assumed to have temporary (intermediate) storages to transfer flows between sub-periods.



**Figure 8-9.** Temporary flow buffering facilities in a single reservoir nodal catchment network and the time of use nodal tree processing.

Step 1 explains how to construct the composite time of use demand curves for different day and night upstream release combinations, and the step 2 subsequently forms the marginal value surface for the intermediate storage. Finally, step 3 describes how to construct the above marginal water value surfaces if a flow splitting method is applied to imply release delays using the extended time-of-use CDDP procedure.

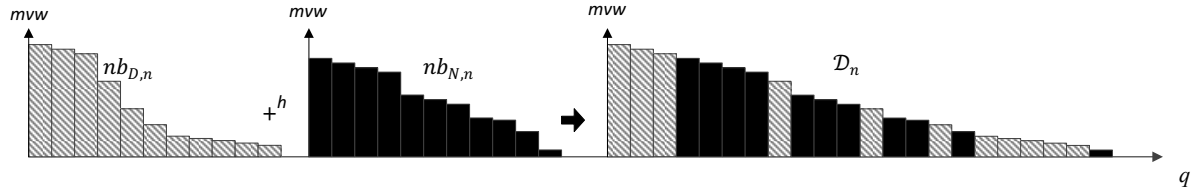
The next section describes how to construct the composite nodal time-of-use demand curve firstly ignoring the storage upper bounds and later when storage capacity limits are binding.

Let  $x_i^D$  and  $x_i^N$  represent upstream sub-period releases.  $\mathcal{D}_i$  denotes the composite nodal time-of-use demand curve.  $\delta_i^{max}$  denotes the upper bounds of the intermediate storage  $i$ .

### 8.3.5. Constructing the Demand Curve for Release from the Time-of-use Demand Curves

Let  $nb_{D,n}, nb_{N,n}$  represent the day and night demand curves for water at the node  $n$  (a leaf node). Ideally, all flows from upstream would be available to meet both  $nb_D$  and  $nb_N$ . But in the real world, there are limits on what can be transferred via intermediate storage to the more desirable sub-period (typically, day). With unrestricted intermediate storage at node  $n$ , node  $n-1$  would see a net demand curve formed by combining the day and night demand curves for node  $n$ . Let  $\mathcal{D}_n = nb_D + {}^h nb_N$  denote the intermediate storage unrestricted cumulative time-of-use demand curve for flow at a node  $n$  formed

by horizontally adding time-of-use demand curves for flows (see Figure 8-10). For example, if  $nb_{D,n}$  and  $nb_{N,n}$  denote day and night sub-period demand curves at the last node  $n$  respectively, inverse addition of these demand curves will produce the composite time-of-use demand curve for flows  $\mathcal{D}_n$  at node  $n$ .



**Figure 8-10.** Composite time-of-use demand curve for flows representing day and night demands of node  $n$ .  $\mathcal{D}_n$  denotes the full bid stack when the intermediate storage constraints are not binding.

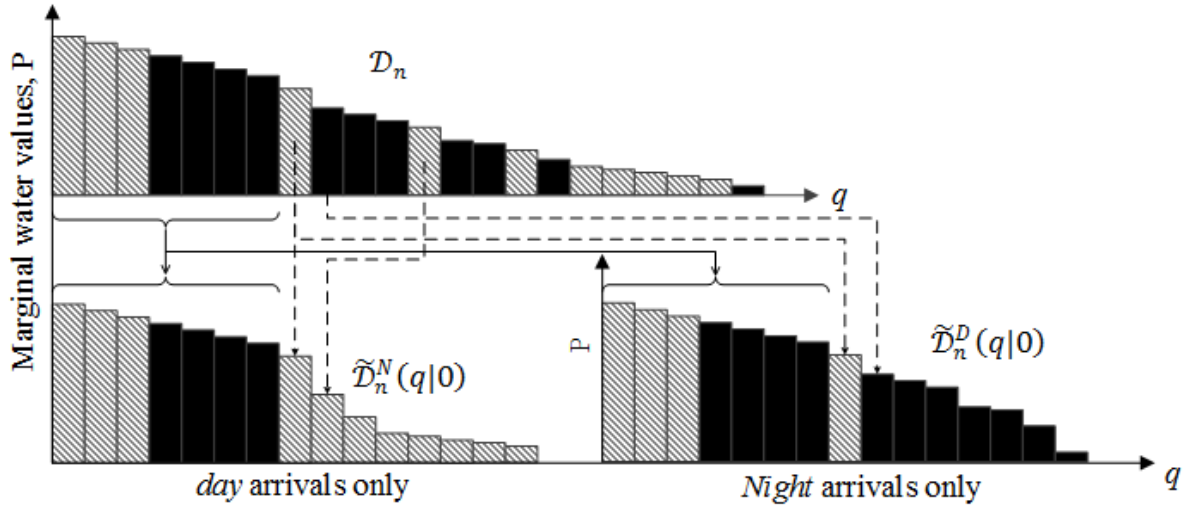
With no delay between nodes  $n - 1$  and  $n$ , each step of the combined net demand curve could be met with a flow in the sub-period of that demand, or with a flow in the other sub-period which is stored until needed. For this unrestricted case the marginal value is read directly from  $\mathcal{D}_n$  based on the combined day-night flow. The upstream net-demand curves will be the same for day and night.

### 8.3.6. Constructing the Marginal Water Value Surface: Corner Solutions

When the intermediate storage has a restrictive capacity (e.g.  $\delta_n^{max}$ ), we may not be able to represent the composite day-night demand curve as a merit ordered stack of day and night demands. Storage capacity constraints together with sub-period upstream release determine how many demands to be met during that sub-period, or stored for the next sub-period.

Suppose  $(x^D = 0, 1, 2, 3, \dots, x^N = 0, 1, 2, 3, \dots)$  denote any day and night upstream releases. Upstream release flows in a particular sub-period (e.g., day with  $x^D$ ) are in principle allocated to meet the time-of-use demands in that sub-period (e.g., day). Suppose there are relatively high demands in the other sub-period (e.g., night). The day release flows could meet some of that sub-period's demands via the intermediate storage. This sub-period flow transfer continues until the upper capacity bound is constrained.  $\tilde{\mathcal{D}}_n$  denotes the updated composite demand curve subject to the upper storage bounds  $x^D, x^N \leq \delta_n^{max}$  for any incremental sub-period arriving flow (an arriving flow enters into an arc in any sub-period and a releasing flow leaves an arc in either sub-period).

For example, let us arbitrarily assume an intermediate storage capacity of  $\delta^{max} = 4$  units. We will show how to construct the updated demand curves  $\tilde{D}$ , for each successive upstream sub-period incremental flow. To begin with we assume all flows arrive in a single subperiod, day or night only.



**Figure 8-11.** Time-of-use demand curves with incremental day / night incoming flow arrivals.

Let us consider the demand curve  $D_n$  in in the first row of Figure 8-10, which repeats the composite *dcr* of Figure 8.9, and consider how we might utilise day-time arrivals. The first three day arrival units meet the high (day) demands (hashed rectangles). The fourth incoming unit is allocated for the first night bid (solid black rectangle) and will be transferred via the intermediate storage. We continue the process of either meeting day demands or transferring flow units to meet night demands until the transfer reaches the upper storage bounds. Beyond this no storage transfer is feasible because the upper storage constraint is binding. Thus, any upstream day flows can only be used to meet day-time demands, or spilled, as shown in the LHS diagram, in the second row of Figure 8-12: the “ $\tilde{D}(q|0)$ ” demand curve for daytime arrivals, assuming night-time release of 0”. The RHS demand curve diagram is constructed similarly, showing the “demand curve for night-time arrivals, assuming daytime release of 0”. These curves represent the corner solutions of problem. That is the initial row and column marginal water values.

The demand curves constructed here assume no flow in the other sub-period. To account for flows in both sub-periods the algorithm next systematically constructs marginal water value curves considering sub-period incoming releases one at a time starting with the corner solutions. For the interest of clarity,

we further explain this step using a numerical example. To set the platform, the following executes the previous step to create numerical values attached to a set of composite time-of-use marginal water values.

### 8.3.7. CDDP Procedure for Filling-in and Corner Solutions

This section presents the time-of-use CDDP procedure that finds corner marginal water values and the filling-in marginal water values of a nodal demand surface.

#### Notation

$i = 0, 1, 2, 3 \dots$ : Integer

$r^D, r^N$ : Flows releasing from an arc in a sub-period (day, night)

$r^{max}$ : Maximum release in a cycle.

$nb_D, nb_N$ : Day and night marginal water values

$x^D, x^N$ : Arriving inflows into an arc in a sub-period (day, night)

$\delta^{DN}, \delta^{ND}$ : Day-night and night-day transfer storages

$\delta^{max}$ : Maximum intermediate storage

#### Initialization

Form  $\tilde{D}$  using  $nb_N$  &  $nb_D$

Set  $x^N = x^D = 0$ ,  $\delta^{ND} = \delta^{DN} = 0, i = 0$

#### Beginning of procedure

While  $i < (\delta^{max} + r^{max})$  Do

If  $\tilde{D}(i)$  is  $r^D$  then

$mwv(x^N, i) \leftarrow \tilde{D}(i)$

$\delta^{ND} = \delta^{ND} + 1$

If  $\delta^{ND} \leq \delta^{max}$  then

$mwv(x^N, j) \leftarrow \tilde{D}(j)$  for  $j \leq i$

Else  $mwv(x^N, j) \leftarrow nb_D(j)$  for  $j > i$  or *spill*

$x^N = x^N + 1$

Else  $\tilde{D}(i)$  is  $r^N$

$$mwv(i, x^D) \leftarrow \tilde{D}(i)$$

$$\delta^{DN} = \delta^{DN} + 1$$

If  $\delta^{DN} \leq \delta^{max}$  then

$$mwv(j, x^D) \leftarrow \tilde{D}(j) \text{ for } j \leq i$$

Else  $mwv(j, x^D) \leftarrow nb_N(j) \text{ for } j > i$  or *spill*

$$x^N = x^N + 1$$

End if  $i = i + 1$

End do

End of procedure.

The next section is about a numerical example for constructing the marginal water value surfaces for the two sub-period release flows at the leaf node  $n$ .

### 8.3.8. Numerical Illustration of the Time-of-use CDDP Procedure

This section explains how to find corner solutions and filling-in solutions of the demand surface for the leaf node using the above CDDP procedure assuming hypothetical data. This section assumes no release flow delays.

#### 8.3.8.1. How to Find Corner Solutions?

To start with, the algorithm constructs the composite time-of-use demand curve for an *unrestricted* intermediate storage with the assumption that there is no delay. Suppose a set of consumptive (e.g. urban use) day and night demands for flows are attached to this node. The third row of Figure 8-12 forms the composite time-of-use demand curve by re-arranging the prices of the first (night demands) and second (day demands) rows in their merit order.

The composite time-of-use demand curve  $D_n$  is constructed from individual sub-period bid stacks shown by two free standing night and day price lists (rows 1 and 2). Row 3 contains composite time-of-use marginal water values ignoring any storage upper bounds. Row 4 and 5 denote the net sub-period (day-night, night-day) transfer volumes (see Figure 8-12).



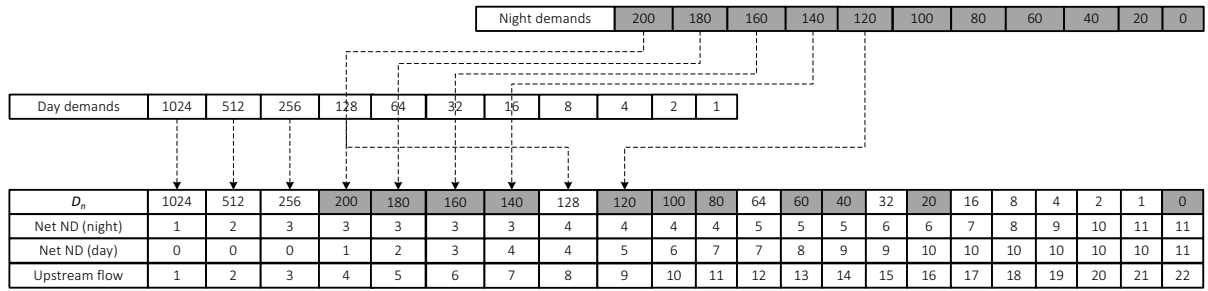


Figure 8-12. Constructing the composite time-of-use demand curve for flows and the day-night transfer schedules.

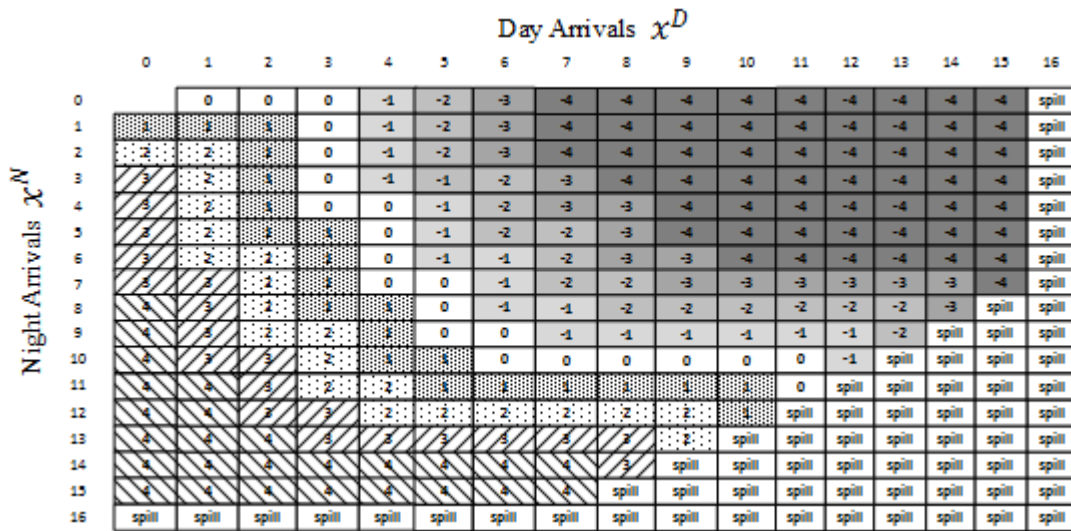
The storage transfer schedule is prepared assuming all upstream flows arrive only in the night (see row number 4). That is, we determine the storage required to meet the demand in the order shown in row 3 if all flows arrive at night. In this example, the first three night-time incoming flow units will be transferred to the intermediate storage to meet the three highest value daytime demands, valued at 1024, 512 and 256. The fourth increment meets the first night-time demand, valued at ‘200’. The next three flow units also meet the night-time demands, valued at 180, 160 and 140 respectively. Now the algorithm comes to the next cell ‘128’ which is a day-time demand. So the 8<sup>th</sup> flow increment will be transferred to the storage.

Similarly, the fifth row shows the net day-to-night transfer schedule assuming all upstream releases occur during the day. Once more, the first three day demands are met by the incoming flows and the fourth unit is transferred to meet the first night demand. Subsequent flow increments (e.g., 2, 3, and 4) will be transferred to meet the rest of the night-time demands, valued at 180, 160, and 140 respectively. The algorithm completes the two rows (4 and 5) in a similar manner to construct the net night-day and net day-night transfer schedules.

### 8.3.8.2. How to find fill-in solutions?

Generally, the procedure has to (1) consider different sub-period release combinations and (2) the storage capacity bounds when deciding the net sub-period transfers (or storage balance). Let us now assume an upper bound for the intermediate storage. Suppose that the maximum capacity of intermediate storage is  $\delta_{max} = 4$ , and that upstream release in each sub-period starts with 0 flow units, and could incrementally increase to a maximum flow of 11 units. The time-of-use CDDP now computes the 2 dimensional night-to-day and day-to-night flow transfer schedule for different sub-

period releases. This is shown in Figure 8-14. For the example, the first row (which partly corresponds to the 5<sup>th</sup> row in Figure 8-12) can be interpreted as night transfers when the only upstream flows arrive during the day time. However, this transfer of flows is stopped beyond the 7<sup>th</sup> day arrival unit because the storage constraint becomes binding. The net day-night transfers are shown by negative integers in Figure 8-13. Thus, all the day time arrivals beyond 7 units are allocated for day demands in the composite demand curve  $D_n$  and 16<sup>th</sup> unit onwards will be spilled (see row 1). Further, the second row of Figure 8-13 denotes night transfers when arrivals are occurring in the day-time but given one unit of night flow. The second iteration  $i = 2$  corresponds to the construction of the second row if the release is to meet a day demand.

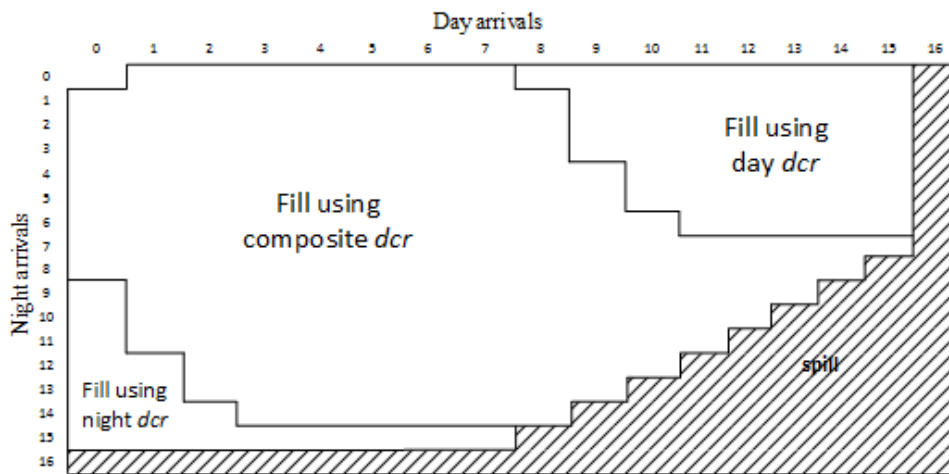


**Figure 8-13.** Storage balance diagram

The CDDP procedure systematically fills all the cells in the storage balance (volume) matrix based on the composite demand curve  $D_n$ . Positive values (in hashed and dotted cells) represent the night to day transfers and negative values (in shaded cells) indicate the day to night transfers. We have not illustrated the storage balances beyond 16 units arriving in either sub-period, as any arrivals beyond 15 units will be spilled. Procedure increments  $i$  to its next integer value to construct remaining rows in the matrix.

By carefully observing the transfer volume patterns, we could identify a more efficient method to fill these points without simply repeating the above procedure. That is, the composite demand curve  $D_n$  is

not needed to fill the cells once the storage constraint binds. At that point the algorithm only has to use respective sub-period demand curve to complete the remaining cells in the matrix (see Figure 8-14).



**Figure 8-14.** Efficient construction of the storage schedule

So that we could use the same approach to construct the time-of-use marginal water value surfaces for the intermediate storage.

Figure 8-15 illustrates the marginal demand surface matrix  $\varphi_n$ . Conceptually, the procedure has to construct two marginal water value surfaces for each sub-period node. However, the two surfaces coincide perfectly on each other because both surfaces are constructed from the same composite demand curve  $D_n$ . That is the procedure constructs the same marginal water value surface regardless as to whether it is constructed by row or column. The grey coloured cells denote night bid prices and uncoloured cells denote day bid prices. The algorithm systematically selects appropriate marginal values from the composite time-of-use marginal water value set (see Row 3 in Figure 8-12) for particular sub-period incremental flow arrivals.

		Day arrivals																						
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Night arrivals	0		1024	512	256	200	180	160	140	128	64	32	16	8	4	2	1	0	0	0	0	0	0	0
	1	1024	512	256	200	180	160	140	128	120	64	32	16	8	4	2	1	0	0	0	0	0	0	0
	2	512	256	200	180	160	140	128	120	100	64	32	16	8	4	2	1	0	0	0	0	0	0	0
	3	256	200	180	160	140	128	120	100	80	64	32	16	8	4	2	1	0	0	0	0	0	0	0
	4	200	180	160	140	128	120	100	80	64	60	32	16	8	4	2	1	0	0	0	0	0	0	0
	5	180	160	140	128	120	100	80	64	60	40	32	16	8	4	2	1	0	0	0	0	0	0	0
	6	160	140	128	120	100	80	64	60	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0
	7	140	128	120	100	80	64	60	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0	0
	8	128	120	100	80	64	60	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0	0	0
	9	120	100	80	64	60	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0	0	0	0
	10	100	80	64	60	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0	0	0	0	0
	11	80	64	60	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0
	12	60	60	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	40	40	32	20	16	8	4	2	1	0	0	0	0	0	0	0	0	6	0	0	0	0	0
	14	20	20	20	16	8	4	2	1	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0
	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

**Figure 8-15.** Time-of-use marginal water value matrix for incremental night and day flow arrivals at node  $n$ .

In the example, the cell (4, 8) in  $\varphi_n$  interprets the marginal water value corresponding to the 4<sup>th</sup> incremental day-time flow arrival, given the 8 units of night-time arrivals subject to storage capacity limits. The first row and first column of the matrix represent the highest possible day and night sub-period demand curves, respectively. In both cases, the first three upstream day flows meet the demands (day) ‘1024’, ‘512’ and ‘256’, and the fourth to seventh units meet the highest valued night-time demands transferred. In the daytime arrival case, the capacity constraint  $\delta_{max} = 4$  is then binding and no more units could be transferred. As a result, any further day flow arrivals could only meet daytime demands. Similarly, in the case of no day releases (see column 1), night arrivals beyond 8 units can only meet night-time demands. The night-time flow arrival increments 1, 2, 3 and 8 are transferred to the storage to meet the day-time demands valued at ‘1024’, ‘512’, ‘256’ and ‘128’ respectively. In the example, the cells or the grid points  $\{(4, 0), (4, 1), (4, 2), \dots, (4, 22)\}$  in  $\varphi_n$  indicate the set of night time marginal water values or the night-time demand curve given 4 units of day-flows subject to the storage bounds.

The next step explains additional processing step requires if a flow splitting method is used to represent the release delays. Hence the algorithm has to employ an interpolating technique to compute each marginal water value.

### 8.3.9. CDDP Procedure with Release Flow Delays and Intermediate Storage Water Values

This step explains how to construct the marginal value surface for intermediate storage of water,  $\tilde{\varphi}_n$  using a suitable day-night flow splitting ratio that represents release delays. This simply combines the weighted marginal water values of the above demand surface and the current surface as they occur at different points in the process. Then the following is described for the sake of completeness.

For example, applying a symmetrical day-to-day and day-to-night transfer weight (e.g. 0.5:0.5) could be computationally simple and straight forward compared to other weights. Suppose transfer weights take the ratio of 0.75:0.25. For example, high day release (12, 0) is split to 9 (day) and 3 (night) arrivals. The night flow arrivals meet night demands '200', '180' and '160' respectively. Four day arrivals can be transferred to meet night demands '140', '120', '100' and '80' and remaining day arrivals meet day demands from '1024' to '64'. Similarly, release combination (16,0) is split into (12,4) and computing  $\tilde{\varphi}_n(12,4)$  may not straight forward as in the previous case because the two corresponding sub-period marginal water values are not equal. The last marginal water value values of the day, and the night sub-periods are '8' and '60' respectively. Hence, the algorithm updates the marginal value using a simple interpolation technique:  $\tilde{\varphi}_n(16,0) = 0.75 \varphi_n(12,4) + 0.25 \varphi_n(9,4) = 21$ . Assume a unit increment of day arrival from (12,0). According to the flow splitting rule, this extra unit is formed from 0.25 night flow units (e.g.,  $\varphi_n(9,4)$ ) and 0.75 day flow units (e.g.,  $\varphi_n(10,4)$ ). So the uses numerical techniques to determine the new marginal water value:  $\tilde{\varphi}_n(13,0) = 0.25 \varphi_n(9,4) + 0.75 \varphi_n(10,4) = 39$ . Each and every point of the marginal value surface for storage delays  $\tilde{\varphi}_n$  could be computed step by step for incremental day/night arrivals.

### 8.3.10. Processing Upstream Nodes and Arcs

The CDDP version developed in the previous section constructed a two-dimensional net demand surfaces at node  $n$  using a one-dimensional composite demand curve. The CDDP algorithm would then proceed to pass these demand surfaces up to node  $n - 1$ . The day and night marginal water value surfaces would first be adjusted by the non-consumptive demands between the nodes  $n - 1$  and  $n$ . Then the algorithm processes the time-of-use consumptive/distributary demands (if any) at node  $n - 1$ . With that, the procedure now has the two marginal water value surfaces (probably different to each other) for

the two sub-period nodes ( $n - 1$ ). Unlike in the situation for node  $n$ , the procedure now has to deal with two different marginal water value *surfaces* rather than a single 1-D composite demand *curve* to compute the sub-period flow transfers if there is an intermediate storage. Therefore the upstream processing (beyond node  $n$ ) is likely to be computationally complicated and expensive because the CDDP procedure has to deal with a series of 2-D surfaces for each sub-period node on its way to the reservoir node.

In summary, these two dimensional constructions seem overly complex when the outcome sought is a one-dimensional demand curve for release from the reservoir. The time-of-use CDDP approach could require additional computational resources with the power of the number of sub-periods and nodes to produce large amounts of useful (or sometimes redundant) hypothetical nodal solutions regarding optimal trade-offs within the system. The value of such detailed insights to the final outcome appears questionable, and so the CDDP approach does not appear to be tractable or useful in this circumstance.

A parametric LP can be solved without having to deal with multi-dimensional curves as in the above CDDP problem. The computational effort is restricted as there is just a single parameter to adjust. Thus, a parametric LP could be a suitable candidate to solve this problem efficiently to produce the required single dimensional demand curve, as explained by Read and George (1998). Thus, the cyclic sub-period approach could be useful for an LP<sup>144</sup>.

#### **8.4. Chapter Conclusions**

The first part of this chapter, formulated the release delay and storage transfer problem using a deterministic LP model assuming the run-of-river type hydropower plants with relatively small and temporary storage facilities. An intermediate storage stores water for the next sub-period use but not beyond that. The LP deals with time-of-use demands and release delays. A cyclic sub-period LP model seems practically appealing over the sequential sub-period LP model. Then we develop some CDDP computational routines to address the intermediate storage and release delay issues. Unfortunately, these developments were ultimately unsuccessful. Main concern was the dimensionality problem of the sub-period solutions and subsequently, the amount of computational resource usage compared with

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<sup>144</sup> On the other hand, the modeller could increase the number of sub-periods (e.g., hourly, 6 hourly) within a short planning horizon (e.g., two days, one week) and then use the results to construct *dcrs* for the CDDP model.

other convex optimization techniques. However, the time-of-use CDDP development does provide some insights into the structure of solutions in such a model.

# 9.

## Conclusions and Perspectives

### 9.1. Introduction

In this thesis we investigate market-based mechanisms to allocate storable surface water among competing users in a single or two-reservoir mixed-use catchment. Here, we outline a summary of our research, following the order of the thesis chapters and highlighting the implications of key developments, possible model extensions, and areas for further research.

### 9.2. LP/SLP-based Water Market Clearing Models

Chapter 3 developed a market clearing formulation to manage hydrology dependent surface water supplies, where consumptive and/or non-consumptive use occurs in a network, with storage. Participants bid, presumably reflecting their marginal use values, are assumed to be cleared by a benefit-maximizing optimization, such as Linear Programming (LP) or stochastic Linear Programming (SLP). With an SLP, one could make the system modelling as complex as desired, and could also make participant bids conditional on a spectrum of hydrology (e.g. reservoir inflow with rainfall events in the catchment, temperature), and/or economic factors.

Section 3.3 lays out a deterministic multi-period LP primal/dual centrally coordinated water market clearing model. This helps us to present a price analysis and discuss issues such as allocating costs and payments associated with shared resources and impacts of uncertainty. The aggregated net demand curve reflects the maximum quantity that each participant is willing to supply or consume, at a particular marginal cost/benefit. In Section 3.5, we were able to formulate centrally coordinated market



model (primal and dual) that clears water under uncertainty, based on marginal water values expressed by participants.

Using the price analysis in Section 3.3.3, we were able to show how the resource constraints included in the LP/SLP models affect market prices. The above creates both locational and temporal price differences, and causes the market to accumulate a “settlement surplus” of rents associated with resource constraints such as storage bounds, arc flow bounds and inflow. Hence, the market design should include mechanisms to deal with the prices associated with these constraints.

Finally, Chapter 3 used a Shapley value-based method to apportion infra-marginal benefits between multiple non-consumptive users on the same arc in the water market context. We were able to show that the method minimises incentives for some users to take advantage of their physical positioning in the network relative to users who are prepared to pay more to increase flows. But, detailed investigation of gaming issues lies outside the scope of this thesis.

### **9.3. Water Hedging Instruments**

Chapter 4 focuses on market arrangements to manage surface water supplies, where consumptive and/or non-consumptive use occurs in a network, with storage under uncertain inflows referring to the dual LP/SLP models formulated in Chapter 3. Constraints and bounds create price differences between locations, and time periods, and cause the market to accumulate a “settlement surplus” of rents associated with resource constraints. Chapter 4 then draws on the Financial Transmission Right (FTR) concepts developed for electricity markets to outline a general structure of financial hedging instruments. To the best of our knowledge, this kind of hedging has not previously been studied in the water market literature. We describe a range of financial products, based on a typical spot physical water market clearing, that hedgers would be free to trade in secondary markets or in an integrated market. A specific hedging instrument is developed for each component in the reservoir system, and we examine the extent to which these financial instruments can be supported by the revenue collected by the market manager as “settlement surplus”, as FTRs are supported in electricity markets.

First, a Financial Inflow Right (FIR) is priced as a Financial Water Right, which is a generic kind of market trading instrument. An FIR gives a right to a proportion of future inflows and this leaves the

holder facing the volume uncertainty. An FIR provides the holder the rights to water expected to flow into a storage reservoir. Fixed volume FIRs expose the SO if the expected inflows do not realise. An index-based FIR provides the holder the right to receive a volume of inflows (or FWRs), with that volume being proportional to the inflow received. So the holder faces the volume risks herself, and the SO does not. The participants would be required to hold FIRs together with the other system capacity rights such as FSRs and ACRs to meet their trading requirements.

Second, a Financial Storage Right (FSR)/Financial Storage Option (FSO) can be used to hedge against temporal price risks associated with the supply system constraint rents. Financial Storage Rights (FSRs) refund inter-temporal storage marginal price differences and could be in the form of an obligation-inclusive right or an option.

The obligation inclusive FSRs would protect participants against exposure to supply system rent uncertainty. Deterministic FSRs could be funded by storage capacity congestion rents. In the case of stochastic FSRs, we were only able to show revenue adequacy in expectation. For stochastic FSRs, though, the expected future revenue adequacy is only met for the conditional expected residual FSR pay-out requirement, over the future market horizon<sup>145</sup>. Moreover, it is possible to issue fresh FSRs, at any given time, up to the unsubscribed reservoir capacity for the any future period in the remaining market horizon. If one could write off/up losses/gains on existing FSRs, expected future revenue adequacy is met for the portfolio of new and residual FSRs. Theoretically, this implies that the SO has the same assurance of expected future revenue adequacy for its portfolio of new and residual FSRs, as if she was starting fresh. In reality, though, the SO has less opportunity act in such a purely rational manner. Instead, the FSR issuance policy may be significantly impacted by the gains or losses accumulated to date. But, detail analysis of this issue lies outside the scope of Section 4.3.

Fixed volume FSRs proposed in Chapter 4 are fundamentally incompatible with indexed based FIRs. Unfortunately, a combination of the two will not fully insulate holders from the impact of inflow uncertainties. Section 4.3.3 quotes Read (2016), who suggests that this is only what we should expect, because the owner of physical storage capacity could not expect any greater certainty than that provided

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<sup>145</sup> Where the conditional expectations of both rents and pay-outs are taken at the same event node.

by these hedging instruments. “True” marginal water value is not actually known at the time these FSRs are settled. Read proposes the “retrospective” storage rights (RSRs) to cover the implications of the incompatibility between FIRs and FSRs. The RSR valuation and settlement can be deferred until the end of the horizon, when all uncertainty has been resolved. As a result, SO would not be exposed to risks. This implies that the actual water value at any node along that path will also be known, depending on whether storage actually did reach its bounds.

Section 4.3.3.2 also explores the concept of FSR-option contracts, representing a right to store water, with no obligation to do so. Again, this is more akin to the rights implied by ownership of physical storage capacity. But, detailed investigation of such concepts lies outside the scope of this thesis.

Finally, like in the case of supply system rents, the SO collects transmission/distribution system rents associated with arc capacity constraints, in the settlement surplus. Section 4.4 focuses on distribution system rents and rights. Instead of using node-to-node FDRs similar to FTRs, this section proposes Arc Capacity Rights (ACRs). An ACR could be defined as analogues to the Flow Gate Right (FGRs) in electricity markets. ACRs could be applied to hedge against the flow congestion price risks, for an arc in the network.

In the absence of loop flow problems, ACRs are equivalent to node-to-node rights between adjacent nodes and a series of ACRs could construct a “node-to-node” right. Section 4.4 discussed the implications of ACRs in the context of three scenarios.

- Consumptive benefits only scenario: The settlement surplus can only fund node-to-node ACRs up to the minimum of the arc capacities, in the relevant direction. The negative rents are associated with minimum flow requirements in the forward direction.
- Non-consumptive costs and consumptive benefits scenario: The ACR hedges against both the capacity constraint rent risk and the variation in the marginal non-consumptive cost. In general, the revenue adequacy is met because it is unlikely to have any “environmental” flow requirement in the pumped channel.
- Non-consumptive benefits and consumptive benefits scenario: The revenue adequacy is met provided that the SO has not issued too many ACRs. In both cases, the revenue adequacy may be improved by setting the strike price to a higher value but, it cannot be guaranteed for

contract volumes outside the physical flow range without knowing the offer prices. Section 4.4.3 also recommends the possibility of using put/non-standard exotic option ACRs to handle situations such as minimum in-stream flows, linked to both electricity and water prices, perhaps packaged into obligation-inclusive ACR rights.

Lastly, while Chapter 4 considered a mix of rights that could be created and traded independently, they could also be combined to form “slice of system” rights that give proportional shares of inflow, storage, and release capacity, allowing development of a “swing option” based virtual reservoir management approach, as described by Barroso et al., (2012). This method creates contractual mechanisms and then improves the level of competition when the incumbent parties will be reluctant to accept the solutions produced by a central optimization.

The contractual mechanisms developed in this chapter deserve consideration by policy makers as these mechanisms provide a wide range of choices to enhance the benefits of competition in multi-used hydro catchments.

#### **9.4. Intra-period Single Reservoir Nodal Market Clearing Models**

The contribution of Chapter 5 is the development of new intra-period market clearing pre-computation models for allocating water among consumptive and non-consumptive uses located over a tree structured water network. Unlike the previous CDDP developments reported in the literature, the presented intra-period CDDP procedure determines the optimal solution by re-ordering and/or adjusting marginal benefits at every node and arc across the network. This moves the demand curves towards the reservoir and finally constructs the demand curve for release (*dcr*) for a single reservoir multi-nodal mixed-use catchment.

The nodal CDDP developed generates a relatively large amount of market information at user, node and arc levels that can be used to backtrack the entire process (if required) to investigate price changes across space and time from the point of market clearing. The strength of this approach is the ability to incorporate the underlying complex reservoir network system components into the optimization model,

while preserving the simplicity of the market clearing algorithm. So market models could be implemented in any single reservoir multi-use catchment<sup>146</sup>.

The intra-period CDDP pre-computes the whole intra-period *dcr*, for use in higher level optimisation and/or simulation models, unlike an SLP which only produces a solution for the current start point. It decomposes the optimization into a sequence of single period trade-offs between the benefits of immediate release and then to be used in the stochastic CDDP to determine the expected benefits of storage. So it could be easily plugged into the single reservoir stochastic CDDP model, to construct the beginning-of-period *dcs* for a single reservoir network system, under different release policy assumptions.

In combination, the intra-period and inter-period models provide a set of market mechanisms to trade storable surface water in the present, through a spot market, and also to determine how much to store for the future, in a stochastic environment. By way of contrast, current “smart” markets, e.g. for electricity, are typically cleared using deterministic optimization models (e.g., Linear Programming (LP)). Long-term inter-temporal storage and inflow uncertainty are typically ignored because there is no inter-temporal storage within the market infrastructure. But water systems are different, in this respect, and participants are likely to want at least indicative future water allocations to be produced by inter-temporal market clearing models. In that context dealing with uncertainty becomes a major issue for market clearing. That is, this thesis has chosen the storage to be included within the market<sup>147</sup>.

Finally, in Section 5.7, we were able to deploy the intra-period CDDP procedure to iteratively estimate and adjust the spatial return flow contributions to the downstream tributaries in the current period depending on the upstream consumptive and distributary off-takes.

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<sup>146</sup> As discussed in Chapter 3, the intra-period CDDP optimises a single reservoir water network and this can be used for clearing a market generating a large amount of useful market information. This produces all of the information required in the inter-period CDDP sub-problems without re-optimising or parametric programming of the CDDP applied over space rather than time. This motivated us to develop intra-period CDDP procedure even though an LP-based method can easily model multiple reservoirs and a non-tree network.

<sup>147</sup> Instead, one could create a deterministic water market model with participants owned and privately managed storages.

### **9.5. Single Reservoir Nodal CDDP Model for Environmental Applications**

Chapter 6 shows how to adapt the intra-period multi-node CDDP algorithm, using node/arc level manipulations, to address uses with water-mixing requirements.

If water is returned to the river after being used for cooling a thermal power plant, cooling and temperature control flow demands may be represented as parallel arcs assuming that an upstream flow unit can be diverted to any arc of our choice (e.g., controlled splitting). But other types of flow splitting methods such as constant, and variable are discussed.

In some cases, recreational, environmental and power generation, benefits may all arise simultaneously from the same flow. We observe that the controlled flow splitting method could be applied in a variety of situations in which flow mixing is important. When the flow is split in a constant ratio, where the splitting ratio is defined as the probability, each tiny increment would flow one way or the other. The resulting combined net-demand curve is found by vertically adding adjusted net-demand curves for the parallel arcs flows. The splitting ratio could change based on the upstream flow level. So the above method can be extended for higher number of splits.

The novelty here lies mainly in the application of an offer based flow splitting method to the issue of diverting environmental flows in a water market context. Better estimated offer curves, or well calibrated splitting devices, could give the policy maker precise control, over how the environmental flows are diverted. The offer based flow splitting technique could be applied in similar situations such as dilution of polluted water flows. We were able to demonstrate how net benefits can be increased by accounting for the demand curve for cooling water, across all hydrology conditions and time periods.

### **9.6. Two-reservoir CDDP Model**

In Chapter 7 the intra/inter period CDDP framework is extended to assess the marginal value of water stored in two inter-connected reservoirs in a mixed-use catchment. As before, optimization of the jointly operated two-reservoir system is decomposed into a set of sub-problems in order to study the nature of demand surfaces underlying each sub-problem.

- A “low level” intra-period CDDP constructs the demand surface for “transfer” on the inter-reservoir chain, which is constructed in the preceding step by reducing the sub-trees leading from the inter-reservoir nodes applying the single reservoir nodal CDDP developed in Chapter 5.

- Then a “higher level” (inter-period) CDDP could determine the trade-off between future storage demands, the current period (net) releases to the inter-reservoir chain, and other direct sub-trees.

Firstly, the inter-reservoir chain sub-problem seeks to maximise the inter-reservoir chain benefits by maximising benefits strictly of the set of nodes and arcs that form the chain between the two reservoirs including the reservoir nodes and accounting for benefits from the subtrees connected to nodes but not those connected to the reservoirs. The scale of this problem depends on the number of inter-reservoir nodes and the number of sub-trees leading from them.

Secondly, the inter-reservoir chain transfer problem provides the maximum possible inter-reservoir chain benefit given release from the upstream reservoir, and outflow into the downstream reservoir. The scale of this problem can significantly increase if a large number of inter-reservoir consumptive demands are to be processed. A CDDP solution method constructs the demand surface for transfer in “strips”, where each strip corresponds to either a critical downstream marginal water value that induces a downstream versus inter-reservoir demand swap, or a critical marginal price difference between the two reservoirs.

Finally, the notion of CDDP curve adding could be performed implicitly via a contour based approach that produces the demand surface for storages.

Price-quantity manipulation in four dimensional spaces sometimes makes it challenging to illustrate how the above two methods form respective demand surfaces. Having said that, the above concepts could be better understood as “strip” based demand surfaces for transfer, and as projected ‘manifolds’ of the demand surfaces on to different planes. The notion of creating the demand surface for transfer deserves consideration as providing a greater insight to the intra-period two-reservoir optimization problem. We note that a contour based demand curve adding method has been developed, and could be improved further, to allow some higher level understanding of the two-reservoir trade-off future storage demands, the current period (net) releases to the inter-reservoir chain, and other direct sub-trees.

On the other hand, to achieve the most efficient implementation of two-reservoir CDDP, issues such as generating non-critical contours, consistently tracking guidelines for contours containing areas, degenerate solutions, keeping track of accurate demand surface approximations, and stochastic implementation require further research.

### **9.7. Multi-dimensional Intra-period CDDP Extensions**

Chapter 8 explores multi-dimensional CDDP extensions to deal with intra-period time-of-use bidding, and the modelling of transportation/release delays in a water market setting, and temporary, relatively small, intermediate storages.

However, developing a CDDP beyond the leaf node is not practical possible due to the dimensionality problem of the sub-period solutions. We have continued to develop the sub-period CDDP models despite being unsuccessful progressing beyond the leaf node due to the dimensionality issues of the sub-period solutions, as they provide useful insights about time of use of water to modelers and river-chain operators.

By way of contrast, an equivalent LP formulation could solve the short-term nodal chain optimization problem with transportation delays, without having to deal with dimensionality issues. We were able to develop the cyclic sub-period LP to model the time-of-use demands and release delays.

The release delay CDDP procedure creates time-of-use demand curves for all nodes, accounting for flow delays but assuming “run-of-river” operations, with no intermediate storage capacity at the nodes. The release delay CDDP procedure builds up a multi-dimensional nodal marginal value surface by considering incremental sub-period flow arrivals under the intermediate storage capacity constraints. We also construct a time-of-use demand surface.

Our conclusion is that the developments in this chapter have failed to show a significant promise, in terms of efficiently solving the time-of-use release and storage delay problems and it consumes considerable computational resources in implementation. Therefore, it is not recommended that this method is pursued further due to the growing complexity of multi-dimensional time-of-use models, not to mention the large amount of information (sometimes redundant) it generates.

### **9.8. General Conclusions and Research Directions**

One major contribution of this thesis is the development of a range of financial instruments that can be used to hedge risk due to hydrological or economic uncertainty, and could either be traded in a centrally coordinated water market, or in secondary markets. We have also developed intra-period nodal CDDP models in this thesis to perform offer-based constrained water allocation among price-taking consumptive, non-consumptive and environmental users located at the nodes and arcs across a tree



structured network, in a single reservoir catchment. And, we have shown how to extend that nodal CDDP to construct the demand surfaces for catchments with two reservoirs.

Further extensions have been studied, but with only mixed success. Dimensionality issues preclude development in some directions, and the most efficient implementation approach is unclear in others. We are motivated, though, to explore avenues to use the stochastic CDDP to model/simulate how a market responds to stochastic inflows influenced by el niño-la niña weather patterns. This might be modelled, perhaps, by assuming a hidden Markov chain. We also note that, while the current model includes hydro generators among its non-consumptive users, the electricity market price is still treated as external. A fully integrated electricity-water market model with a constrained river network model could be usefully investigated in the future.

Finally, our study also suggests broad research directions such as market power issues in a water contract market, issues of auction design, trading water under uncertainty, locational and temporal management of storages, infrastructure investment, and asset ownership issues. Future research might investigate this by using both standard oligopoly models and simulating the market assuming self-learning users. Such models could, perhaps, be based on the works of Newbery (2012), and Krause & Andersson (2006). An agent based method could be used to simulate market behaviour using our proposed models by iteratively modelling individual bids/bid adjustments. Such methods could study issues such as gaming, and risk aversion in spot/forward and derivative markets. Further, this could extend to investigate issues such as asset ownership regimes (private, public, and joint) and ownership structures (outright, lease, and share).



## References

- Afualo, V. I., & McMillan, J. (1997). *Auctions of rights to public property* (Vol. 97): Graduate School of International Relations and Pacific Studies, University of California, San Diego.
- Alvey, T., Goodwin, D., Ma, X. W., Streiffert, D., & Sun, D. (1998). A security-constrained bid-clearing system for the New Zealand Wholesale Electricity Market. *IEEE Transactions on Power Systems*, 13(2), 340-346.
- Ambec, S., & Doucet, J. A. (2003). Decentralizing hydro power production. *Canadian Journal of Economics/Revue canadienne d'économique*, 36(3), 587-607.
- Anandalingam, G., & Nam, K. (1997). Conflict and cooperation in designing international telecommunication networks. *Journal of the Operational Research Society*, 48(6), 600-611.
- Anderson, E. J., & Philpott, A. B. (2002). Using supply functions for offering generation into an electricity market. *Operations Research*, 50(3), 477-489.
- Anderson, T. L., & Snyder, P. (1997). *Water markets: Priming the invisible pump*: Cato Institute.
- Archer, A., Feigenbaum, J., Krishnamurthy, A., Sami, R., & Shenker, S. (2004). Approximation and collusion in multicast cost sharing. *Games and Economic Behavior*, 47(1), 36-71.
- Archibald, T., McKinnon, K., & Thomas, L. (1997). An aggregate stochastic dynamic programming model of multireservoir systems. *WATER RESOURCES RESEARCH*, 33(2), 333-340.
- Archibald, T., McKinnon, K., & Thomas, L. (2006a). Modeling the operation of multireservoir systems using decomposition and stochastic dynamic programming. *Naval Research Logistics (NRL)*, 53(3), 217-225.
- Archibald, T., McKinnon, K., & Thomas, L. (2006b). Modelling the operation of multireservoir systems using decomposition and stochastic dynamic programming. *Naval Research Logistics*, 53(3), 217-225.
- Archibald, T. W., McKinnon, K. I. M., & Thomas, L. C. (1995). On the generation of markov decision processes. *Journal of the Operational Research Society*, 354-361.
- Armstrong, M., Cowan, S., & Vickers, J. (1994). *Regulatory reform: economic analysis and British experience* (Vol. 20): MIT press.

- Arriaza, M., Gómez-Limón, J. A., & Upton, M. (2002). Local water markets for irrigation in southern Spain: a multicriteria approach. *Australian Journal of Agricultural and Resource Economics*, 46(1), 21-43.
- Baker, K. A. (2014). Coordination of Resources Across Areas for the Integration of Renewable Generation: Operation, Sizing, and Siting of Storage Devices.
- Baldick, R. (1995). The generalized unit commitment problem. *IEEE Transactions on Power Systems*, 10(CONF-940702--).
- Bannister, C. H., & Kaye, R. J. (1991). A rapid method for optimization of linear systems with storage. *Operations Research*, 39(2), 220-232.
- Barros, M. T. L., Zambon, R. C., Barbosa, P. S. F., & Yeh, W. (2008). Planning and operation of large-scale water distribution systems with preemptive priorities. *Journal of Water Resources Planning and Management*.
- Barroso, L., Granville, S., Pereira, M., Read, E., & Jackson, P. Virtual Models for Reservoir Management in Competitive Markets: Technical Report, available at: [www.psr-inc.com](http://www.psr-inc.com).
- Barroso, L. A., Granville, S., Jackson, P. R., Pereira, M. V. F., & Read, E. G. (2012). *Overview of Virtual Models for Reservoir Management in Competitive Markets*. Paper presented at the Proceedings 4th IEEE/Cigré International Workshop on Hydro Scheduling in Competitive Markets, Bergen, Norway.
- Barroso, L. A., Granville, S., Trinkenreich, J., Pereira, M. V. F., & Lino, P. (2003). *Managing hydrological risks in hydro-based portfolios*. Paper presented at the Power Engineering Society General Meeting, 2003, IEEE.
- Barroso, L. A., Porrua, F., Thome, L. M., & Pereira, M. V. F. (2007). Planning for big things in Brazil. *IEEE Power and Energy Magazine*, 5(5), 54-63.
- Battle, C., Barroso, L. A., & Uribe, M. (2009). Long-term energy auctions design criteria and flaws in Latin America. The case of Peru: IIT Working Paper, Instituto de Investigación Tecnológica, Universidad Pontificia Comillas, Madrid.
- Battle, C., Barroso, L. A. N., & Pérez-Arriaga, I. J. (2010). The changing role of the State in the expansion of electricity supply in Latin America. *ENERGY POLICY*, 38(11), 7152-7160.

- Batstone, S., & Scott, T. (1998). *Long-term contracting in a deregulated Electricity Industry: Simulation results from a Hydro Management Model*. Paper presented at the Proceedings of the 33rd Annual Conference, Mason, A., Henderson, S and Philpott, A., eds., ORSNZ.
- Bauer, C. J. (2004). Results of Chilean water markets: Empirical research since 1990. *WATER RESOURCES RESEARCH*, 40(9).
- Bauer, C. J. (2015). Water conflicts and entrenched governance problems in Chile's market model. *Water Alternatives*, 8(2).
- Bear, J., Levin, O., & Buras, N. (1964). Optimal utilization of aquifers as elements of water-resource systems *Hydraulic Laboratory PN: TECHNION*.
- Becker, N. (1995). Value of moving from central planning to a market system: lessons from the Israeli water sector. *AGRICULTURAL ECONOMICS*, 12(1), 11-21.
- Becker, N., & Easter, K. W. (1997). Water diversion from the Great Lakes: is a cooperative approach possible? *International Journal of Water Resources Development*, 13(1), 53-66.
- Bellman, R. (1957). A Markovian decision process: DTIC Document.
- Berger, T., Birner, R., McCarthy, N., Díaz, J., & Wittmer, H. (2007). Capturing the complexity of water uses and water users within a multi-agent framework. *Water Resources Management*, 21(1), 129-148.
- Berry, C. A., Hobbs, B. F., Meroney, W. A., O'Neill, R. P., & Stewart, W. R. (1999). Understanding how market power can arise in network competition: a game theoretic approach. *Utilities Policy*, 8(3), 139-158.
- Bertram, G. (2006). Restructuring the New Zealand electricity sector 1984-2005. *Electricity Market Reform: An International Perspective*, Elsevier, 203-234.
- Bertsekas, D. P., & Tseng, P. (1994). Partial Proximal Minimization Algorithms for Convex Pprogramming. *SIAM Journal on Optimization*, 4(3), 551-572.
- Bichler, M., Gupta, A., & Ketter, W. (2010). Research commentary-designing smart markets. *Information Systems Research*, 21(4), 688-699.
- Birol, E., Karousakis, K., & Koundouri, P. (2006). Using economic valuation techniques to inform water resources management: A survey and critical appraisal of available techniques and an application. *Science of the total environment*, 365(1), 105-122.

- Bjornlund, H. (2004). Formal and informal water markets: Drivers of sustainable rural communities? *WATER RESOURCES RESEARCH*, 40(9).
- Bjornlund, H., & McKay, J. (1998). Factors affecting water prices in a rural water market: a South Australian experience. *WATER RESOURCES RESEARCH*, 34(6), 1563-1570.
- Bjornlund, H., & McKay, J. (2002). Aspects of water markets for developing countries: experiences from Australia, Chile, and the US. *Environment and Development Economics*, 7(04), 769-795.
- Bjornlund, H., Nicol, L., & Klein, K. K. (2007). Challenges in implementing economic instruments to manage irrigation water on farms in southern Alberta. *Agricultural water management*, 92(3), 131-141.
- Booker, J. F., & Young, R. A. (1994). Modeling intrastate and interstate markets for Colorado River water resources. *Journal of Environmental Economics and Management*, 26(1), 66-87.
- Boshier, J. F., Manning, G. B., & Read, E. G. (1983). Scheduling releases from New Zealand's hydro reservoirs.
- Bower, J., & Bunn, D. W. (2000). Model-based comparisons of pool and bilateral markets for electricity. *The Energy Journal*, 1-29.
- Bower, J., Bunn, D. W., & Wattendrup, C. (2001). A model-based analysis of strategic consolidation in the German electricity industry. *ENERGY POLICY*, 29(12), 987-1005.
- BPA. (1999). Slice of the System Product: Product Description.
- Brennan, D. (2008). Missing markets for storage and the potential economic cost of expanding the spatial scope of water trade. *Australian Journal of Agricultural and Resource Economics*, 52(4), 471-485.
- Brooks, R. E. (2003). Optimizing complex natural gas models.
- Brown, C., & Carriquiry, M. (2007). Managing hydroclimatological risk to water supply with option contracts and reservoir index insurance. *WATER RESOURCES RESEARCH*, 43(11).
- Bunn, D. W. (2000). Forecasting loads and prices in competitive power markets: the technology of power system competition. *Proceedings of the IEEE*, 88(2), 163-169.
- Bunn, D. W., & Oliveira, F. S. (2001). Agent-based simulation-an application to the new electricity trading arrangements of England and Wales. *IEEE transactions on Evolutionary Computation*, 5(5), 493-503.

- Burke, S. M., Adams, R. M., & Wallender, W. W. (2004). Water banks and environmental water demands: case of the Klamath project. *WATER RESOURCES RESEARCH*, 40(9).
- Bushnell, J. (1998). *Water and power: Hydroelectric resources in the era of competition in the western US*.
- Bushnell, J. B., & Stoft, S. E. (1996a). Electric grid investment under a contract network regime. *Journal of Regulatory Economics*, 10(1), 61-79.
- Bushnell, J. B., & Stoft, S. E. (1996b). Grid investment: Can a market do the job? *The Electricity Journal*, 9(1), 74-79.
- Bushnell, J. B., & Stoft, S. E. (1997). Improving private incentives for electric grid investment. *Resource and Energy Economics*, 19(1), 85-108.
- Calatrava, J., & Garrido, A. (2006). Difficulties in adopting formal water trading rules within users' associations. *Journal of Economic Issues*, 40(1), 27-44.
- Carbone, R., & Sweigart, J. R. (1976). Equity and selective pollution abatement procedures. *Management Science*, 23(4), 361-370.
- Casseboom, P. D., & Read, E. G. (1987). Dual dynamic programming for coal stockpiling. *Proc. Operations Research Society of New Zealand*, 15-18.
- Chang, G. W., Aganagic, M., Waight, J. G., Medina, J., Burton, T., Reeves, S., & Christoforidis, M. (2001). Experiences with mixed integer linear programming based approaches on short-term hydro scheduling. *Power Systems, IEEE Transactions on*, 16(4), 743-749.
- Chao, H., & Peck, S. (1996). A market mechanism for electric power transmission. *Journal of Regulatory Economics*, 10(1), 25-59.
- Chao, H., & Peck, S. (1997). An institutional design for an electricity contract market with central dispatch. *The Energy Journal*, 85-110.
- Chao, H., & Peck, S. (1998). An institutional design for an electricity contract market with central dispatch *Designing Competitive Electricity Markets* (pp. 185-207): Springer.
- Chao, H., Peck, S., Oren, S. S., & Wilson, R. (2000). Flow-based transmission rights and congestion management. *The Electricity Journal*, 13(8), 38-58.

- Chapman, D. V., & Organization, W. H. (1996). Water quality assessments: a guide to the use of biota, sediments and water in environmental monitoring.
- Chattopadhyay, D. (1995). An energy brokerage system with emission trading and allocation of cost savings. *Power Systems, IEEE Transactions on*, 10(4), 1939-1945.
- Cheng, W., Hsu, N. S., Cheng, W., & Yeh, W. (2009). A flow path model for regional water distribution optimization. *WATER RESOURCES RESEARCH*, 45(9).
- Coase, R. H. (1960). The problem of social cost *Classic Papers in Natural Resource Economics* (pp. 87-137): Springer.
- Colby, B. G. (1989). Estimating the value of water in alternative uses. *Nat. Resources J.*, 29, 511.
- Crampes, C., & Moreaux, M. (1999). Competition in Power Generation with Heterogeneous Technologies.
- Culy, J. G., Read, E. G., & Wright, B. (1996). Structure and regulation of the New Zealand electricity sector *International Comparison of Electricity Regulation* (pp. 312-365): Cambridge University Press.
- Daellenbach, H. G., & Read, E. G. (1976). *Survey on optimization for the long-term scheduling of hydro-thermal power systems*. Paper presented at the ORSNZ Conference, New Zealand.
- Dantzig, G. B. (1955). Linear programming under uncertainty. *Management Science*, 1(3-4), 197-206.
- Day, C. J., Hobbs, B. F., & Pang, J. (2002). Oligopolistic competition in power networks: a conjectured supply function approach. *IEEE Transactions on Power Systems*, 17(3), 597-607.
- de Maere d'Aertrycke, G., Shapiro, A., & Smeers, Y. (2013). Risk exposure and Lagrange multipliers of nonanticipativity constraints in multistage stochastic problems. *Mathematical Methods of Operations Research*, 77(3), 393-405.
- De Matos, V. L., Philpott, A. B., & Finardi, E. C. (2015). Improving the performance of stochastic dual dynamic programming. *Journal of Computational and Applied Mathematics*, 290, 196-208.
- De Wolf, D., & Smeers, Y. (2000). The gas transmission problem solved by an extension of the simplex algorithm. *Management Science*, 46(11), 1454-1465.
- Dempster, M. A. H. (1988). On stochastic programming ii: dynamic problems under risk\*. *Stochastics: An International Journal of Probability and Stochastic Processes*, 25(1), 15-42.
- Deng, S. J., & Oren, S. S. (2006). Electricity derivatives and risk management. *Energy*, 31(6), 940-953.



- Dentcheva, D., Ruszczyński, A., & Shapiro, A. (2009). Lectures on stochastic programming. *SIAM, Philadelphia*, 10.
- Dettmer, R. (2002). Living with NETA [New Electricity Trading Arrangements]. *IEE Review*, 48(4), 32-36.
- Dinar, A., Howitt, R. E., Rassenti, S. J., & Smith, V. L. (1998). Development of water markets using experimental economics *Markets for Water* (pp. 259-275): Springer.
- Dinar, A., Ratner, A., & Yaron, D. (1992). Evaluating cooperative game theory in water resources. *Theory and decision*, 32(1), 1-20.
- Dinar, A., Rosegrant, M. W., & Meinzen-Dick, R. (2004). *Water Allocation Mechanisms: Principles and Examples*.
- Dinar, A., & Subramanian, A. (1997). *Water pricing experiences: An international perspective*: The World Bank.
- Drayton-Bright, G. R. (1997). Coordinating energy and reserves in a wholesale electricity market.
- Drayton, G. R., & Read, E. G. (1996). Using LP to form a market for spinning reserve. *Proceedings of the Operational Research Society of New Zealand*, 119-124.
- Dupačová, J., Gaivoronski, A., Kos, Z., & Szantai, T. (1991). Stochastic programming in water management: A case study and a comparison of solution techniques. *EUROPEAN JOURNAL OF OPERATIONAL RESEARCH*, 52(1), 28-44.
- Dye, S., Read, E. G., Read, R. A., & Starkey, S. R. (2012). *Easy Implementations of Generalised Stochastic CDDP Models for Market Simulation Studies* Paper presented at the 4th IEEE and Cigré International Workshop on Hydro Scheduling in Competitive Markets, Bergen, Norway.
- Easter, K. W., Rosegrant, M. W., & Dinar, A. (1998). *Markets for water: Potential and performance* (Vol. 15): Springer Science & Business Media.
- Edwards, B. K., Flaim, S. J., & Howitt, R. E. (1999). Optimal provision of hydroelectric power under environmental and regulatory constraints. *Land Economics*, 267-283.
- Eisner, M. J., & Olsen, P. (1975). Duality for Stochastic Programming Interpreted as LP in  $L_p$ -Space. *SIAM Journal on Applied Mathematics*, 28(4), 779-792.

- Fabra, N., Von der Fehr, N., & Harbord, D. (2002). Modeling electricity auctions. *The Electricity Journal*, 15(7), 72-81.
- Faria, E., Barroso, L. A., Kelman, R., Granville, S., & Pereira, M. V. F. (2009). Allocation of firm-energy rights among hydro plants: An Aumann–Shapley approach. *Power Systems, IEEE Transactions on*, 24(2), 541-551.
- Fleten, S.-E. (2000). Portfolio management emphasizing electricity market applications. A stochastic programming approach.
- Forsund, F. R. (2007). *Hydropower economics* (Vol. 112): Springer Science & Business Media.
- Gabriel, S. A., Manik, J., & Vikas, S. (2003). Computational experience with a large-scale, multi-period, spatial equilibrium model of the North American natural gas system. *Networks and Spatial Economics*, 3(2), 97-122.
- Garcia, A., Reitzes, J. D., & Stacchetti, E. (2001). Strategic pricing when electricity is storable. *Journal of Regulatory Economics*, 20(3), 223-247.
- Garman, M. B., & Barbieri, A. (1997). Ups and downs of swing. *Energy Power Risk Management*, 2(1).
- Garrick, D., Siebentritt, M. A., Aylward, B., Bauer, C. J., & Purkey, A. (2009). Water markets and freshwater ecosystem services: Policy reform and implementation in the Columbia and Murray-Darling Basins. *Ecological Economics*, 69(2), 366-379. doi: 10.1016/j.ecolecon.2009.08.004
- Garrido, A. (1998). Economics of water allocation and the feasibility of water markets in agriculture. *Sustainable Management of Water in Agriculture: Issues and Policies*, 57-80.
- Garrido, A. (2000). A mathematical programming model applied to the study of water markets within the Spanish agricultural sector. *Annals of Operations Research*, 94(1), 105-123. doi: 10.1023/a:1018965016134
- Garrido, A. (2007). Water markets design and evidence from experimental economics. *Environmental and Resource Economics*, 38(3), 311-330.
- Gassmann, H. I., & Ireland, A. M. (1995). Scenario formulation in an algebraic modelling language. *Annals of Operations Research*, 59(1), 45-75.

- Georgakakos, A. P., Yao, H. M., & Yu, Y. Q. (1997). Control model for hydroelectric energy-value optimization. *JOURNAL OF WATER RESOURCES PLANNING AND MANAGEMENT-ASCE*, 123(1), 30-38. doi: 10.1061/(ASCE)0733-9496(1997)123:1(30)
- Gibbons, D. C. (1986). *The Economic Value of Water* (Washington, DC, Resources for the Future): Inc.
- Gibbons, D. C. (2013). *The economic value of water*: Routledge.
- Gilbert, R. J., & Kahn, E. P. (1996). Competition and institutional change in US electric power regulation. *International comparisons of electricity regulation*.
- Goor, Q., Kelman, R., & Tilmant, A. (2010). Optimal multipurpose-multireservoir operation model with variable productivity of hydropower plants. *Journal of Water Resources Planning and Management*, 137(3), 258-267.
- Gorenstin, B. G., Campodonico, N. M., Costa, J. P., & Pereira, M. V. F. (1991). *Stochastic optimization of a hydro-thermal system including network constraints*. Paper presented at the Power Industry Computer Application Conference, 1991. Conference Proceedings.
- Granville, S., Oliveira, G. C., Thomé, L. M., Campodónico, N., Latorre, M. L., Pereira, M. V. F., & Barroso, L. A. N. (2003). *Stochastic optimization of transmission constrained and large scale hydrothermal systems in a competitive framework*. Paper presented at the Power Engineering Society General Meeting, 2003, IEEE.
- Green, R. (1996). Increasing competition in the British electricity spot market. *The journal of Industrial economics*, 205-216.
- Green, R. J. (1996). Increasing competition in the British electricity spot market. *The journal of Industrial economics*, 205-216.
- Green, R. J., & Newbery, D. M. (1992). Competition in the British electricity spot market. *Journal of political economy*, 929-953.
- Griffin, R. C., & Characklis, G. W. (2002). Issues and trends in Texas water marketing. *Water Resources Update*, 121(1), 29-33.
- Griffin, R. C., & Hsu, S. (1993). The potential for water market efficiency when instream flows have value. *American Journal of Agricultural Economics*, 292-303.

- Hadjigeorgalis, E., & Lillywhite, J. (2004). The impact of institutional constraints on the Limarí River Valley water market. *WATER RESOURCES RESEARCH*, 40(5).
- Hall, W. A., & Buras, N. (1961). The dynamic programming approach to water-resources development. *Journal of Geophysical Research*, 66(2), 517-520.
- Hamilton, J. R., Whittlesey, N. K., & Halverson, P. (1989). Interruptible water markets in the Pacific Northwest. *American Journal of Agricultural Economics*, 63-75.
- Hanak, E. (2002). *California's Water Market, by the numbers*: Public Policy Institute of California San Francisco.
- Harou, J. J., Pulido-Velazquez, M., Rosenberg, D. E., Medellín-Azuara, J., Lund, J. R., & Howitt, R. E. (2009). Hydro-economic models: Concepts, design, applications, and future prospects. *Journal of Hydrology*, 375(3), 627-643.
- Harvey, S. M., & Hogan, W. W. (2002). Loss hedging financial transmission rights. *Center for Business and Government, Harvard University*, 15.
- Harvey, S. M., Hogan, W. W., & Pope, S. L. (1996). Transmission capacity reservations implemented through a spot market with transmission congestion contracts. *The Electricity Journal*, 9(9), 42-55.
- Hearne, R. R., & Donoso, G. (2014). Water Markets in Chile: Are They Meeting Needs? *Water markets for the 21st century* (pp. 103-126): Springer.
- Hearne, R. R., & Easter, K. W. (1995). *Water allocation and water markets: an analysis of gains-from-trade in Chile* (Vol. 315): World Bank Publications.
- Heinz, I., Pulido-Velazquez, M., Lund, J. R., & Andreu, J. (2007). Hydro-economic modeling in river basin management: implications and applications for the European water framework directive. *Water Resources Management*, 21(7), 1103-1125.
- Higle, J. L. (2005). Stochastic programming: optimization when uncertainty matters. *Cole Smith J (ed) Tutorials in operations research*, 30-53.
- Higle, J. L., & Wallace, S. W. (2003). Sensitivity analysis and uncertainty in linear programming. *Interfaces*, 33(4), 53-60.
- Hobbs, B. F. (2001). Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets. *IEEE Transactions on Power Systems*, 16(2), 194-202.

- Hobbs, B. F., & Helman, U. (2003). Complementarity-based equilibrium modeling for electric power markets.
- Hogan, W. W. (1992). Contract networks for electric power transmission. *Journal of Regulatory Economics*, 4(3), 211-242.
- Hogan, W. W. (1993). A competitive electricity market model. *Harvard Electricity Policy Group*.
- Hogan, W. W. (2000). Flowgate rights and wrongs. *John F. Kennedy School of Government, Harvard University*.
- Hogan, W. W. (2001). *Designing Market Institutions for Electric Network Systems: Reforming the Reforms in New Zealand and the US*. Paper presented at the Paper for The Utility Convention, Auckland, New Zealand.
- Hogan, W. W. (2002). Electricity market restructuring: reforms of reforms. *Journal of Regulatory Economics*, 21(1), 103-132.
- Hogan, W. W., Read, E. G., & Ring, B. J. (1996). Using mathematical programming for electricity spot pricing. *International Transactions in Operational Research*, 3(3-4), 209-221.
- Hogan, W. W., Rosellón, J., & Vogelsang, I. (2010). Toward a combined merchant-regulatory mechanism for electricity transmission expansion. *Journal of Regulatory Economics*, 38(2), 113-143.
- Hogan, W. W., & Toulouse, F. R. (2008). *Electricity market design: coordination, pricing and incentives*. Paper presented at the ERCOT energized conference.
- Hollinshead, S. P., & Lund, J. R. (2006). Optimization of environmental water purchases with uncertainty. *WATER RESOURCES RESEARCH*, 42(8).
- Hotelling, H. (1931). The economics of exhaustible resources. *The Journal of Political Economy*, 137-175.
- Howitt, R. E., Sexton, R. J., & Chatterjee, B. (1998). The Optimal Joint Provision of Water for Irrigation and Hydropower. *Journal of Environmental Economics and Management*, 36(3), 295-295.
- Høyland, K., & Wallace, S. W. (2001). Generating scenario trees for multistage decision problems. *Management Science*, 47(2), 295-307.
- Hsu, M. (1997). An introduction to the pricing of electric power transmission. *Utilities Policy*, 6(3), 257-270.

- Hsu, N. S., & Cheng, K. W. (2002). Network flow optimization model for basin-scale water supply planning. *Journal of Water Resources Planning and Management*, 128(2), 102-112.
- Hsu, N. S., Cheng, W., Cheng, W., Wei, C., & Yeh, W. (2008). Optimization and capacity expansion of a water distribution system. *Advances in Water Resources*, 31(5), 776-786.
- Hughes, N., Gibbs, C., Dahl, A., Tregeagle, D., Sanders, O., & Goesch, T. (2013). Storage rights and water allocation arrangements in the Murray–Darling basin. *Australian Bureau of Agricultural and Resource Economics and Sciences Technical Report*, 13.
- Hughes, N., & Goesch, T. (2009). *Management of Irrigation Water Storages: Carryover Rights & Capacity Sharing*.
- Hull, J. C. (2006). *Options, futures, and other derivatives*: Pearson Education India.
- I.M., S., Hoekstra, A. Y., & Savenije, H. H. G. (2000). The Water Value-Flow Concept. *Value of Water Research Report Series*(3).
- Hunt, D., Read, E.G., Jackson, P.R., Barroso, L. and Granville, S. (2012). *Tasmanian Market Reform: Commentary on Panel's Preferred Options* . Concept Consulting Report to Aurora Energy, Tasmania, Australia
- Israel, M. S., & Lund, J. R. (1999). Priority preserving unit penalties in network flow modeling. *Journal of Water Resources Planning and Management*, 125(4), 205-214.
- Jacobs, J., Freeman, G., Grygier, J., Morton, D., Schultz, G., Staschus, K., & Stedinger, J. R. (1995). SOCRATES: A system for scheduling hydroelectric generation under uncertainty. *Annals of Operations Research*, 59(1), 99-133.
- Jacoby, H., Murgai, R., & ur Rehman, S. (2001). *Monopoly power and distribution in fragmented markets: the case of groundwater* (Vol. 2628): World Bank Publications.
- Jaillet, P., Ronn, E. I., & Tompaidis, S. (2004). Valuation of commodity-based swing options. *Management Science*, 50(7), 909-921.
- Jamasb, T., & Pollitt, M. (2005). Electricity market reform in the European Union: review of progress toward liberalization & integration. *The Energy Journal*, 11-41.

- Jenkins, M. W., Lund, J. R., Howitt, R. E., Draper, A. J., Msangi, S. M., Tanaka, S. K., Ritzema, R. S., & Marques, G. F. (2004). Optimization of California's water supply system: Results and insights. *Journal of Water Resources Planning and Management*, 130(4), 271-280.
- Ji-keng, L., Yi-xin, N. I., & Wu, F. F. (2002). A SURVEY OF MARKET POWER IN RELATION WITH ELECTRICITY MARKET STRUCTURE [J]. *Power System Technology*, 11(26), 70-76.
- Johnson, S. A., Stedinger, J. R., Shoemaker, C. A., Li, Y., & Tejada-Guibert, J. (1993). Numerical solution of continuous-state dynamic programs using linear and spline interpolation. *Operations Research*, 41(3), 484-500.
- Joskow, P. L. (1985). Vertical integration and long-term contracts: The case of coal-burning electric generating plants. *Journal of Law, Economics, & Organization*, 33-80.
- Joskow, P. L. (1987). Contract duration and relationship-specific investments: Empirical evidence from coal markets. *The American Economic Review*, 168-185.
- Joskow, P. L. (2008). *Lessons Learned from the Electricity Market Liberalization*: Massachusetts Institute of Technology, Center for Energy and Environmental Policy Research.
- Joskow, P. L., & Tirole, J. (2000). Transmission rights and market power on electric power networks. *The Rand Journal of Economics*, 450-487.
- Jowett, I. G. (1997). Instream flow methods: a comparison of approaches. *Regulated Rivers: Research & Management*, 13(2), 115-127.
- Junqueira, M., Costa Jr, L., Barroso, L. A., Oliveira, G. C., Thomé, L. M., & Pereira, M. V. F. (2007). An Aumann-Shapley approach to allocate transmission service cost among network users in electricity markets. *Power Systems, IEEE Transactions on*, 22(4), 1532-1546.
- Kahn, A. E., Cramton, P. C., Porter, R. H., & Tabors, R. D. (2001). Uniform pricing or pay-as-bid pricing: a dilemma for California and beyond. *The Electricity Journal*, 14(6), 70-79.
- Kall, P., & Wallace, S. W. (1994). *Stochastic programming*. Chichester; New York: Wiley.
- Kall, P., & Wallace, S. W. (1995). *Stochastic Programming* (Wiley Interscience Series in Systems and Optimization).
- Kaminski, V. (1997). The challenge of pricing and risk managing electricity derivatives. *The US Power Market*, 3, 149-171.

- Kaminski, V., & Gibner, S. (1995). Exotic options. *Managing energy price risk*.
- Kelman, M. P. N. C. R. (1998). Long-term hydro scheduling based on stochastic models. *EPSOM*, 98, 23-25.
- Kerr, A. L. (2003). Stochastic utility maximising dynamic programming applied to medium-term reservoir management.
- Kerr, A. L., & Read, E. G. (1997). Short-term Hydro Scheduling using Integer Programming: Management and Modelling Issues. *University of Canterbury*.
- Khazaei, J., Zakeri, G., & Oren, S. S. (2013). Market clearing mechanisms under demand uncertainty: Citeseer.
- Kim, Y., Eum, H., Lee, E., & Ko, I. (2007). Optimizing operational policies of a Korean multireservoir system using sampling stochastic dynamic programming with ensemble streamflow prediction. *Journal of Water Resources Planning and Management*, 133(1), 4-14.
- Kirschen, D. S., & Strbac, G. (2004). *Fundamentals of power system economics*: John Wiley & Sons.
- Klemperer, P. (2001). Collusion and predation in auction markets. *Available at SSRN 260188*.
- Klemperer, P. D., & Meyer, M. A. (1989). Supply function equilibria in oligopoly under uncertainty. *Econometrica: Journal of the Econometric Society*, 1243-1277.
- Krause, T., & Andersson, G. (2006). Evaluating congestion management schemes in liberalized electricity markets using an agent-based simulator. In *2006 IEEE Power Engineering Society General Meeting* (pp. 8-pp). IEEE.
- Kucukmehmetoglu, M., & Guldmann, J. (2004). International water resources allocation and conflicts: the case of the Euphrates and Tigris. *Environment and Planning A*, 36(5), 783-802.
- Labadie, J. W. (2004). Optimal operation of multireservoir systems: State-of-the-art review. *Journal of Water Resources Planning and Management*, 130(2), 93-111.
- Labadie, J. W., Baldo, M. L., Israel, M. S., & Lund, J. R. (2001). Priority Preserving Unit Penalties in Network Flow Modeling. *Journal of Water Resources Planning and Management*, 127(1), 67-69.
- Lamond, B. F., & Boukhtouta, A. (1996). Optimizing Long-term Hydro-power Production Using Markov Decision Processes. *International Transactions in Operational Research*, 3(3-4), 223-241.



- Lane, M. N., & Littlechild, S. C. (1976). Weather-dependent pricing for water resources in the Texas High Plains. *WATER RESOURCES RESEARCH*, 12(4), 599-605.
- Larson, R. E., & Keckler, W. G. (1969). Applications of dynamic programming to the control of water resource systems. *Automatica*, 5(1), 15-26. doi: 10.1016/0005-1098(69)90050-8
- Lasdon, L. S. (1970). Optimization theory for large systems: *Courier Corporation*.
- Lee, J. H., & Labadie, J. W. (2007). Stochastic optimization of multireservoir systems via reinforcement learning. *WATER RESOURCES RESEARCH*, 43(11), W11408.
- Lence, B. J., Latheef, M. I., & Burn, D. H. (1992). Reservoir management and thermal power generation. *Journal of Water Resources Planning and Management*, 118(4), 388-405.
- Li, T., Shahidehpour, M., & Keyhani, A. (2004). Market power analysis in electricity markets using supply function equilibrium model. *IMA Journal of Management Mathematics*, 15(4), 339-354.
- Lino, P., Barroso, L. A. N., Pereira, M. V. F., Kelman, R., & Fampa, M. H. C. (2003). Bid-Based Dispatch of Hydrothermal Systems in Competitive Markets. *Annals of Operations Research*, 120(1), 81-97. doi: 10.1023/A:1023322328294
- Little, J. D. C. (1955). The use of storage water in a hydroelectric system. *Operations Research*, 3(2), 187-197.
- Littlechild, S. C., & Owen, G. (1973). A simple expression for the Shapley value in a special case. *Management Science*, 20(3), 370-372.
- Littlechild, S. C., & Thompson, G. F. (1977). Aircraft landing fees: a game theory approach. *The Bell Journal of Economics*, 186-204.
- Löhndorf, N., Wozabal, D., & Minner, S. (2013). Optimizing trading decisions for hydro storage systems using approximate dual dynamic programming. *Operations Research*, 61(4), 810-823.
- Loomis, J. B. (1987). Balancing public trust resources of Mono Lake and Los Angeles' water right: An economic approach. *WATER RESOURCES RESEARCH*, 23(8), 1449-1456.
- Loucks, D. P., Stedinger, J. R., & Haith, D. A. (1981). *Water resource systems planning and analysis*. Englewood Cliffs, N.J: Prentice-Hall.
- Lund, J. R., & Guzman, J. (1999). Derived operating rules for reservoirs in series or in parallel. *Journal of Water Resources Planning and Management*, 125(3), 143-153.

- Lyons, K., Fraser, H., & Parmesano, H. (2000). An introduction to financial transmission rights. *The Electricity Journal*, 13(10), 31-37.
- Ma, X. W., Sun, D., & Cheung, K. (1999). Energy and reserve dispatch in a multi-zone electricity market. *IEEE Transactions on Power Systems*, 14(3), 913-919.
- Mankiw, N. G., & Taylor, M. P. (2006). *Microeconomics*. : Cengage Learning EMEA.
- Manning, R., & Gallagher, D. (1982). Optimal water pricing and storage: The effect of discounting. *WATER RESOURCES RESEARCH*, 18(1), 65-70.
- Massé, P., & Boutteville, R. (1946). *Les réserves et la régulation de l'avenir dans la vie économique*: Hermann & Cie.
- Mastropietro, P., Barroso, L. A., & Batlle, C. (2015). Power transmission regulation in a liberalised context: an analysis of innovative solutions in South American markets. *Utilities Policy*, 33, 1-9.
- Mastropietro, P., Barroso, L. A. N., & Batlle, C. (2015). Power transmission regulation in a liberalised context: an analysis of innovative solutions in South American markets. *Utilities Policy*, 33, 1-9.
- Mastropietro, P., Batlle, C., Barroso, L. A. N., & Rodilla, P. (2014). Electricity auctions in South America: Towards convergence of system adequacy and RES-E support. *Renewable and Sustainable Energy Reviews*, 40, 375-385.
- McAfee, R. P., & McMillan, J. (1987). Auctions and bidding. *Journal of economic literature*, 699-738.
- McCabe, K. A., Rassenti, S. J., & Smith, V. L. (1989). Designing 'smart' computer-assisted markets: an experimental auction for gas networks. *European Journal of Political Economy*, 5(2), 259-283.
- McCabe, K. A., Rassenti, S. J., & Smith, V. L. (1991). Smart computer-assisted markets. *Science (New York, N.Y.)*, 254(5031), 534-538.
- McCarl, B. A., & Parandvash, G. H. (1988). Irrigation development versus hydroelectric generation: can interruptible irrigation play a role? *Western Journal of Agricultural Economics*, 267-276.
- Medellín-Azuara, J., Harou, J. J., Olivares, M. A., Madani, K., Lund, J. R., Howitt, R. E., Tanaka, S. K., Jenkins, M. W., & Zhu, T. (2008). Adaptability and adaptations of California's water supply system to dry climate warming. *Climatic Change*, 87(1), 75-90.
- Michelsen, A. M., Booker, J. F., & Person, P. (2000). Expectations in water-right prices. *International Journal of Water Resources Development*, 16(2), 209-219.

- Michelsen, A. M., & Young, R. A. (1993). Optioning agricultural water rights for urban water supplies during drought. *American Journal of Agricultural Economics*, 1010-1020.
- Micola, A. R., Banal-Estañol, A., & Bunn, D. W. (2008). Incentives and coordination in vertically related energy markets. *Journal of Economic Behavior & Organization*, 67(2), 381-393.
- Miller, K. A., & Hall, D. (1996). Water banking to manage supply variability. *Advances in the economics of environmental resources. Volume 1: Marginal cost rate design and wholesale water markets.*, 185-210.
- Miller, K. A., & Hall, D. C. (1996). Water banking to manage supply variability. *Advances in the economics of environmental resources. Volume 1: Marginal cost rate design and wholesale water markets.*, 185-210.
- Mobasheri, F., & Harboe, R. C. (1970). A two-stage optimization model for design of a multipurpose reservoir. *WATER RESOURCES RESEARCH*, 6(1), 22-31.
- Momoh, J. A. (2008). *Electric power system applications of optimization*: CRC Press.
- Moreno, R., Barroso, L. A. N., Rudnick, H., Mocarquer, S., & Bezerra, B. (2010). Auction approaches of long-term contracts to ensure generation investment in electricity markets: Lessons from the Brazilian and Chilean experiences. *ENERGY POLICY*, 38(10), 5758-5769.
- Moreno, R., Bezerra, B., Barroso, L. A. N., Mocarquer, S., & Rudnick, H. (2009). *Auctioning adequacy in South America through long-term contracts and options: from classic pay-as-bid to multi-item dynamic auctions*. Paper presented at the 2009 IEEE Power & Energy Society General Meeting.
- Mulvey, J. M., & Ruszczyński, A. (1995). A new scenario decomposition method for large-scale stochastic optimization. *Operations Research*, 43(3), 477-490.
- Murphy, F. H., & Rosenthal, E. C. (2006). Allocating the added value of energy policies. *The Energy Journal*, 143-156.
- Murphy, J. J., Dinar, A., Howitt, R. E., Rassenti, S. J., & Smith, V. L. (2000). The Design of "Smart" Water Market Institutions Using Laboratory Experiments. *Environmental and Resource Economics*, 17(4), 375-394.

- Murphy, J. J., Dinar, A., Howitt, R. E., Rassenti, S. J., Smith, V. L., & Weinberg, M. (2009). The design of water markets when instream flows have value. *Journal of environmental management*, 90(2), 1089-1096. doi: 10.1016/j.jenvman.2008.04.001
- Murphy, J. J., & Stranlund, J. K. (2003a). An experimental analysis of compliance behavior in emissions trading programs: some preliminary results.
- Murphy, J. J., & Stranlund, J. K. (2003b). A Laboratory Investigation of Compliance Behavior under Transferable Permits and Fixed Standards: Some Preliminary Results: Mimeo, University of Massachusetts-Amherst.
- Murray, D. M., & Yakowitz, S. J. (1979). Constrained differential dynamic programming and its application to multireservoir control. *WATER RESOURCES RESEARCH*, 15(5), 1017-1027.
- Nandalal, K. D. W., & Bogárdi, J. (2007). *Dynamic programming based operation of reservoirs: applicability and limits*. Cambridge: Cambridge University Press.
- Nemirovski, A., Juditsky, A., Lan, G., & Shapiro, A. (2009). Robust stochastic approximation approach to stochastic programming. *SIAM Journal on Optimization*, 19(4), 1574-1609.
- Newberry, D. (2012). The robustness of agent-based models of electricity wholesale markets. Available at [www.econ.cam.ac.uk/dae/repec/cam/pdf/cwpe1228.pdf](http://www.econ.cam.ac.uk/dae/repec/cam/pdf/cwpe1228.pdf).
- Newbery, D. (2002). Problems of liberalising the electricity industry. *European Economic Review*, 46(4), 919-927.
- O'Neill, R. P., Williard, M., Wilkins, B., & Pike, R. (1979). A mathematical programming model for allocation of natural gas. *Operations Research*, 27(5), 857-873.
- Olivares, M. A. (2008). *Optimal hydropower reservoir operation with environmental requirements*. UNIVERSITY OF CALIFORNIA DAVIS.
- Oren, S. S. (1997). Economic inefficiency of passive transmission rights in congested electricity systems with competitive generation. *The Energy Journal*, 63-83.
- Oren, S. S. (2005). Generation adequacy via call options obligations: safe passage to the promised land. *The Electricity Journal*, 18(9), 28-42.
- Oven-Thompson, K., Alercon, L., & Marks, D. H. (1982). Agricultural vs. hydropower tradeoffs in the operation of the High Aswan Dam. *WATER RESOURCES RESEARCH*, 18(6), 1605-1613.

- Pepper, W., & Lo, G. (1999). *Application of linear program to gas market and pipeline model in the state of Victoria, Australia*. Paper presented at the INFORMS conference, Cincinnati.
- Pepper, W., Ring, B., Read, E., & Starkey, S. (2012). Implementation of a scheduling and pricing model for natural gas *Handbook of Networks in Power Systems II* (pp. 3-35): Springer.
- Pepper, W., Ring, B. J., Read, E. G., & Starkey, S. R. (2012). Implementation of a scheduling and pricing model for natural gas *Handbook of Networks in Power Systems II* (pp. 3-35): Springer.
- Pereira, M., & Pinto, L. (1991). Multi-stage stochastic optimization applied to energy planning. *Mathematical Programming*, 52(1), 359-375.
- Pereira, M. V. F., & Pinto, L. (1985). Stochastic optimization of a multireservoir hydroelectric system: A decomposition approach. *WATER RESOURCES RESEARCH*, 21(6), 779-792.
- Pérez-Arriaga, I., Gómez, T., Olmos, L., & Rivier, M. (2011). *Transmission and distribution networks for a sustainable electricity supply*: Cheltenham, UK: Edward Edgar Publishing.
- Philbrick, C. R., & Kitanidis, P. K. (1998). Optimal conjunctive-use operations and plans. *WATER RESOURCES RESEARCH*, 34(5), 1307-1316.
- Philpott, A. B. (2013). *The competitive price of stored water*. Paper presented at the CMSS Summer Workshop, Auckland.
- Philpott, A. B., Ferris, M., & Wets, R. (2013). Equilibrium, uncertainty and risk in hydro-thermal electricity systems. *Mathematical Programming*, 1-31.
- Philpott, A. B., & Guan, Z. (2013). Models for estimating the performance of electricity markets with hydro-electric reservoir storage: Technical report, Electric Power Optimization Centre, University of Auckland.
- Philpott, A. B., & Pritchard, G. (2004). Financial transmission rights in convex pool markets. *Operations Research Letters*, 32(2), 109-113.
- Pinto, A. A., Raffensperger, J. F., Cochrane, T. A., & Read, E. G. (2012). Proposed smart market design for sediment discharge. *Journal of Water Resources Planning and Management*, 139(1), 96-108.
- Ponnambalam, K., & Adams, B. J. (1996). Stochastic Optimization of Multi Reservoir Systems Using a Heuristic Algorithm: Case Study From India. *WATER RESOURCES RESEARCH*, 32(3), 733-741.

- Porrua, F., Chabar, R., Thomé, L. M., Barroso, L. A. N., & Pereira, M. V. F. (2009). *Incorporating large-scale renewable to the transmission grid: Technical and regulatory issues*. Paper presented at the 2009 IEEE Power & Energy Society General Meeting.
- Prabodanie, R. A. R., Raffensperger, J., & Milke, M. W. (2010). A pollution offset system for trading non-point source water pollution permits. *Environmental and Resource Economics*, 45(4), 499-515.
- Prabodanie, R. A. R., & Raffensperger, J. F. (2007). *Cleaning the water: A smart market for nitrates*. Paper presented at the MODSIM 2007 Int. Congress on Modelling and Simulation, Modelling and Simulation Society, Australia and New Zealand.
- Pritchard, G., Philpott, A. B., & Neame, P. J. (2005). Hydroelectric reservoir optimization in a pool market. *Mathematical Programming*, 103(3), 445-461.
- Pritchard, G., Zakeri, G., & Philpott, A. B. (2010). A single-settlement, energy-only electric power market for unpredictable and intermittent participants. *Operations Research*, 58(4-part-2), 1210-1219.
- Qureshi, M. E., Shi, T., Qureshi, S. E., & Proctor, W. (2009). Removing barriers to facilitate efficient water markets in the Murray-Darling Basin of Australia. *Agricultural water management*, 96(11), 1641-1651.
- Raffensperger, J., Milke, M., & Read, G. (2009). A Deterministic Smart Market Model for Ground Water.
- Rassenti, S. J., Smith, V. L., & Wilson, B. (2002). Demand-side bidding will reduce the level and volatility of electricity prices. *INDEPENDENT REVIEW-OAKLAND-*, 6(3), 441-446.
- Read, E., & Sell, D. (1989). Pricing and operation of transmission services: Short run aspects. *Turner, A.,(ed.), 1989, Principles for pricing electricity transmission*.
- Read, E. G. (1979). Optimal operation of power systems.
- Read, E. G. (1982). *Approaches to stochastic reservoir modelling*: College of Business Administration, University of Tennessee.
- Read, E. G. (1989a). A dual approach to stochastic dynamic programming for reservoir release scheduling. *Dynamic Programming for Optimal Water Resources Systems Analysis (AO Esogbue, Ed.)*, 361-372.

- Read, E. G. (1989b). Dynamic Programming for Optimal Water Resources System Management, chap. A Dual Approach to Stochastic Dynamic Programming for Reservoir Release Scheduling: Prentice Hall NY.
- Read, E. G. (1997). Transmission pricing in New Zealand. *Utilities Policy*, 6(3), 227-235.
- Read, E. G. (2010). Co-Optimization of Energy and Ancillary Service Markets. In S. Rebennack, M. V. F. Pereira, P. M. Pardalos & N. A. Iliadis (Eds.), *Handbook of Power Systems II* (Vol. Vol I, pp. p.307-327). Dordrecht: Springer.
- Read, E.G. (2016). Notes on Revenue adequacy for Financial Storage Rights in a Water Market. Personal Communication.
- Read, E. G., & Boshier, J. F. (1986). Biases in stochastic reservoir scheduling models. *Hobbs BF, Hopenstal A, (1989). Is optimization optimistically biased*, 152-160.
- Read, E. G., & Chattopadhyay, D. (1999). *Electricity market models: Lessons from Australasian experience*. Paper presented at the The 34th Annual Conference of the Operational Research Society of New Zealand.
- Read, E. G., Culy, J. G., Halliburton, T. S., & Winter, N. L. (1987). A simulation model for long-term planning of the New Zealand power system. *Operational research. North Holland, New York*, 493-507.
- Read, E. G., & George, J. A. (1990). Dual dynamic programming for linear production/inventory systems. *Computers and Mathematics with Applications*, 19(11), 29-42. doi: 10.1016/0898-1221(90)90146-b
- Read, E. G., George, J. A., & McGregor, A. D. (1994). Dual dynamic programming with lagged variables. *ORSNZ Proceedings*, 148-153.
- Read, E. G., & Hindsberger, M. (2010). Constructive dual DP for reservoir optimization. *Handbook of Power Systems I*, 3-32.
- Read, E. G., & Jackson, P. R. (2013). Experience with FTRs and Related Concepts in Australia and New Zealand *Financial Transmission Rights* (pp. 305-332): Springer.
- Read, E. G., & Miller, R. (2011). *Possible FTR developments*. Paper presented at the EPOC winter workshop, Auckland.

- Read, E. G., Ring, B. J., Starkey, S. R., & Pepper, W. (2012). An LP Based Market Design for Natural Gas. *Handbook of Networks in Power Systems II*, 77-113.
- Reznicek, K., & Cheng, T. (1991). Stochastic modelling of reservoir operations. *EUROPEAN JOURNAL OF OPERATIONAL RESEARCH*, 50(3), 235-248.
- Richter, B. D., Baumgartner, J. V., Powell, J., & Braun, D. P. (1996). A method for assessing hydrologic alteration within ecosystems. *Conservation biology*, 1163-1174.
- Richter, C. W., & Sheblé, G. B. (1998). Genetic algorithm evolution of utility bidding strategies for the competitive marketplace. *IEEE Transactions on Power Systems*, 13(1), 256-261.
- Richter, C. W., Sheblé, G. B., & Ashlock, D. (1999). Comprehensive bidding strategies with genetic programming/finite state automata. *IEEE Transactions on Power Systems*, 14(4), 1207-1212.
- Richter Jr, C. W., Jayantilal, A., & Kumar, J. (2001). *Improving market participant strategies with FTR options*. Paper presented at the Power Industry Computer Applications, 2001. PICA 2001. Innovative Computing for Power-Electric Energy Meets the Market. 22nd IEEE Power Engineering Society International Conference on.
- Ringler, C., & Cai, X. (2006). Valuing fisheries and wetlands using integrated economic-hydrologic modeling—Mekong River Basin. *Journal of Water Resources Planning and Management*, 132(6), 480-487.
- Rockafellar, R. T., & Wets, R. (1976). Stochastic convex programming: basic duality. *Pacific Journal of Mathematics*, 62(1), 173-195.
- Rockafellar, R. T., & Wets, R. (1991). Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research*, 16(1), 119-147.
- Rogers, P., De Silva, R., & Bhatia, R. (2002). Water is an economic good: How to use prices to promote equity, efficiency, and sustainability. *Water Policy*, 4(1), 1-17.
- Rosegrant, M. W., & Binswanger, H. P. (1994). Markets in tradable water rights: potential for efficiency gains in developing country water resource allocation. *World development*, 22(11), 1613-1625.



- Rosegrant, M. W., Ringler, C., McKinney, D. C., Cai, X., Keller, A., & Donoso, G. (2000). Integrated economic-hydrologic water modeling at the basin scale: The Maipo River basin. *AGRICULTURAL ECONOMICS*, 24(1), 33-46.
- Rosellón, J., & Kristiansen, T. (2013). *Financial Transmission Rights*: Springer.
- Rudkevich, A. (1999). Supply function equilibrium in power markets: learning all the way. *TCA technical paper*, 1299, 1702.
- Rudkevich, A., Hausman, E., Tabors, R. D., Bagnall, J., & Kopel, C. (2005). *Loss hedging rights: a final piece in the LMP puzzle*. Paper presented at the System Sciences, 2005. HICSS'05. Proceedings of the 38th Annual Hawaii International Conference on.
- Rudnick, H., Barroso, L. A. N., Llaens, D., Watts, D., & Ferreira, R. (2012). Flexible connections: Solutions and challenges for the integration of renewables in south america. *IEEE Power and Energy Magazine*, 10(2), 24-36.
- Saad, M., & Turgeon, A. (1988). Application of principal component analysis to long-term reservoir management. *WATER RESOURCES RESEARCH*, 24(7), 907-912.
- Saguan, M., Keseric, N., Dessante, P., & Glachant, J. M. (2006). *Market power in power markets: Game theory vs. agent-based approach*. Paper presented at the 2006 IEEE/PES Transmission & Distribution Conference and Exposition: Latin America.
- Saliba, B., & Bush, D. B. (1987). *Water markets in theory and practice: market transfers, water values, and public policy*: Westview Press.
- Schleyer, R. G. (1996). Chilean water policy: the role of water rights, institutions and markets. *International Journal of Water Resources Development*, 12(1), 33-48.
- Schweppe, F. C., Tabors, R. D., Caraminis, M. C., & Bohn, R. E. (1988). Spot pricing of electricity.
- Scott, T. J. (1998). *Hydro reservoir management for an electricity market with long-term contracts*. (Dissertation/Thesis).
- Scott, T. J., & Read, E. G. (1996). Modelling hydro reservoir operation in a deregulated electricity market. *International Transactions in Operational Research*, 3(3), 243-253.
- Seifi, A., & Hipel, K. W. (2001). Interior-point method for reservoir operation with stochastic inflows. *Journal of Water Resources Planning and Management*, 127(1), 48-57.

- Sen, S., & Higle, J. L. (1999). An introductory tutorial on stochastic linear programming models. *Interfaces*, 29(2), 33-61.
- Seyam, I. M., Hoekstra, A. Y., & Savenije, H. H. G. (2002). Calculation methods to assess the value of upstream water flows and storage as a function of downstream benefits. *Physics and Chemistry of the Earth, Parts A/B/C*, 27(11), 977-982.
- Shafer, J. M., Labadie, J. W., & Jones, E. B. (1981). Analysis of firm water supply under complex institutional constraints. *Water Resources Bulletin*, 17(3).
- Shane, R. M., & Gilbert, K. C. (1982). TVA hydro scheduling model: practical aspects. *J. Water Resour. Plann. Manage. Div., Proc. Am. Soc. Civ. Eng.;(United States)*, 108.
- Shapiro, A. (2011). Analysis of stochastic dual dynamic programming method. *EUROPEAN JOURNAL OF OPERATIONAL RESEARCH*, 209(1), 63-72.
- Sioshansi, F. P., & Pfaffenberger, W. (2006). *Electricity market reform: an international perspective*: Elsevier.
- Solow, R. M. (1974). Intergenerational equity and exhaustible resources. *The review of economic studies*, 29-45.
- Starkey, S. R. (2014). *Water Allocation Under Uncertainty – Potential Gains from Optimisation and Market Mechanisms*. (Dissertation/Thesis), University of Canterbury. Management, Marketing and Entrepreneurship.
- Starkey, S. R., Dye, S., Read, E. G., & Read, R. A. (2012). *Stochastic vs. Deterministic Water Market Design: Some Experimental Results*. Paper presented at the 4th IEEE and Cigré International Workshop on Hydro Scheduling in Competitive Markets, Bergen, Norway.
- Starkey, S. R., Dye, S., Read, G., Raffensperger, J. F., & Ring, B. J. (2011). *Enhancing Urban Water Allocation: An Evolving Framework for Trading Stochastically*. Paper presented at the 19th Triennial IFORS Conference, Melbourne, Australia.
- Stedinger, J. R., Sule, B. F., & Loucks, D. P. (1984). Stochastic dynamic programming models for reservoir operation optimization. *Water Resour. Res.*, 20(11), 1499-1505. doi: 10.1029/WR020i011p01499

- Steeger, G., Barroso, L. A., & Rebennack, S. (2014). Optimal bidding strategies for hydro-electric producers: A literature survey. *IEEE Transactions on Power Systems*, 29(4), 1758-1766.
- Stoft, S. (2002). Power system economics. *Journal of Energy Literature*, 8, 94-99.
- Stoft, S., Belden, T., Goldman, C., & Pickle, S. (1998). Primer on electricity futures and other derivatives. *Ernest Orlando Lawrence Berkeley National Laboratory Report LBNL-41098*, Berkeley, CA, USA.
- Sun, J., & Tesfatsion, L. (2007). Dynamic testing of wholesale power market designs: An open-source agent-based framework. *Computational Economics*, 30(3), 291-327.
- Tejada-Guibert, J. A., Johnson, S. A., & Stedinger, J. R. (1993). Comparison of two approaches for implementing multireservoir operating policies derived using stochastic dynamic programming. *WATER RESOURCES RESEARCH*, 29(12), 3969-3980.
- Tejada-Guibert, J. A., Johnson, S. A., & Stedinger, J. R. (1995). The value of hydrologic information in stochastic dynamic programming models of a multireservoir system. *WATER RESOURCES RESEARCH*, 31(10), 2571-2579.
- Tellidou, A. C., & Bakirtzis, A. G. (2007). Agent-based analysis of capacity withholding and tacit collusion in electricity markets. *IEEE Transactions on Power Systems*, 22(4), 1735-1742.
- Thobani, M. (1997). Formal water markets: why, when, and how to introduce tradable water rights. *The World Bank Research Observer*, 12(2), 161-179.
- Tietenberg, T. (2003). The tradable-permits approach to protecting the commons: Lessons for climate change. *Oxford Review of Economic Policy*, 19(3), 400.
- Tilmant, A., Goor, Q., & Pinte, D. (2009). Agricultural-to-hydropower water transfers: sharing water and benefits in hydropower-irrigation systems. *Hydrology and Earth System Sciences*, 13(7), 1091-1101.
- Tilmant, A., & Kelman, R. (2007). A stochastic approach to analyze trade-offs and risks associated with large-scale water resources systems. *WATER RESOURCES RESEARCH*, 43(6).
- Tilmant, A., Pinte, D., & Goor, Q. (2008). Assessing marginal water values in multipurpose multireservoir systems via stochastic programming. *WATER RESOURCES RESEARCH*, 44(12).

- Tirole, J. (1988). *The theory of industrial organization*: MIT press.
- Tisdell, J. G. (2010). Acquiring water for environmental use in Australia: an analysis of policy options. *Water Resources Management*, 24(8), 1515-1530.
- Tisdell, J. G., & Harrison, S. R. (1992). Estimating an optimal distribution of water entitlements. *WATER RESOURCES RESEARCH*, 28(12), 3111-3117.
- Travers, D. L., & Kaye, R. J. (1998). Dynamic dispatch by constructive dynamic programming. *IEEE Transactions on Power Systems*, 13(1), 72-78. doi: 10.1109/59.651616
- Tsukamoto, Y., & Iyoda, E. (1996). Allocation of fixed transmission cost to wheeling transactions by cooperative game theory. *Power Systems, IEEE Transactions on*, 11(2), 620-629.
- Tsur, Y. (2009). On the economics of water allocation and pricing. *Annu. Rev. Resour. Econ.*, 1(1), 513-536.
- Tully, F. R., & Kaye, R. J. (1997). Unit Commitment In Competitive Electricity Markets Using Genetic Algorithms. *The transactions of the Institute of Electrical Engineers of Japan. B, A publication of Power and Energy Society*, 117(6), 815-821.
- Turgeon, A. (1981a). A decomposition method for the long-term scheduling of reservoirs in series. *WATER RESOURCES RESEARCH*, 17(6), 1565-1570.
- Turgeon, A. (1981b). Optimal short-term hydro scheduling from the principle of progressive optimality. *WATER RESOURCES RESEARCH*, 17(3), 481. doi: 10.1029/WR017i003p00481
- Tzoannos, J. (1977). An empirical study of peak-load pricing and investment policies for the domestic market of gas in Great Britain. *Applied Economics*, 9(2), 133-153.
- Unsihuay-Vila, C., Marangon-Lima, J. W., Perez-Arriaga, I. J., & Balestrassi, P. P. (2010). A model to long-term, multiarea, multistage, and integrated expansion planning of electricity and natural gas systems. *Power Systems, IEEE Transactions on*, 25(2), 1154-1168.
- Van Der Vlerk, M. H. (2005). On multiple simple recourse models. *Mathematical Methods of Operations Research*, 62(2), 225-242.
- Vaux, H. J., & Howitt, R. E. (1984). Managing water scarcity: an evaluation of interregional transfers. *WATER RESOURCES RESEARCH*, 20(7), 785-792.

- Velásquez, J., Restrepo, P. J., & Campo, R. (1999). Dual dynamic programming: A note on implementation. *WATER RESOURCES RESEARCH*, 35(7), 2269-2271.
- Ventosa, M., Baíllo, Á., Ramos, A., & Rivier, M. (2005). Electricity market modeling trends. *ENERGY POLICY*, 33(7), 897-913. doi: 10.1016/j.enpol.2003.10.013
- Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance*, 16(1), 8-37.
- Visudhiphan, P., & Ilic, M. D. (1999). *Dynamic games-based modeling of electricity markets*. Paper presented at the Power engineering society 1999 winter meeting, IEEE.
- Wallace, S. W., & Fleten, S. E. (2003). Stochastic programming models in energy. *Handbooks in operations research and management science*, 10, 637-677.
- Wang, J., Yuan, X., & Zhang, Y. (2004). Short-term scheduling of large-scale hydropower systems for energy maximization. *Journal of Water Resources Planning and Management*, 130(3), 198-205.
- Wang, L. Z., Fang, L. P., & Hipel, K. W. (2007). Mathematical programming approaches for modeling water rights allocation. *JOURNAL OF WATER RESOURCES PLANNING AND MANAGEMENT-ASCE*, 133(1), 50-59.
- Waterfind. (2013). *Waterfind Water Market Specialists*. Retrieved from <http://www.waterfind.com.au>
- Watkins, D. W., McKinney, D. C., Lasdon, L. S., Nielsen, S. S., & Martin, Q. W. (2000). A scenario-based stochastic programming model for water supplies from the highland lakes. *International Transactions in Operational Research*, 7(3), 211-230.
- Weidlich, A., & Veit, D. (2008). A critical survey of agent-based wholesale electricity market models. *Energy Economics*, 30(4), 1728-1759.
- Wheeler, S., Bjornlund, H., Zuo, A., & Shanahan, M. (2010). The changing profile of water traders in the Goulburn-Murray Irrigation District, Australia. *Agricultural water management*, 97(9), 1333-1343.
- Wolfram, C. D. (1999). Measuring duopoly power in the British electricity spot market. *AMERICAN ECONOMIC REVIEW*, 805-826.
- Wong, S., & Fuller, J. D. (2007). Pricing energy and reserves using stochastic optimization in an alternative electricity market. *IEEE Transactions on Power Systems*, 22(2), 631-638.

- Wu, F. F., Varaiya, P., Spiller, P., & Oren, S. S. (1996). Folk theorems on transmission access: Proofs and counterexamples. *Journal of Regulatory Economics*, 10(1), 5-23.
- Wurbs, R. A. (1993). Reservoir-system simulation and optimization models. *Journal of Water Resources Planning and Management*, 119(4), 455-472.
- Wystup, U. (2007). *FX options and structured products*: John Wiley & Sons.
- Xu, W., Xianfeng, H., Guohua, F., & Fei, K. (2011). Optimal Operation of Multi-Objective Hydropower Reservoir with Ecology Consideration. *Journal of Water Resource and Protection*, 2011.
- Yakowitz, S. (1982). Dynamic programming applications in water resources. *WATER RESOURCES RESEARCH*, 18(4), 673-696.
- Yang, M., & Read, E. G. (1999). A Constructive Dual DP for a Reservoir Model with Correlation. *WATER RESOURCES RESEARCH*, 35(No. 7).
- Yang, Y. (1991). Identification of an optimal operating policy for a multipurpose reservoir.
- Yeh, W. (1985). Reservoir Management and Operations Models'. *WATER RESOURCES RESEARCH*, 21(12), 1797-1818.
- Young, G. K. (1967). Finding reservoir operating rules. *Journal of the Hydraulics Division*, 93(6), 297-322.
- Young, M., McColl, J., & Fisher, T. (2007). Urban water pricing: How might an urban water trading scheme work? *The University of Adelaide: Droplet*(5).
- Young, R. A., & Loomis, J. B. (2014). *Determining the economic value of water: concepts and methods*: Routledge.
- Zaman, A. M., Malano, H. M., & Davidson, B. (2009). An integrated water trading–allocation model, applied to a water market in Australia. *Agricultural water management*, 96(1), 149-159.
- Zheng, Q. P., Rebennack, S., Iliadis, N. A., & Pardalos, P. M. (2010). Optimization models in the natural gas industry *Handbook of Power Systems I* (pp. 121-148): Springer.
- Zingales, L. (1995). What determines the value of corporate votes? *The Quarterly Journal of Economics*, 1047-1073.

## Appendix V

Table A5.1. Weekly inflow patterns

	Number of weeks												
Hydrology state	1	2	3	4	5	6	7	8	9	10	11	12	13
Low	48	60	60	48	40	56	40	40	48	36	60	32	48
Medium	72	64	64	48	40	60	72	60	52	40	64	44	52
High	84	80	80	60	72	60	72	72	64	72	64	48	64
	Number of weeks												
Hydrology state	14	15	16	17	18	19	20	21	22	23	24	25	26
Low	40	32	32	28	28	20	30	36	36	12	16	24	12
Medium	44	44	36	40	32	24	40	36	36	16	16	28	16
High	48	48	40	48	40	40	48	40	40	20	32	32	20
	Number of weeks												
Hydrology state	27	28	29	30	31	32	33	34	35	36	37	38	39
Low	8	8	8	8	8	12	8	12	8	12	24	20	12
Medium	8	12	12	12	8	16	8	12	8	16	28	28	12
High	16	16	16	16	12	20	12	16	12	20	32	28	16
	Number of weeks												
Hydrology state	40	41	42	43	44	45	46	47	48	49	50	51	52
Low	8	16	20	32	44	56	20	36	44	36	60	68	64
Medium	12	20	24	48	48	64	24	40	48	40	72	72	72
High	16	32	36	56	56	72	32	40	52	64	76	88	84

## Appendix VI

Table A6.1. Marginal water values for Scenario-1 and Scenario-2

No. of weeks	1	2	3	4	5	6	7	8	9	10	11	12	13
Scenario-1	83	67	59	57	54	53	52	52	50	50	49	48	48
Scenario-2	65	62	50	49	44	43	42	41	40	40	39	39	39

No. of weeks	14	15	16	17	18	19	20	21	22	23	24	25	26
Scenario-1	47	47	47	47	46	46	46	46	45	45	44	43	43
Scenario-2	38	37	37	37	37	36	36	35	35	35	35	35	35

No. of weeks	27	28	29	30	31	32	33	34	35	36	37	38	39
Scenario-1	43	43	43	42	42	42	42	42	41	41	41	40	40
Scenario-2	34	34	34	34	34	34	34	33	33	33	33	33	33

No. of weeks	40	41	42	43	44	45	46	47	48	49	50	51	52
Scenario-1	40	40	39	39	39	38	38	36	36	36	36	35	35
Scenario-2	33	32	32	32	32	31	31	31	31	31	31	30	30